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Applications of neutrosophic rings whose elements are regular

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Abstract

In this paper, we develop some rings in algebra by studying their neutrosophic property. We have also studied several relationships between these rings using the concept of neutrosophic. The neutrosophic center of neutrosophic regular ring is also neutrosophic regular ring. Neutrosophic subring of neutrosophic regular ring is also neutrosophic regular ring.

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1. Introduction

As a mathematics system, we consider a regular concept is very important in Abstract Algebra. It is known that every ring need two binary operations. It is also known that there are several types of the rings, including the regular ring, which is the subject of our study in this paper A definition of regular ring in [1, 2]. We may choose [2] for some applications of regular ring with several graphs. The authors [3, 9, 12], studied one type of regular structure namely regular semigroup with some properties and some result of group. In 1980 the author smarndache presented neutrosophic theory. The basic principle of neutrosophic

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concept in logic where this logic approach to three cases namely truth (T), indeterminacy (I) and falsity (F). As a deal with indeterminacy, we refer to fuzzy set [14]. Fuzzy set with more information in [4, 10]. Neutrosophic semigroup and neutrosophic rings [5]. Neutrosophic multiplication module in [6]. Note that neutrosophic of some algebra like BCK-algebra and Ω-algebra in [6]. Hasan and Mohammad presented new results neutrosophic set in [7]. Vildan, Banu and Halis, studied neutrosophic submodules of a modules [8]. Some facts of submoduls and extending module in [9, 15]. More informations of neutrosophic concept in [10]. In [12] Neutrosophic sets and its applications to decision making. To show more about the rings can see [13, 15, 16, 17]. In this paper we present new results of neutrosophic regular ring with several applications about the topic.

2. Preliminaries

In this part, we review the most important basic tools that are directly related to our study and that we use later in the paper.

Definition 2.1: [13]. We say $(G,*)$ is neutrosophic group if $(< G \cup I >) = \{\gamma + \beta I, \gamma, \beta \ \in G\}$, $(G \cup I)$ refere to neutrosophic group.

Remark 2.2: [11]. Let $(< G \cup I >)$ be a group. So $(G \cup I) \supseteq (G,*)$ and $(G \cup I)$ is not a group.

Definition 2.3: [13] We say $\varrho(I)$ is neutrosophic ring and $(< \varrho(I), >) = \{\gamma + \beta I, \gamma, \beta \ \in \varrho\}$.

Remark 2.4: If $\varrho(I)$ is a ring, so ϱ is a ring.

Theorem 2.5: [13]. *If $\varrho(I)$, is a ring so ϱ is also ring with commute property.*

Definition 2.6: [12]. A set $\mathcal{B}$ has neutrosophic on the universe X and define by $\mathcal{B} = \{< \gamma, t_{\mathcal{B}}(\gamma), I_{\mathcal{B}}(\gamma), f_{\mathcal{B}}(\gamma) >, \gamma \ \in X \}$.

Remark 2.7: Let Å, $\mathcal{B}$ are two sets on X. So Å $\subseteq \mathcal{B}$ iff $Å(\gamma) \leq \mathcal{B}(\gamma)$. Note that $t_{Å}(\gamma) \leq \ t_{\mathcal{B}}(\gamma), i_{Å}(\gamma) \leq i_{\mathcal{B}}(\gamma)$ and $t_{Å}(\gamma) \geq \ f_{\mathcal{B}}(\gamma)$.

Definition 2.8: [7]. Let Å and $\mathcal{B}$ are two sets on X and has indeterminacy property. So, $C = Å \cup \mathcal{B}$ known as $C(\gamma) = Å(\gamma) \vee \mathcal{B}(\gamma) \ \in Å(\gamma) \vee \mathcal{B}(\gamma) = (t_{Å}(\gamma) \vee \ t_{\mathcal{B}}(\gamma) \ , \ i_{Å}(\gamma) \vee i_{\mathcal{B}}(\gamma), f_{Å}(\gamma) \wedge f_{\mathcal{B}}(\gamma) \ )$.

3. The Main Results

During this main result about regular ring will appear and their properties.

Definition 3.1: We say $\emptyset \neq Å$ is a neutrosophic set which generated by Å, I denoted by $(Å(I))$, where I denoted to indeterminacy.

Remark 3.2: Let Å be a collection of all points(elements) as a. A neutrosophic set Å is $T_{Å}$, $I_{Å}$ and $F_{Å}$ where $T_{Å}$ is the truth, $I_{Å}$ is the indeterminacy and $F_{Å}$ is the falsity.

Note: $T_{Å}(a)$, $I_{Å}(a)$ and $F_{Å}$ are subset of [0,1]: $Å = \{< \gamma, (T_{Å}(\gamma), Å(\gamma), F_{Å}(\gamma) > ; \gamma \in U; U \text{ is universal set}\}$

Definition 3.3: Neutrosophic element means s and I are generated this concept and denoted by sI in general.

Theorem 3.4: *Let $\varrho(I)$ be a netrosophic ring. If $\varrho_D(I)$ is a neutrosophic division ring $\varrho(I)$ is neutrosophic regular.*

Proof: If an element $0 \neq sI$ is neutrosophic of $\varrho(I)$, so $\varrho_D(I)$is a division ring. Hence each neutrosophic element aI has multiplicative inverse. So $I \in \varrho_D(I) \ni (\gamma I)(\gamma^{-1}I) = (\gamma^{-1}I)(\gamma I) = eI$, where eI is multiplicative identity. Hence $(\gamma I)(\gamma^{-1}I)(\gamma I) = \gamma I((\gamma I)((\gamma^{-1}I)) = (\gamma I)(eI) = (\gamma I)$.Therefore γI is regular neutrosophic element of $\varrho(I)$. Thus $\varrho(I)$ is regular.

Proposition 3.5: Let ϱ(I) be a Boolean. So, ϱ(I) is a neutrosophic regular.

Proof: If $\varrho(I)$ is a neutrosophic Boolean ring with $aI \in \varrho(I)$. But from definition a neutrosophic Boolean ring, we abtain $(\omega I)(\gamma I) = \gamma(I)$. Hence $(\omega I)(\omega I)(\omega I) = (\omega I)(\gamma I) = (\gamma\omega)(\omega I)$. Therefore every element of $\varrho(I)$ is regular. Hence $\varrho(I)$ is neutrosophic regular.

In the next proposition, there is a clear relationship between neutrosophic property of $\varrho(I)$ and commutative property.

Proposition 3.6: Let $\varrho_n(I)$ neutrosophic regular ring. If $\varrho_n(I)$ has identity with(.), then $\varrho(I)$ is a commutative ring.

Proof: If $0 \neq sI$ is element of $\varrho(I)$. Since $\varrho(I)$ is regular, $(\sigma I)(\delta I)(\sigma I) = \sigma I$; $(\delta I) \in \varrho(I)$and $(\delta I) = \sigma^{-1}$. Note that $(\sigma I\delta I)(\sigma I) = \sigma I(\delta I\sigma I)$, because $\varrho(I)$is neutrosophic ring, so $\sigma I\delta I = \delta I\sigma I = eI$ where eI is neutrosophic multiplicative identity is a unique. Hence $\sigma I\delta I = \delta I\sigma I$. Thus $\varrho(I)$ is commutative neutrosophic ring.

Remark 3.7: In the next theorem, we study neutrosophic multiplicative inverse concept with neutrosophic regular ring in order to prove that the center of neutrosophic regular ring is also neutrosophic regular ring.

Theorem 3.8: *For $\varrho(I)$ is a regular ring, if xI element in $\varrho(I)$ has neutrosophic inverse, then the center (C($\varrho(I)$)) of of neutrosophic regular ring is also neutrosophic regular*

Proof: If that $C(\varrho(I))$ represent the center of neutrosophic regular ring $\varrho(I)$, so $C(\varrho(I)) \neq \emptyset$. $C(\varrho(I)) \subseteq \varrho(I)$ Now we need to show that $C(\varrho(I))$ is neutrosophic regular ring. To satisfy our goal, we must prove that $C(\varrho(I))$commutative neutrosophic group.:

Closure: Let $\omega I, \varepsilon I \in C(\varrho(I))$. So the commute is holds: $\omega IrI = rI\omega I, \varepsilon IrI = rI\varepsilon I$; $rI \in \varrho(I)$. Also, $\omega IrI + \varepsilon IrI = (\omega I + \varepsilon I)rI = rI$.

Hence, $rI\omega I = rI(\omega I + \omega I)$. But $\omega IrI = rI\omega I$. Then, $\varepsilon IrI = rI\varepsilon I$.

Therefore, $\omega IrI + \varepsilon IrI = (rI\omega I + rI\varepsilon I)$. So, $(\omega I + \varepsilon I)rI = rI(\omega I + \varepsilon I$.

So, $\omega I + \varepsilon I \in C(\varrho(I))$. So, (+) in C($\varrho(I)$)is a neutosophic binary operation.

(ii) Associative: Suppose that $\omega I, \varepsilon I, \rho I \in C(\varrho(I))$. Also $(\omega I + \varepsilon I) + \rho I, \omega I + (\varepsilon I + \rho I) \in C(\varrho(I))$. But $\omega I, \varepsilon I, \rho I \in C(\varrho(I))$. So $\omega IrI = rI\omega I, \varepsilon IrI = rI\varepsilon I$ and $\rho IrI = rI\rho I$.

Hence

$$\omega IrI + \varepsilon IrI + \rho IrI = (\omega I + \varepsilon I)rI + \rho IrI = \big((\omega I + \varepsilon I) + \rho I\big)rI \ldots (1).$$

$$rI\omega I = rI\varepsilon I + rI\rho I = rI(\omega I + \varepsilon I) + rI\rho I = rI\big((\omega I + \varepsilon I) + \rho I\big) \ldots (2).$$

$$\omega IrI + \varepsilon IrI + \rho IrI = \omega IrI + (\varepsilon I + \rho I)rI = (\omega I + ((\varepsilon I + \rho I)rI \ \ldots (3).$$

$$rI\omega I + rI\varepsilon I + rI\rho I = rI\omega I + rI(\varepsilon I + \rho I) = rI\ (\omega I + ((\varepsilon I + \rho I) \ldots (4).$$

Since $\omega I\delta I = \delta I\omega I \ and \ \rho I\delta I = \delta I\rho I \ and \ \varepsilon I\delta I = \delta I\varepsilon I, so \ \omega I\delta I + \varepsilon I\delta I + \rho I\delta I = \delta I\omega I + \delta I\varepsilon I + \delta I\varepsilon I + \delta IcI$. From (1) and (2); $((\omega I + \varepsilon I) + \rho I)\delta I = \delta I((\omega I + \varepsilon I) + \rho I)$. From (2) and (4); $\omega I + (\varepsilon I + \rho I))rI = \delta I((\omega I + (\varepsilon I + \rho I))$. Therefore, $(\omega I + \varepsilon I) + \rho I \in C(\varrho(I))$, and $\omega I + (\varepsilon I + \rho I) \in C(\varrho(I))$. Hence $\omega I\delta I + \varepsilon I\delta I + \rho I\delta I = \delta I\omega I + \delta I\varepsilon I + \delta I\rho I$. Thus $((\omega I + \varepsilon I) + \rho I))\delta I = \delta I(\omega I + (\varepsilon I + \rho I))$, with $(\omega I + \varepsilon I) + \rho I \in C(\varrho(I))$ and $\omega I + (\varepsilon I + \rho I) \in C(\varrho(I))$. Hence $(\omega I + \varepsilon I) + \rho I = \omega I + (\varepsilon I + \rho I)$.

(iii) To prove that $0I$ is a identity element in $C(\varrho(I))$, we must show that $0I \in C(\varrho(I))$. Suppose that: $0I = (rI)(0I)$, $\forall\ (rI) \in \varrho(I); 0I \in C(\varrho(I))$. Since $0I \in C(\varrho(I))$ with $C(\varrho(I) \subseteq \varrho(I)$, then + is commutative. Hence $\sigma I + 0I$ and $0I + \sigma I \in C(\varrho(I))$.

Now to prove that $0I$ is identity element in $C(\varrho(I))$. Suppose that $\sigma I \in C(\varrho(I))$, so $(\sigma I)(\delta I) = (\delta I)(\sigma I), \forall\ \delta I \in \varrho(I)$. Hence

$$(\sigma I)(\delta I) + (0I)(\delta I) = \big((\sigma I) + (0I)\big) = ((0I) + (\sigma I))(\delta I).$$

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