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Local antimagic vertex coloring of a Myceilski of graphs

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Abstract

This paper presents an investigation into the fundamental properties associated with the chromatic mean and variance within specific standard graphs, alongside the application of the Mycielski transformation to select graph structures. Furthermore, our study delves into the practical implications of local antimagic vertex coloring, emphasizing its significance in the fields of network security and anomaly detection.

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1. Introduction

In this paper, we exclusively deal with finite, simple, and undirected graphs. When we mention a graph $G = (V, E)$, we are referring to a finite, simple, undirected graph that does not contain any isolated edges (K_2 components). Hartsfield and Ringel were the first to introduce the concept of antimagic graph labeling in 1990, as referenced in their works [2, 3]. Consider a graph denoted as $G = (V, E)$, and let there be a bijection $\phi(i): E \rightarrow 1,2,\ldots,|E|$. For each vertex $u \in V(G)$, we define the weight $w(u) = \sum_{e\in E(G)} \phi(e)$, where E(u) denotes the set of edges connected to vertex u.

Consider a connected graph denoted as $G = (V, E)$, where $|V| = n$ and $|E| = m$. Within this context, a bijection $\phi: E \rightarrow 1,2,\ldots,m$ is designated as a "local antimagic labeling" if, for any pair of adjacent vertices u and v in the graph, their associated weights $w(u)$ and $w(v)$ must be distinct. c with $E(u)$ representing the set of edges incident to vertex u. It's important to emphasize that any local antimagic labeling naturally results in a valid vertex coloring of the graph. In this coloring, each vertex v is assigned a color equivalent to its weight, denoted as w(v).

The "local antimagic chromatic number," represented as $\chi_{la}(G)$, is defined as the minimum number of distinct colors required to properly color the vertices of the graph, considering all possible colorings derived from local antimagic labelings, as outlined in [1]. For additional information on the Local antimagic chromatic numbers of graphs, involved readers can explore related results in references such as [6, 7, 8, 9]. In their work [1], Arumugam et al. provided precise values for the local antimagic chromatic numbers of several specific graphs, including P_n (Path with n vertices), C_n (Cycle with n vertices), F_n, $K_{m,n}$ (Complete bipartite graph with m and n having the same remainder when divided by 2), $K_{2,n}$, W_n (Wheel graph with n being a multiple of 4.), and $L(n)$ (A graph formed by adding a vertex to every edge of the Star graph S_n). For further insights into graph labeling techniques, consult references[18, 19, 20, 21].

This paper not only explores graph-theoretical findings but also introduces fundamental statistical concepts such as mean and variance, providing values for these parameters across various standard graphs [10, 11]. The focus on local antimagic vertex coloring properties in Mycielskian graphs derived from specific standards is a key aspect. Bridging graph theory and statistics, the study defines a local antimagic vertex k-coloring denoted as $C = c_1, c_2, c_3, \ldots, c_k$ for graph G. The paper introduces a real-valued function ϕ(x) as the probability mass function for the count of vertices assigned a specific color. This function quantifies the likelihood of encountering vertices with a given color, providing insights into both graph properties and statistical analysis within the realm of graph theory [12, 13].

1.1 *Definition* [15] Let's consider a graph G with a certain proper k-coloring denoted by $C = \{c_1, c_2, c_3, \ldots, c_k\}$. For a particular color c_i chosen from C, let ϕ(x) represent the probability mass function (p.m.f) signifies the likelihood of assigning a particular color to vertices within graph G. (i) The coloring mean of this k-coloring C on graph G, denoted as $\mu_C G$ or simply $\mu(G)$, is defined as the expected value E(i) given by the sum of the product of each color index and its respective probability: $\mu_C G = E(i) = \sum_{i=1}^{k} i \ \phi(x)$ (ii) The coloring variance of C is denoted by $\sigma_C^2(G)$, representing the variability or spread of the colors assigned to the vertices in G. It is defined as the sum of squares of the differences between each color index and the coloring mean, weighted by their probabilities: $\sigma_C^2(G) = V(i) = \sum_{i=1}^{k} (i-\mu)^2 \phi(x)$. If the context is evident, we can use the terms "coloring mean" and "coloring variance" to describe these parameters for graph G in relation to coloring C.

This research paper focuses on investigating the local antimagic labeling properties of Mycielski graphs, aiming to unveil their behavior in the context of Mycielski graph transformations. Originating from Mycielski's work on identifying triangle-free graphs with substantial chromatic numbers, our exploration delves into the realm of local antimagic labeling applied to these graphs. This study is grounded in the transformation techniques introduced by Mycielski, documented in [5, 4], as we seek triangle-free graphs with elevated chromatic numbers [7]. Let's consider an initial graph denoted as G with a set of vertices $V(G) = \{v_1, v_2, v_3, \dots, v_n\}$. We implement the following steps to transform graph G : (i) Create a new set of vertices, denoted as U = $\{u_1, u_2, u_3, \dots, u_n\}$. Establish connections between each vertex u_i in set U and the vertices v_j if and only if the consistent vertex v_i in graph G is adjacent to vertex v_j. (ii)Additionally, a new vertex u is introduced, and edges are created to connect it to all vertices within set U. The resulting graph, derived from these procedures, is referred to as the 'Mycielski Graph' or 'Mycielskian of G', commonly represented as μ(G) and documented in [17]. To simplify notation, particularly concerning the powers of graphs, we represent the Mycielski graph of a graph G as "M(G)". In the previous work by N.K. Sudevh, extensive structural analyses of various powers of Mycielski graphs were conducted, along with a detailed examination of vertex adjacency patterns in these graphs, as documented in [16].

Furthermore, the study identified independent sets within various powers of the Mycielski graphs linked with paths and cycles. Building upon these prior investigations, our paper delves into several aspects. In Section 2, we explore the 'Chromatic Mean and Variance of certain standard graphs'. Section 3 is dedicated to the investigation of Local antimagic vertex coloring parameters within the Mycielskian graphs of specific Standard graph classes. In Section 4, we demonstrate the potential applications of Mycielski graphs and their local antimagic chromatic numbers in the realms of network security and anomaly detection.

2. The local antimagic chromatic mean and variance of some graphs

Lemma 2.1: *The local antimagic chromatic mean number of a path P_n is*

$$\mu_{la}(P_n) = \begin{cases} \frac{3n+2}{2n} & when\ n\ is\ even \\ \frac{3(n+1)}{2n} & when\ n\ is\ odd \end{cases}$$

and the local antimagic chromatic variance of a path P_n is

$$\sigma_{la}(P_n) = \begin{cases} \frac{2n^3+14n^2+n-10}{8n^3} & when\ n\ is\ even \\ \frac{n^3+8n^2-18n+9}{4n^3} & when\ n\ is\ odd \end{cases}$$

Proof: Consider the path P_n. It has n vertices. The Path Graph (P_n), we have $\chi_{la}(P_n) = 3$[1]. We have the following cases,

Case (i): When n is even, we have $\theta(c_1) = \frac{n}{2}$, $\theta(c_2) = \frac{n}{2} - 1$ and $\theta(c_3) = 1$.

Now, define the probability mass function,

$$\phi(x) = \begin{cases} \frac{n}{2n} & if \quad x = 1 \\ \frac{n-2}{2n} & if \quad x = 2 \\ \frac{1}{n} & if \quad x = 3 \end{cases}$$

Then, $\sum_{x=1}^{3} \phi(x) = \frac{n}{2n} + \frac{n-2}{2n} + \frac{1}{n} = 1$ and hence $\phi(x)$ is the p.m.f of P_n under the coloring concerned. Therefore, the local antimagic chromatic mean of P_n is given by $\mu\chi(P_n) = 1 \times \frac{n}{2n} + 2 \times \frac{n-2}{2n} + 3 \times \frac{1}{n} = \frac{3n+2}{2n}$. Also, the local antimagic chromatic variance of P_n is given by

$$\sigma_\chi^2(P_n) = \sum (i - \mu_\chi(P_n))^2 \phi(x)$$

$$= (1 - \frac{3n+2}{2n}). \frac{n}{2n} + (2 - \frac{3n+2}{2n}). \frac{n-2}{2n} + (3 - \frac{3n+2}{2n}). \frac{1}{n}$$

$$= \frac{2n^3+14n^2+n-10}{8n^3}$$

Lemma 2.2: The local antimagic chromatic mean of a Cycle C_n is

$$\mu_{la}(C_n) = \begin{cases} \frac{3(n+1)}{2n} & when\ n\ is\ even \\ \frac{3n-1}{n} & when\ n\ is\ odd \end{cases}$$

and the local antimagic chromatic variance of a Cycle C_n is

$$\sigma_{la}(C_n) = \begin{cases} \frac{3(n+1)}{2n} & when\ n\ is\ even \\ \frac{3n-1}{n} & when\ n\ is\ odd \end{cases}$$

Proof: The proof proceeds in a manner akin to the structure of Theorem 1.

Lemma 2.3: The local antimagic chromatic mean of the friendship graph F_n is

$$\mu_{la}(F_n) = \begin{cases} \frac{3(n+1)}{2n} & when\ n\ is\ even \\ \frac{3n-1}{n} & when\ n\ is\ odd \end{cases}$$

and the Local antimagic chromatic variance of the Friendship graph F_n is,

$$\sigma_{la}(F_n) = \begin{cases} \frac{3(n+1)}{2n} & when\ n\ is\ even \\ \frac{3n-1}{n} & when\ n\ is\ odd \end{cases}$$

Proof: Consider the friendship graph F_n. It has $2n + 1$ vertices. The friendship graph F_n, we have $\chi_{la}(F_n) = 3$[1]. We have $\theta(c_1) = n$, $\theta(c_2) = n$ and $\theta(c_3) = 1$.

Now, define the probability mass function,

$$\phi(x) = \begin{cases} \frac{n}{2n+1} & if \quad x = 1 \\ \frac{n}{2n+1} & if \quad x = 2 \\ \frac{1}{2n+1} & if \quad x = 3 \end{cases}$$

Then, $\sum_{i=1}^{3} \phi(x) = \frac{n}{2n+1} + \frac{n}{2n+1} + \frac{1}{2n+1} = 1$ and later $\phi(x)$ is the P.M.F of F_n under the coloring concerned. Therefore, the local antimagic chromatic mean of F_n is given by $\mu\chi(F_n) = 1 \times \frac{n}{2n+1} + 2 \times \frac{n}{2n+1} + 3 \times \frac{1}{2n+1} = \frac{3(n+1)}{2n+1}$.

Also, the local antimagic chromatic variance of F_n is given by

$$\sigma_\chi^2(P_n) = \sum (i - \mu_\chi(F_n))^2 \phi(x)$$

$$= (1 - \frac{3(n+1)}{2n+1})^2 . \frac{n}{2n+1} + (2 - \frac{3(n+1)}{2n+1})^2 . \frac{n}{2n+1} + (3 - \frac{3(n+1)}{2n+1})^2 . \frac{1}{2n+1}$$

$$= \frac{2n^3+11n^2+5n}{(2n+1)^3}$$

Lemma 2.4: *The mean of the local antimagic chromaticity for a Complete bipartite graph $K_{2,n}$ is*

$$\mu_{la}(K_{2,n}) = \begin{cases} \frac{3(n+1)}{2n} & when\ n\ is\ even \\ \frac{3n-1}{n} & when\ n\ is\ odd \end{cases}$$

and the local antimagic chromatic variance of a path $K_{2,n}$ is

$$\sigma_{la}(K_{2,n}) = \begin{cases} \frac{3(n+1)}{2n} & when\ n\ is\ even \\ \frac{3n-1}{n} & if\ when\ is\ odd \end{cases}$$

Proof: Consider the Complete bipartite graph $K_{2,n}$. It has $n+2$ vertices. The Complete bipartite graph $K_{2,n}$, we have $\chi_{la}(K_{2,n}) = 3$[1],

We have $\theta(c_2) = 1$, $\theta(c_1) = n$, and $\theta(c_3) = 1$.

Now, define the probability mass function,

$$\phi(x) = \begin{cases} \frac{n}{n+2} & if \quad x = 1 \\ \frac{1}{n+2} & if \quad x = 2 \\ \frac{1}{n+2} & if \quad x = 3 \end{cases}$$

Then, $\sum_{i=1}^{3} \phi(x) = \frac{n}{n+2} + \frac{1}{n+2} + \frac{1}{n+2} = 1$ and hence $\phi(x)$ is the p.m.f of $K_{2,n}$ under the coloring concerned. Therefore, the local antimagic chromatic mean of $K_{2,n}$ is given by $\mu\chi(K_{2,n}) = 1 \times \frac{n}{n+2} + 2 \times \frac{1}{n+2} + 3 \times \frac{1}{n+2} = \frac{n+5}{n+2}$.

Also, the local antimagic chromatic variance of $K_{2,n}$ is given by

$$\sigma_\chi^2(K_{2,n}) = \sum (i - \mu_\chi(K_{2,n}))^2 \phi(x)$$

$$= (1 - \frac{n+5}{n+2})^2 \frac{n}{n+2} + (2 - \frac{n+5}{n+2})^2 . \frac{1}{n+2} + (3 - \frac{n+5}{n+2})^2 . \frac{1}{n+2} = \frac{5n^2-11n+2}{n^3+6n^2+12n+8}$$

Consider a star graph S_n of order $n+1$, where v represents the central vertex and v_i, for $1 \le i \le n$, denote its leaves. Let's denote the resulting graph by $L(n)$, which is obtained by adding a vertex to each edge vv_i, where $1 \le i \le n-1$, of the star S_n.[1]

3. Local antimagic labeling of the Mycielski of graphs

Throughout this study, we define a few key sets and vertices: We use the set $v = v_1, v_2, v_3, \dots, v_n$ to represent the set of vertices in graph 'G' and its Mycielskian transformation (denoted as $M(G)$). We introduce a new set of independent vertices, denoted as $u = u_1, u_2, u_3, \dots, u_n$, within the Mycielskian graph $M(G)$. There is a specific vertex w that we consider, which is connected to the vertices in set u within the Mycielskian graph $M(G)$. In our first research outcome, we aim to determine the local antimagic chromatic parameters for Mycielskians of paths.

Lemma 3.1: *Mycielski graph of the Path Graph $M(P_n)$ admit local antimagic labeling.*

Proof: Consider the Mycielski graph of the path graph, denoted as $M(P_n)$. It's worth noting that for $n \ge 2$, the Mycielski graph of the path graph admits a local antimagic labeling. Observe the following pattern in the values of vertices and their corresponding local antimagic chromatic numbers:

Table 1

$\chi_{la}(M(P_n))$

$V(P_n)$	$V(M(P_n))$	$\chi_{la}(M(P_n))$
2	5	3
3	7	4
4	9	5
5	11	5
6	13	8
7	15	8
8	17	9
9	19	8
…	. Etc.,	Etc.,

For larger values of n, finding a general formula for $\chi_{la}(M(P_n))$ becomes challenging. Table 1 represents the values of $\chi_{la}(M(P_n))$.

Lemma 3.2: *For Myceilski of Cycle Graph $M(C_n)$, we have $\chi_{la}(M(C_n)) = n + 2$*

Proof: According to Theorem 2.1, the set of vertices u denotes the largest independent set in the Mycielskian graph of C_n. We designate a color $c_1 = n\ g(e) + (n+2)$ for each vertex within u, where $g(e)$ denotes the count of edges in the graph G. As previously mentioned, every vertex in v shares adjacency with at a minimum, one vertex in u, thereby preventing any vertices in v from being assigned the color c_1. To continue, let's examine the following functions. $M(C_n)$ can be shaded with n colors, say $c'_2 = 2g(e) + (n+1) + 3i \quad for \quad i = 2,3,4, \ldots n-1$, $c''_2 = 2(2n-i) + g(e) \quad for \quad i = 1$, $c'''_2 = 2(2n-i) + g(e) + (2(n-2)+1)$ for $i = n$, each color category includes individual vertices. Then all the colors of v_i is distinct. Given that w is connected to every vertex in u and w has no connections to vertex in set v, we assign the color $c_3 = \frac{n(n+1)}{2}$ to w. If $\theta(c_i)$ represents the size of the color class of c_i, then, $\theta(c_1) = n$, $\theta(c'_2) = n-2$, $\theta(c''_2) = 1$, $\theta(c'''_2) = 1$ and $\theta(c_3) = 1$. Hence, the edge labeling function corresponding to this is formulated as:

$\phi(v_i v_{i+1}) = (2f(v_1 v_2) + 1) + (2(i-2) - 1)$ for $i = 1,2,3, \ldots, n$

$\phi(u_i v_{i+1}) = i + n \quad for \quad i = 1,2, \ldots, n-1$; $\phi(u_i v_1) = i + n$ for $i = n$

$\phi(v_i u_{i+1}) = (E(G) - 2) - 2(i-1)$ for $i = 1,2, \ldots, n-1$; $\phi(v_i u_1) = g(e) \quad for \quad i = n$; $(v_i v_{i+1}) = 2(i-1) + 2n + 1 \quad for \quad i = 1,2, \ldots, n-1$

$\phi(x)(v_i v_1) = 2(i-1) + 2n + 1 \quad for \quad i = n$; $\phi(x)(wu_i) = i \quad \forall \quad i$

Case 1: We have $\theta(c_1) = n$, $\theta(c'_2) = n-2$, $\theta(c''_2) = 1$, $\theta(c'''_2) = 1$ and $\theta(c_3) = 1$. Now, define the probability mass function

$$\phi(x) = \begin{cases} \frac{n}{2n+1} & if \quad i = 1 \\ \frac{n-2}{2n+1} & if \quad i = 2 \\ \frac{1}{2n+1} & if \quad i = 3 \\ \frac{1}{2n+1} & if \quad i = 4 \\ \frac{1}{2n+1} & if \quad i = 5 \end{cases}$$

Then, $\sum_{i=1}^{5} \phi(x) = \frac{n}{2n+1} + \frac{n-2}{2n+1} + \frac{1}{2n+1} + \frac{1}{2n+1} + \frac{1}{2n+1} = 1$ and later $\phi(x)$ is the p.m.f of $M(C_n)$ under the coloring concerned. Therefore, the Local antimagic Chromatic mean of $M(C_n)$ is specified by $\mu\chi(M(C_n)) = 1 \times (\frac{n}{2n+1}) + 2 \times (\frac{n-2}{2n+1}) + 3 \times (\frac{1}{2n+1}) + 4 \times (\frac{1}{2n+1}) + 5 \times (\frac{1}{2n+1}) = \frac{3n+8}{2n+1}$. Also, a local antimagic Chromatic variance of $M(C_n)$ is,

$\sigma_{\chi}^{2}(M(C_n)) = \sum (i - \mu_{\chi}(M(C_n)))^2 \phi(x) = (1 - \frac{3n+8}{2n+1}). \frac{n}{2n+1} + (2 - \frac{3n+8}{2n+1}). \frac{n-2}{2n+1} + (3 - \frac{3n+8}{2n+1}). \frac{1}{2n+1} + (4 - \frac{3n+8}{2n+1}). \frac{1}{2n+1} + (5 - \frac{3n+8}{2n+1}). \frac{1}{2n+1} = \frac{2n^3+32n^2-71n+14}{8n^3+12n^2+4}$

Lemma 3.3: *For the Mycielski graph of the complete graph $M(k_n)$, where n is even, we have $\chi_{la}(M(k_n)) = n + 1$.*

Proof: Let G be the Mycielski graph obtained from the complete graph $M(k_n)$ with $2n + 1$ vertices and $3m + n$ edges. To show the existence of a local antimagic labeling of $M(k_n)$ using $n + 1$ colors. Consider the original graph with a vertex set $v_i = \{v_1, v_2, \dots, v_n\}$, and create a copy of $M(k_n)$ denoted by $u_i = \{u_1, u_2, \dots, u_n\}$, along with an additional vertex w. The degree of vertices in v_i is $2(n - 1)$, the degree of vertices in u_i is n, and the degree of w is n. Now label the edges of $M(k_n)$ using the following edge labeling function:

1. Split the edge set $G(e)$ into n sets: $S_1, S_2, \dots, S_{n+1}$.
2. The set $S_1 = \{1,2, \dots, n\}$ will be labeled on the incident edges in v_i. Since the degree of v_i is $2(n - 1)$, we label the elements in S_1 on the incident edges in v_i that are not adjacent to u_i and w.
3. The remaining edge sets $S_2, S_3, \dots, S_{n+1}$, each contains n elements. We label these sets on the incident edges in u_i. Specifically, label u_1 with elements $x_1, y_n, z_1, \dots, k_n$; label u_2 with elements $x_2, y_{n-1}, z_2, \dots, k_{n-1}$; and so on. Label u_n with elements $x_n, y_1, z_n, \dots, k_1$. Then, the weight of u_i, denoted by $w(u_i)$, is c_1, where $c_1 = x_1 + y_n + z_1 + \dots + k_n$. In other words, the sum of the labels assigned to the edges connected to vertices in u_i.
4. Some incident edges of w are the same as incident edges of u_i, so we label S_{n-1} to avoid conflicts. The weight of w, denoted by $w(w)$, is c_2.
5. Next, we consider vertex v_1. v_1 is not adjacent to u_1, so we label S_n to the incident edges in v_1. That is, we label the elements $k_2, k_3, \dots, k_n$ on the

incident edges in v_1. The weight of v_1, denoted by $w(v_1)$, is c_2, which is the same as the weight of w.

6. The remaining edges are labeled by incident edges of u_i. The remaining weights of $v_2, v_3, \dots, v_n$ are distinct and denoted by $C_1{}', C_2{}', \dots, C_{n-1}{}'$.
7. In this way, all edges are labeled, and we have $w(v_i) + w(u_i) + w(w) = n + 1$. Hence, the Local antimagic chromatic number of the Mycielski graph derived from the Complete graph equals $n + 1$. As previously stated in theorems, the node set U represents the main independent set within $M(k_n)$, allowing us to assign color $c_1$to all U vertices. Next, one node in set V along with the node in U can be colored using color c_2, while all remaining vertices in V must be colored distinctly, denoted as $c_3, c_4, \dots, c_{n+1}$. Now, by defining a function, we have: $(c_1) = n$, $\theta(c_2) = 2$, $\theta(c_3) = 1$, $\theta(c_4) = 1$. Now, define the probability mass function,

$$\phi(x) = \begin{cases} \frac{n}{2n+1} & if \quad i = 1 \\ \frac{2}{2n+1} & if \quad i = 2 \\ \frac{n-1}{2n+1} & if \quad i = 3,4,5,6 \dots n+1 \end{cases}$$

Here, $\sum_{x=1}^{n+1} \phi(x) = \frac{n}{2n+1} + \frac{2}{2n+1} + \frac{n-1}{2n+1} = 1$ Therefore, $\phi(x)$ represents the probability mass function of $M(K_n)$. Subsequently, the χ_{la}- Chromatic mean of $M(K_n)$ is expressed as follows:

$$\chi_{la}(M(K_n)) = 1 \times (\frac{n}{2n+1}) + 2 \times (\frac{2}{2n+1}) + 3 \times (\frac{n-1}{2n+1}) \quad = \frac{4n-5}{2n+1}$$

Additionally, the Local antimagic Chromatic variance of $M(K_n)$ is

$\sigma_\chi^2(M(K_n)) = \sum (x - \mu_\chi(M(K_n)))^2 \phi(x) = (1 - \frac{4n-5}{2n+1}). \frac{n}{2n+1} + (2 - \frac{4n-5}{2n+1}). \frac{2}{2n+1} + ((n-1) - \frac{4n-5}{2n+1}). \frac{1}{2n+1} = \frac{n+18}{(2n+1)^2}$

Lemma 3.4: For Myceilski of Star Graph S_n we have $\chi_{la}(S_n) = 4$

Proof: Let G be the Mycielski graph of the Star graph $M(S_n)$ with $2n + 1$ vertices and $3m + n$ edges. To demonstrate the existence of a local antimagic labeling for $M(k_{1,n})$ using 4 colors. Consider the original graph with a vertex set $v_i = \{v_1, v_2, \dots, v_n\}$, and create a copy of $M(k_{1,n})$ denoted by $u_i = \{u_1, u_2, \dots, u_m\}$, along with an additional vertex w. Define the edge labeling function, $\phi(v_i v_n) = i \ for \ i = 1,2,3 \dots, n-1$; $\phi(u_i w) = i + g(e) \quad for \quad i = 1,2, \dots, m$

where $g(e)$ is the number of edges in the Star graph.

$$\phi(u_i v_n) = [m(e) - (n-1)] - (i-1) \quad for \quad i = 1,2, \dots m-1$$

where $m(e)$ is the number of edges in mycielski of the star graph.

$$\phi(v_i u_n) = m(e) - (i-1) \quad for \quad i = 1,2, \dots, n-1$$

Define the vertex labeling function;

$$\phi(v_i) = m(e) + 1 \text{ for} i = 1,2,3 \ldots, n-1$$

$$\phi(u_i) = m(e) + 1 \quad for \quad i = 1,2,\ldots, m-1; \phi(w) = \frac{n(n+1)}{2} + ng(e),$$

$$\phi(v_i) = \frac{(n-1)n}{2} + (m-1)(m(e) - (n-1)) + \frac{(m-2)(m-1)}{2} \quad \text{for } i = n;$$

$$\phi(u_i) = \frac{(n-1)n}{2} + (m-1)\big(m(e) - (n-1)\big) + \frac{(m-2)(m-1)}{2} \quad \text{for} \quad i = n;$$

$\phi(u_i) = (n-1)(m(e)) - \frac{[(n-1)(n-1+1)]}{2} + (2g(e)+1)$. Then the Local antimagic chromatic number of Mycielski graph of the Star graph is 4. As previously stated in the theorems, the set of vertex u represents main independent set in S_n. Thus, We can allocate color c_1 to all vertices in u_i, where $i = 1, 2, 3, \ldots. n-1$ along with v_i, where $i = 1, 2, 3, \ldots., n-1$. Subsequently, the last vertex in v and u can colored using the color c_2 and c_3. Then the remaining vertex w coloured as c_4. Now, define a function we have $\theta(c_1) = \frac{2}{n-1}$, $\theta(c_2) = 1$, $\theta(c_3) = 1$ and $\theta(c_3) = 1$. Now, define the probability mass function

$$\phi(x) = \begin{cases} \frac{2(n-1)}{2(n+1)} & if \quad i = 1 \\ \frac{1}{2n+1} & if \quad i = 2 \\ \frac{1}{2n+1} & if \quad i = 3 \\ \frac{1}{2n+1} & if \quad i = 4 \end{cases}$$

Then, $\sum_{i=1}^{4} \phi(x) = \frac{2(n-1)}{2n+1} + \frac{1}{2n+1} + \frac{1}{2n+1} + \frac{1}{2n+1} = 1$ and thus, the p.m.f of $M(S_n)$ is established based on the provided coloring scheme. As a result, the local antimagic chromatic mean of $M(S_n)$ can be determined,

$\mu\chi\big(M(S_n)\big) = 1 \times \frac{2(n-1)}{2(n+1)} + 2 \times \frac{1}{2n+1} + 3\frac{1}{2n+1} + 4 \times \frac{1}{2n+1} = \frac{2n+7}{2n+1}$. Also, a local antimagic Chromatic variance of $M(S_n)$ is,

$$\sigma_\chi^2\big(M(S_n)\big) = \sum\ (i - \mu_\chi(M(S_n))^2\phi(x) = (1 - \frac{2n+7}{2n+1})^2\, \frac{2(n-1)}{2n+1} + (2 - \frac{2n+7}{2n+1})^2 . \frac{1}{2n+1} + (3 - \frac{2n+7}{2n+1})^2 . \frac{1}{2n+1} + (4 - \frac{2n+7}{2n+1})^2 . \frac{1}{2n+1} = \frac{56n^2 - 16n - 22}{(2n+1)^2}$$

4. Application; Proposed Application: Network Security and Anomaly Detection

In the realm of network security, the ability to identify anomalies in network traffic is paramount for uncovering potential cyber threats. A novel approach involves utilizing Mycielski graphs and their local antimagic chromatic numbers. The process begins by converting network traffic data into a Mycielski graph (M(G)), where nodes represent devices and edges signify communication over time. Local antimagic labeling is then applied to M(G), assigning labels to edges based on specific criteria related to communication patterns [22, 23]. These labels are used to induce a vertex coloring of the graph, with each vertex's color determined by the sum of incident edge labels (communication signal strength). Anomaly detection occurs by monitoring changes in vertex colors over time, signaling unusual communication patterns. Upon detecting an anomaly, the system triggers alerts for further investigation. Security analysts can focus on the implicated devices and communication pathways. Additionally, an adaptive learning mechanism allows the system to evolve its labeling strategies over time, enhancing its ability to detect emerging anomalies. The integration of local antimagic labelings with machine learning techniques aims to create an early warning system for identifying unusual network behavior before it evolves into a full-fledged attack. This speculative application scenario showcases the potential of Mycielski graphs in fortifying network security through advanced anomaly detection as shown in Figure 1.

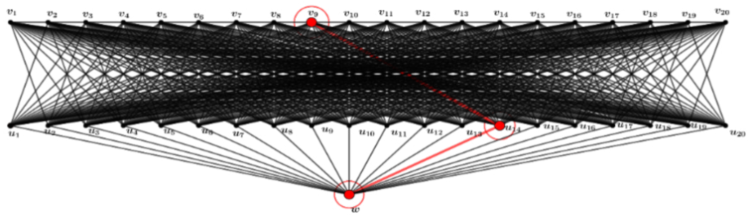

Figure 1
Anomaly Detection in Networking (Mycielski Graph)

5. Conclusion

In conclusion, this paper has uncovered the Chromatic Mean and Variance of Local Antimagic Labeling in Mycielski graphs derived from diverse structures. Our findings reveal the unique property of Local Antimagic Labeling in graphs generated from Paths, Cycles, Complete Graphs, and Stars. Beyond theoretical insights, our study extends practical applications, particularly in Network Security and Anomaly Detection. The demonstrated efficacy of Local Antimagic Labeling opens new avenues for addressing contemporary challenges in these domains. As we conclude, we encourage further exploration into graph structures sharing this labeling property, fostering advancements in both theory and practical applications within the dynamic field of graph theory. The intriguing pursuit of identifying all graphs with this distinctive property remains an open problem, beckoning further research.

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