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Radial radio mean labelling of Grotzsch graph and some graphs

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Abstract

A graph G is undirected with V vertices and E edges. This study introduces radial radio mean labelling of graphs, a novel method of graph labelling. A radial radio mean labelling of the undirected graph G is a function $g : V(G) \to \{1, 2, \ldots, n\}$ that satisfies the requirement $g(u) + g(v)/2$ between any two nodes u, v belongs to G and r(G) is the radius of G. The greatest integer in the range of a radial radio mean labelling g is the span, and it is represented by span (g). The minimal span over all G radial radio mean labelling is known as

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the radial radio mean number rrmn(G). The radial radio mean number of the Grotzsch graph and some graphs have been analyzed in this paper.

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Keywords: *Radial radio labelling, Grotzsch graph, Radial radio mean labelling, Triangular ladder graph, Eccentricity, Radius Diameter.*

1. Introduction

"The notation of graph labelling was first introduced by Rosa [1] in 1960, and many different graph labellings have been defined and studied since then. Graph labelling is the process of mapping a collection of integers, known as labels, to graph elements, which commonly represent vertices, edges, or both, referring to an excellent dynamic survey by Gallian[2]. In the nineteenth century, the term graph labelling was used to study the strait consignment problematic work, with receivers serving as the nodes of the grid. If two vertices are sufficiently close to each other, they are said to be adjacent. The issue of allocating frequencies to radio transmitters is known as the frequency assignment problem, first discussed in 1980 by William Hale [3]. In graph theory, graph labelling refers to the process of assigning integers to vertices, edges, or both based on a condition [4]. This difficulty in assigning frequencies inspired Gaory Chartrand et al. [8] to develop a new type of graph labelling known as radio labelling. A radio labelling of the undirected graph G is a mapping from $g: V(G) \to \{1,2,...,n\}$ so that $d(u,v) + |g(u) - g(v)| \geq 1 + diam(G)$. This inspired Kathiresan.K.M and Vimalajenifer.S to develop the idea of labelling radial radio labels [11]. Several mathematicians developed different labelling techniques in response to problems in real life. A radial radio labelling g of the undirected graph G is a mapping from $g: V(G) \to \{1,2,...,n\}$ sustaining the circumstance, $d(u,v) + |g(u) - g(v)| \geq 1 + r(G)$, where $r(G)$ represents the radius of the graph G and $u,v \in V(G)$. Radial radio condition illustrates this condition. For a radial radio labels, the span of g is the greatest numeral in the range of g. The radial radio number $rr(G)$, is the least width reserved across every radial radio labels of G. That is, the least value. A radial radio mean labelling of the undirected graph G is a function $g : V(G) \to \{1, 2, ..., n\}$ meeting the criteria $d(u,v) + \frac{g(u)+g(v)}{2} \geq 1 + r(G)$, where $r(G)$ denotes the radius of the graph G and *distance of* (u, v) is the distance amongst the vertices. The minimal span added to all radial radio mean labellings of graph G is the span of a radial radio mean labelling 'g'. Additionally, radial radio mean graceful labelling as well as radial radio mean labelling, $V(G) = rrmn(G)$. Here referred a few labelling techniques. [11, 12,13,14]. Mongolian tent and diamond graphs were labelled using radial radio means by Arputha Jose et al. [15]. Motivated by this work, in this paper, radial radio labelling of the Grotzsch graph, Star graph, Triangular ladder graph, and a graph Fl_n determined.

2. Preliminaries

Definition 1: Let v denote the node of G and Let G denote a connected graph. The eccentricity e(v) of v is the longest vertex from u. Thus $e(v) = \max d(u,v)\ \forall u,\ v \in V(G)$ [6].

Definition 2: For a graph G, radius is the least eccentricity of the nodes of G. It is represented by rad(G) [6].

Definition 3: For a graph G, diameter is the extreme eccentricity of the nodes of G. It is represented by dia(G) [7].

Definition 4: For a connected graph G = (V, E), radial radio label is an consignment of positive numerals to the nodes sustaining the rule d(u, v) + | f (u) – f (v) | ≥ 1 + rad(G), for any two dissimilar vertices u, v belongs to vertex set of G, where rad(G) represent the radius of the graph G.

Definition 5: A radial radio mean labelling of G is a function $g : V(G) \to \{1,2,\ldots,n\}$ satisfying the condition $d(u,v) + \left|\frac{g(u)+g(v)}{2}\right| \geq 1 + r(G)$ So that distance of (u, v) represents the distance amongst the nodes u, v and r(G) represents the radius of the graph G."

Definition 6: The Grotzsch graph is a triangular free graph with 11 vertices and 20 edges.

Definition 7: To obtain the triangular ladder tent graph, add a new vertex x above the triangular ladder graph it to each vertex $u_{1,i}$, $1 \leq i \leq 2n$.

Theorem 1: *The radial radio mean number of complete graphs K_n, $rrmn(K_n) = n$.*

Theorem 2: *The radial radio mean number of complete bipartite graphs $K_{m,n}$, $rrmn(K_{m,n}) = m + n$."*

3. Main Results

Theorem 3: *"The radial radio mean labelling of the graph star is n, where n ≥ 2.*

Proof: Let $x, v_1, v_2, ..., v_n$ be the vertices with n edges of the star edges of the star graph as shown in Figure 1. Consider the center vertex as x. The vertices of the star graph are denoted by $f(k) = k + 1, 1 \le k \le n$.......(1). Assume u and v as any two nodes of the star graph.

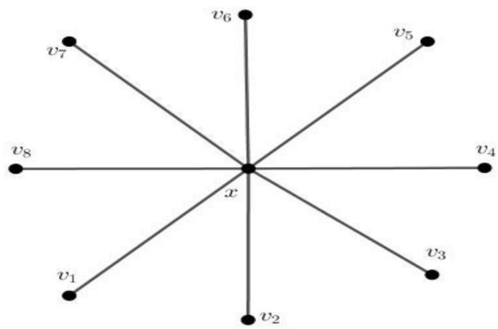

Figure 1

Illustration of Star Graph

Case (i): Suppose the vertex u and the vertex v are adjacent then $d(u, v) = 1$. The vertices $u = x$ and $v = v_{k+1}$, $1 \le k \le n$

Therefore $\left|\frac{f(u)+f(v)}{2}\right| = \left|\frac{1+k+1}{2}\right| = \left|\frac{k+2}{2}\right|$ and $d(u,v) + \left|\frac{f(u)+f(v)}{2}\right| \ge 2$

Case (ii): Suppose the vertex u and the vertex v are non-adjacent then $d(u, v) > 1$ the vertices $u = v_k$ and $v = v_{k+1}$

$$\left|\frac{f(u)+f(v)}{2}\right| = \left|\frac{k+1+k+2}{2}\right| = \left|\frac{2k+3}{2}\right| \text{ and } d(u,v) + \left|\frac{f(u)+f(v)}{2}\right| \ge 2$$

Therefore, the star graph admits radial radio mean labelling.

$$rrmnS(V_n) = n + 1.$$

Theorem 4: *The radial radio mean labelling of Grotzsch graph is at most 2n+1.*

Proof: Let $v_1, v_2, ..., v_n$ be the vertices with $2n + 1$ vertices and $4n$ edges of Grotzsch graph as shown in Figure 2. The vertices of Grotzsch graph G are labeled by the vertex labelling $f(v_i) = i + 1, 1 \le i \le n$.........(2)

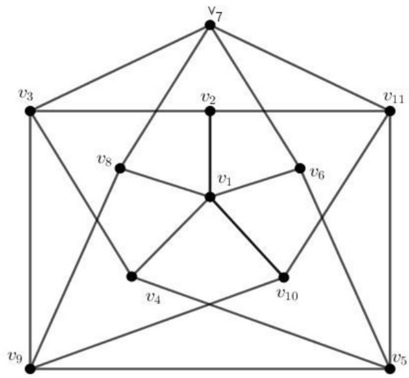

Figure 2

Illustration of Grotzsch Graph

Claim: The vertex labelling function admits radial radio mean labelling. Let the vertex u and the vertex v be any two vertices of the Grotzsch graph G

Case (i) Suppose the vertex u and the vertex v are adjacent then $d(u, v) = 1$. Therefore the vertices u and v are in the form $u = v_k$ and $u = v_{k+1}$, $1 \le k \le 11$. and By the mapping $f(v_k)=k+1$ and $f(v_{k+1}) = k+2$, $1 \le k \le 11$ and $\left|\frac{f(u)+f(v)}{2}\right| = \left|\frac{k+1+k+2}{2}\right| = \left|\frac{2k+3}{2}\right|$ therefore $d(u,v) + \left|\frac{f(u)+f(v)}{2}\right| \ge 3$ is obtained.

Case (ii) If the vertex u and the vertex v are non-adjacent then $d(u, v)$ is more than 1. The nodes u and v are in the form of $u = v_k$ and $u = v_{k+a}$, $a \ne 1,\ 1 \le k \le 11,\ 1 < a \le 11$. By mapping

$$\left|\frac{f(u)+f(v)}{2}\right| = \left|\frac{k+1+k+a+1}{2}\right| = \left|\frac{2k+2+a}{2}\right|, a \ne 1, 1 \le k \le 11, 1 < a \le 11,$$

therefore $d(u,v) + \left|\frac{f(u)+f(v)}{2}\right| \ge 3$

In both cases, it can be seen that $d(u,v) + \left|\frac{f(u)+f(v)}{2}\right| \ge 1 + r(G)$

Therefore the vertex labelling 2 admits radial radio mean labelling.

Theorem 5: *The graph Fl_n admits a radial radio labelling.*

Proof: Let $v_1, v_2, \ldots, v_{2n+1}$ be the vertices of the graph Fl_n with $2n + 1$ vertices and $4n$ edges with $2n+1$ vertices and 4n edges as shown in Figure 3. Label the interior vertex v_1 be 1. The remaining vertices are $v_2, v_3, \ldots, v_n$ labeled by $u_x = x$, where $x = 2, 3, \ldots, 2n + 1$

Claim: To prove the vertex labelling admits the radial radio mean labelling.

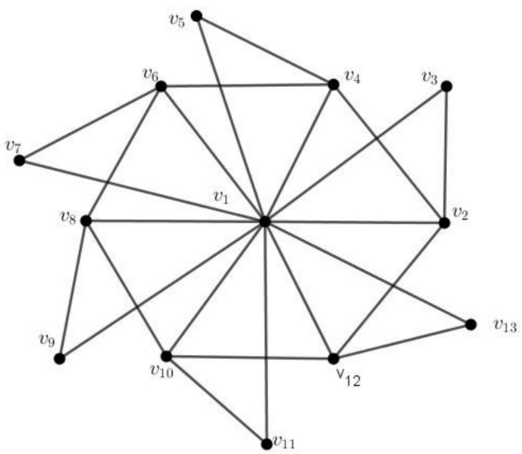

Figure 3

Illustration of Graph *Fln*

Case (i): Let u and v are adjacent vertices and distance of $(u, v) = 1$. The vertices u and v are in the form of $u = v_k$ and $v = v_{k+1}$. By the mapping $\left|\frac{f(u)+f(v)}{2}\right| = \left|\frac{k+k+1}{2}\right| = \left|\frac{2k+1}{2}\right|$. Thus $d(u,v) + \left|\frac{f(u)+f(v)}{2}\right| \geq 2$.

Case (ii): Let u and v are non-adjacent then $d(u,v) > 1$.

The nodes u and v are in the form of $u = v_k$ and $v = v_{k+a}$, $a > 1$.

By the mapping $\left|\frac{f(u)+f(v)}{2}\right| = \left|\frac{k+k+a}{2}\right| = \left|\frac{2k+a}{2}\right|$.

Therefore $d(u,v) + \left|\frac{f(u)+f(v)}{2}\right| \geq 2$. Therefore, graph Fl_n admits radial radio mean labelling in both the cases $d(u,v) + \left|\frac{f(u)+f(v)}{2}\right| \geq 1 + r(G)$.

Theorem 6: *The maximum radial radio mean number for the tent graph is 2n + 1.*

Proof: Let x, $u_{1,1}$, $u_{1,2}$, ..., $u_{1,2n}$, $u_{2,1}$, $u_{2,2}$, ..., $u_{2,2n}$ be the vertex set of the triangular tent graph as illustrated in Figure 4 and 5, respectevelly. Assign the top single vertex x as one. The vertices of the triangular tent graph's top row R_1 are identified by

$$f(u_{1,k}) = 2k,\ 1 \leq k \leq n \tag{3}$$

and the bottom row R_2 vertices labelled by $f(u_{2,k}) = 2k + 1$,

$$1 \leq k \leq n........ \tag{4}$$

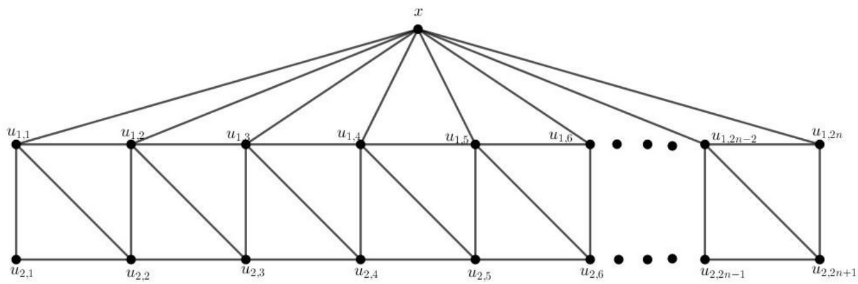

Figure 4

Triangular Ladder Tent Grap

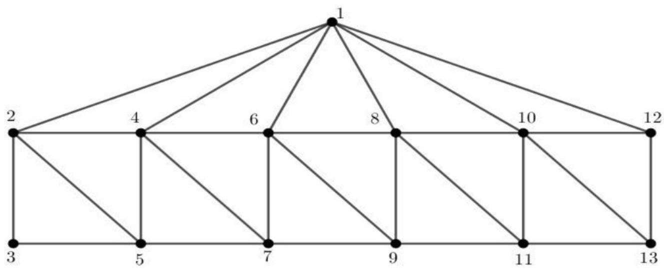

Figure 5

Triangular Ladder Tent Graph

Claim: The vertex labelling of the triangular ladder tent graph admits radial radio mean labeling.

Let u, v represent any two of the triangle ladder tent graph's vertices.

Case (i): Suppose the vertices $u, v \in R_1$ of the triangular ladder tent graph.

Case (i.i): Suppose u and v upholds adjacency , then d(u, v)=1. Let assign the vertices $u = u_{1,2k}$ and $v = u_{1,2k+2}$, $1 \le k \le n$. Therefore by equation 3 $\left|\frac{f(u)+f(v)}{2}\right| = \left|\frac{2k+2k+2}{2}\right| = \left|\frac{4k+2}{2}\right|$. Hence $d(u,v) + \left|\frac{f(u)+f(v)}{2}\right| \ge 3$.

Case (i.ii): Suppose u and v are non -adjacent vertices, then d(u, v)>1. Let assign the vertices $u = u_{1,2k}$ and $v = u_{1,2k+a}$, $a = 2k$, $1 \le k \le n$. Therefore by equation 3 $\left|\frac{f(u)+f(v)}{2}\right| = \left|\frac{2k+2k+a}{2}\right| = \left|\frac{4k+a}{2}\right|$.

Hence $d(u,v) + \left|\frac{f(u)+f(v)}{2}\right| \ge 3$.

Case (ii): Suppose the vertices $u, v \in R_2$ of the triangular ladder tent graph.

Case (ii.i): Suppose u and v upholds adjacency condition, then d(u, v)=1. Let assign the vertices $u = u_{2,2k+1}$ and $v = u_{2,2k+3}$, $1 \le k \le n$. Therefore by equation 4 $\left|\frac{f(u)+f(v)}{2}\right| = \left|\frac{2k+1+2k+3}{2}\right| = \left|\frac{4k+4}{2}\right|$. Hence $d(u,v) + \left|\frac{f(u)+f(v)}{2}\right| \ge 3$.

Case (ii.ii): Suppose u and v are non adjacent vertices, then d(u, v)>1. Let assign the vertices $u = u_{2,2k+1}$ and $v = u_{2,2k+1+a}$, $a = 2k$, $1 \le k \le n$. Therefore by equation 4 $\left|\frac{f(u)+f(v)}{2}\right| = \left|\frac{2k+2k+a}{2}\right| = \left|\frac{4k+a}{2}\right|$.

$$\text{Hence } d(u,v) + \left|\frac{f(u)+f(v)}{2}\right| \ge 3.$$

Case (iii): Suppose the vertex $u \in x$ and v in bottom row of the triangular ladder tent graph.

If v in bottom row R_2, non-adjacent therefore distance between u and v is greater than one. Let assign the vertices $u = x$ and $v = u_{2,2k+1}$, $1 \le k \le n$. Therefore by equation 4 $\left|\frac{f(u)+f(v)}{2}\right| = \left|\frac{x+2k+1}{2}\right| = \left|\frac{x+2k+1}{2}\right|$.

$$\text{Hence } d(u,v) + \left|\frac{f(u)+f(v)}{2}\right| \ge 3.$$

Case (iv): Suppose $u \in R_1$ and $v \in R_2$.

Case (iv.i): If u and v are holds adjacency relation between vertices, then d(u, v)=1. Let assign the vertices $u = u_{1,2k}$ and $v = u_{2,2k+1}$, $1 \le k \le n$. Therefore by equation 4 $\left|\frac{f(u)+f(v)}{2}\right| = \left|\frac{2k+2k+1}{2}\right| = \left|\frac{4k+1}{2}\right|$. Hence $d(u,v) + \left|\frac{f(u)+f(v)}{2}\right| \ge 3$.

Case (iv.ii): If u and v are non-adjacent vertices, then d(u, v)>1. Let assign the vertices $u = u_{1,2k+2}$ and $v = u_{2,2k+1}$, $1 \le k \le n$. Therefore by equation 3 and 4, $\left|\frac{f(u)+f(v)}{2}\right| = \left|\frac{2k+2+2k+1}{2}\right| = \left|\frac{4k+5}{2}\right|$.

Hence $d(u,v) + \left|\frac{f(u)+f(v)}{2}\right| \ge 3$. As a result, in each instance, it is evident that $d(u,v) + \left|\frac{f(u)+f(v)}{2}\right| \ge 1 + r(G)$.

By Radial radio mean labelling the vertex receives the maximum labelling in all cases and it is provided by $f(u_{2,2n+1}) = 2n+1$.

Therefore rrmn(TLTG(n))=2n+1."

4. Applications

Due to its numerous real-world applications, graph labelling is one of the most popular parameters. Radio labelling proved to be an efficient method of determining communication time for sensor networks. It has a wide range of applications for providing valuable mathematical models, including circuit design, communication networks, coding theory, demand and supply scenarios, radar, database management, electrical switchboards, addressing channel assignment processes, social networks, astronomy, X-ray crystallography, and data security. Perhaps we aren't aware that we use graph theory in our daily lives. A lot of the actions we do everyday use graph theory. Graphs are the foundation of many things, thoughts, concepts, actions, and procedures in daily life. We are aware of the interconnectedness of everything in our world. Roads, railroads, and networks connect cities. On the internet, links are used to join web pages together. The various components of a computer chip or an electric circuit are connected, etc. Graph theory may help engineers, scientists, and other professionals analyze, grasp, and optimize complex networks. Almost every branch of research and various aspects of daily life make use of graph theory. Graphs have a wide range of uses, including in criminology, medicine, and even the prevalent teenage problem of speculation. The transmission of diseases between countries or cities can potentially be forecast using graph theory. Graph theory is utilized in biology and medicine to distinguish pharmacological targets, ascertain the purpose of proteins, and ascertain the characteristics of ambiguous capacity. Graph theory is used in traffic light operation, specially in the turning of Green/ Red and the timing between them. As a result, drones reach each of the sub-areas considering vertex $v_1, v_2, \ldots, v_n$ of the city using Grotzsch graph and radial radio mean labelling technique within short span of time. The drone will have the option to fly up to 15 miles and reach the spot within 30 minutes up to 5 kgs. At present, it is using the one-to-one correspondence procedure to deliver necessary things. It reduces the cost and time of using this technique.

4.1 *Delivery of food using drones*

In some emergencies like tsunamis, floods, earthquakes, etc. the government can supply food or necessary things to the people using drones based on this technique to distribute the essentials. Consider the vertices $v_1, v_2, \ldots, v_n$ from the Grotzsch graph representing sub-areas of metropolitan cities like Chennai. The link between each vertex of the graph represents the distance between each of the sub-areas of the metropolitan city.

5. Conclusion

In this paper analyzed the radial radio labelling, radio number, and radio mean number of some graph families in this paper. This technique can easily expanded to other networks.

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