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Mathematical analysis of wavelet-based multi-image compression in medical diagnostics

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Abstract

Accurate diagnosis and well-informed treatment choices are only possible with the help of sophisticated medical imaging technology. The introduction of digital imaging methods has led to a dramatic rise in the volume of data associated with medical images, which in turn has led to difficulties in their storage, transmission, and administration. It is crucial to utilize effective image compression techniques to address these challenges without compromising the diagnostic integrity of the pictures. The wavelet transform has matured into a potent method for striking a good compromise between picture quality and file size reduction while compressing. The safe and efficient transfer of medical image data is a major concern in today's healthcare settings. In this academic investigation, we investigate how wavelet transform-based techniques may be used to enhance medical picture compression. The proposed techniques optimize compression ratios while maintaining diagnostic picture quality by making use of the wavelet transform's multi-resolution and frequency localization features. To address the unique challenges given by medical image collections, various iterations of the wavelet transform and compression techniques are investigated. Through a series of detailed tests involving several medical picture modalities, the effectiveness of

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these technologies is thoroughly evaluated, demonstrating their effectiveness in achieving significant data reduction without sacrificing clinical information.

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1. Introduction

The main goal of this research work is to introduce the uses and properties of wavelet analysis that can be successfully applied in particular areas of interest, such as mathematical physics, information technology, fluid dynamics, optimization theory, and biomedical engineering. In order to address problems in the selected application sectors and their related disciplines, this study makes use of wavelet tools, techniques, and the wavelet transform[1]. Using wavelets or wavelet-based approaches to evaluate data, build mathematical models, and run simulations that are applicable to scientific and technical contexts is the main objective of this research. Additionally, this analytical method has been used to design items[2], create system prototypes, produce industrial commodities, and improve production procedures.

Wavelets' invention is inextricably linked to real-world uses. Although wavelets' complex analytical characteristics previously made them less useful for professional mathematicians, this position is slowly changing [3,4]. Notably, wavelets have achieved outstanding results in the applied fields of image processing and signal analysis. The term "processing" refers to activities like cleansing, filtering (denoising), efficient storing, retrieval, and transmission of time-signal and image data, with compression playing a particularly important role[5,6].

In order to increase accuracy and efficiency, this paper explores a number of encoding and decoding algorithms that revolve around the compression of pictures using multi-wavelets and multi-resolution methods.

2. Literature Survey

Wavelets gained attention in the middle of the 1980s when they were used in signal processing and image reduction. Their efficiency was amply proved by the results of experiments after they were initially thought of as instruments in these realms. Because of their unique characteristics, wavelets are frequently used for applications including picture compression, pattern identification, and signal denoising[7]. Wavelets are effective and potent tools for such tasks because of their capacity for localization in both space and frequency and the availability of quick methods[8]. In essence, wavelets have the potential to lead to a variety of ideas that connect engineering, physics, and both pure and applied mathematics.

Early 1980s saw wavelets rise in popularity and usefulness, mostly because of the work of geophysicist J. Morlet. He understood the need for advanced signal processing methods that go beyond Fourier analysis. Gabor changed the Gaussian window he used, and he came up with the name "wavelet" to describe this updated notion, which is similar to a little wave[9]. Although the results were promising, not everyone in the mathematical world agreed with them. Grossman established a solid foundation for the theory in 1984 by recognizing the potential of Morlet's wavelet[10]. He began conducting active research in the area as a result. Following suit, Yves Meyer succeeded in creating smooth wavelets in 1985.

The axiomatic treatment of Multiresolution analysis by Mallat and Meyer in 1986 marked a turning point in the late 1980s. Daubechies introduced the theory and procedure for producing orthonormal wavelets with compact support shortly after[11, 12], building on Mallat's work in 1987. This important discovery paved the road for creating image processing algorithms and numerically solving differential and integral equations[13, 14]. Wavelet theory has promoted a symbiotic interaction between models and real-world applications in modern times. The purpose of this study is to make a contribution in this area by providing insights on active research projects.

3. Medical Image Compression & its Significance

Nearly every field, including biomedicine, geography, statistics, astronomy, and aerospace engineering, to mention a few, depends heavily on images. Large amounts of data, including photos and outcomes, are saved and sent through computer systems in the field of biomedical engineering[15]. The limitations of the available computational resources, however, impose restrictions in a number of different ways. Image compression develops as a viable method to improve storage capacity and bandwidth utilization [16, 17]. Coding, interpixel, and psycho-visual redundancies are the three main types of data redundancies that are the focus of image compression.

Coding redundancy develops when a picture is represented by subpar code.

Image compression is helpful in the digital age, especially when it comes to the World Wide Web, for accelerating the speed at which images load and are uploaded while also taking up less space on web servers[18, 19]. In order to effectively store a big number of photos in a small amount of space and subsequently decrease total execution time, file size reduction is essential.

4. Image Compression Algorithms

For image compression, several algorithms have been developed, and a large number of papers in national and international publications have been published in this field of study. It is crucial to consider whether an algorithm produces a lossless or lossy rebuilt image before choosing it. When using lossless compression, the decompressed image is an exact reproduction of the original bit stream. This kind of method is especially useful for files like computer programs where even one lost bit makes the file useless[20]. Lossy compression, on the other hand, modifies the bit stream after decompression. In this method, only visually important content is maintained while certain non-critical data is ignored.

5. Proposed wavelet-based Image Compression Technique

Let's start by studying a function with the notation f that has finite energy before delving into the mathematical basis of wavelet-based picture compression. Let's talk about images in their mathematical form using a *L x L* grayscale image with 256 grayscales. For $i \le \alpha \le i+1,\ \ j \le \beta \le j+1$ and $0 \le \alpha, \beta \le L.$ This image is interpreted as a block with the shape of a square, with the constant function $(\alpha, \beta) = p_{i,j} \in N,$ here $0 \le p_{i,j} \le 255$ specified on it. Now, have a look at an orthonormal wavelet basis of $L^2(R),$ represented by the notation $\{g_m\} m \in N$ A. The coefficients of a function f will be $f = \Sigma_{m \in Y} \langle f.g_m \rangle g_m$ in this basis, which can be written as $\langle f.g_m \rangle m \in Y.$ For the sake of simplicity, let h_m stand for all the coefficients. As part of the compression procedure, M coefficients are predetermined. This choice shows that lossy compression techniques are being used. Let $f_M$be the obtained compression and K be the collection of indices for the M coefficients that were chosen. After that $f_M == \Sigma_{m \in Y} \langle h_m g_m \rangle.$ To achieve compression objectives, the wavelet transform may be used in a number of different methods. Fix an orthogonal wavelet basis, say $\{\varphi\},$ in this respect.

Let's get the best estimate for this by reducing the error $\| f - f_M \|^2$. The approximate value is equal to $\Sigma(|\langle f, \varphi_{i,j}\rangle - b_{i,j}|^2)^{0.5}$. Non-zero coefficients M should be manually chosen among those with the highest absolute value, setting $b_{i,j} = \langle f, \varphi_{i,j}\rangle$. This will reduce error. Thus, the required best estimate of f is produced. When the wavelet transform is used, the picture is considered in its 2D form. The discrete wavelet transform (DWT), individually in the horizontal and vertical axes, generates a matrix of wavelet coefficients. These coefficients are then arranged according to a predetermined pattern. For the purpose of transmission, let us assume that original image is formed from set of pixels $m_{i,j}$, here (i, j) are the coordinates of pixel. Let $c_{i,j}$ be the transformed coefficient at (i, j) coordinate and is represented by small number of bits, like 16 bits, indicating integer. Let transformation $T(.)$ represents unitary hierarchical sub-band transformation. Let 2-D array be represented by c that has dimension same as of m.

$$c = T(m) \tag{1}$$

The initial value of the reconstruction vector, represented by the symbol, is zero.In a progressive transmission system, the decoder modifies this vector in accordance with the coded message it has just received.

$$\hat{m} = T^{-1}(\hat{c}) \tag{2}$$

The mean square error (MSE) is used to quantify the degree of distortion:

$$d_{mse}(m - \hat{m}) = \frac{\|m-\hat{m}\|^2}{N} = \frac{1}{N}\Sigma_i\Sigma_j(m_{i,j} - \hat{m}_{i,j})^2 \tag{3}$$

The entire number of image pixels is represented in this context by the sign N. Furthermore, it has been demonstrated that the Euclidean norm is unaffected by a unitary transformation T, which means:

$$d_{mse}(m - \hat{m}) = d_{mse}(c - \hat{c}) = \frac{1}{N}\Sigma_i\Sigma_j(c_{i,j} - \hat{c}_{i,j})^2 \tag{4}$$

It can be understood that if exact value of $c_{i,j}$ is received by decoder, then MSE decreases by $|c_{i,j}|^2 / N$. This suggests that bigger magnitude coefficients should be prioritized when transferring data since they may contain more important information. Figure 1 provides a schematic illustration of the compression process while also illustrating the procedure's overarching concept.

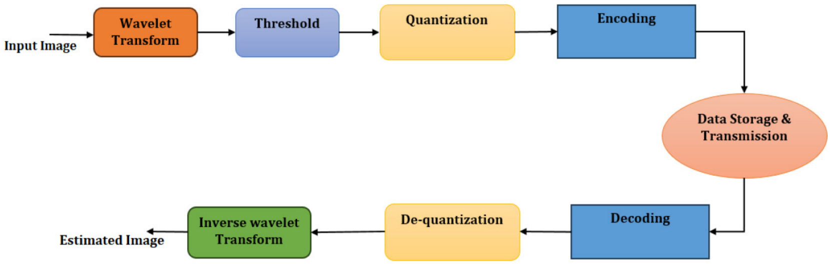

Figure 1

General idea of image compression

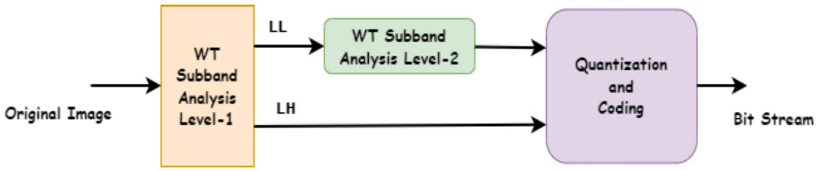

Figure 2

Image compression at encoding scheme based on DWT

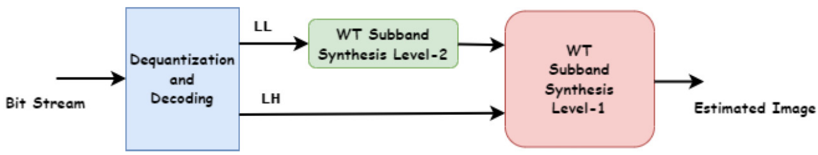

Figure 3

Image decoding scheme based on DWT

Four sub-bands are typically formed as a result of the data decomposition: LL: The vertical and flat low-pass filter elements. HH: Flat and upright high-pass filter elements. LH: Low-pass upright and high-pass flat filter elements. HL: Elements of the high-pass upright and low-pass flat filters. Figures 2 and 3, respectively, show a two-level representation of the analysis and synthesis of a signal.

6. Performance Metrix

Various performance indicators are frequently employed in modern practice to assess the efficiency of picture compression. PSNR, MSE, average difference, normalized cross-correlation, structural content, Laplacian mean square error, and picture quality scale are a few of these.

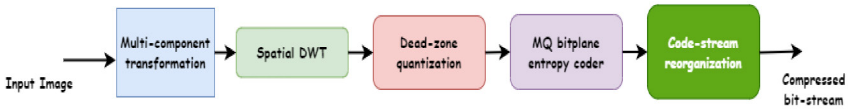

Figure 4

Block diagram of Image compression standard

The following are some of them that are often used and are discussed below: Ratio of Compression (CR) The ratio of the original image's (m) and compressed image's (n) sizes is quantified by this metric. It is specifically denoted as m: n.

Peak Signal to Noise Ratio (PSNR): The definition of PSNR is as follows, assuming that the image's pixels are represented with 8 bits each sample (n = 8).

$$PSNR = 10\log_{10}\left(\frac{(2^n-1)^2}{MSE}\right) = 10\log_{10}\left(\frac{255^2}{MSE}\right) \tag{5}$$

MSE is for Mean Square Error in this context, and n stands for bits per sample.

Mean Square Error (MSE): It is calculated by dividing the square of the differences between the original and reconstructed image by the total number of image components. The formula is as follows:

$$MSE = \frac{1}{mn}\Sigma_{i=o}^{m-1}\Sigma_{j=0}^{n-1}|X(i,j)-X_c(i,j)|^2 \tag{6}$$

It always returns a positive value. It is deemed better if the MSE value is closer to zero.

Maximum Error: As the name suggests, it provides the highest absolute deviation between the original and reconstructed image.

Bits per pixel ratio (BPP): BPP is a quantity that describes how many bits are required to store or transmit each individual pixel in an image.

One of JPEG2000's primary advantages is its scalability, which enables bit plane encoding to truncate the code stream at various locations. This makes it easier to reassemble with customized higher compression levels. Figure 4 presents the JPEG2000 algorithm's basic methodology.

7. Results and Discussion

Let's use a real medical photograph only for illustration to show the qualitative and quantitative effects of using several compression algorithms. Let's think about the fruit image shown in Figure 5. With a maximum of 12 repetitions and an initial 4-level decomposition, we use the Matlab software to apply various compression techniques (already discussed) to an image that uses the Haar wavelet transform. Figures 6 and 7 combine the results of each algorithm's evaluation of the image quality using several quantitative criteria for the intermediate compression frequency.

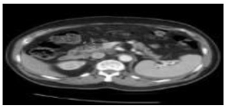

Figure 5

True Medical image

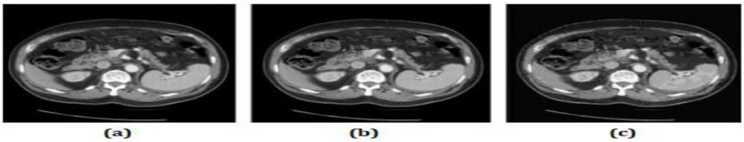

Figure 6

(a) Single (b) double (c) Triple Compressed Medical Image

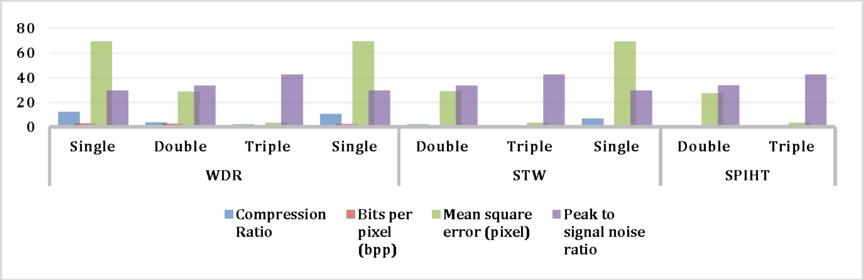

Figure 7

Performance Metrix of different Compression Techniques

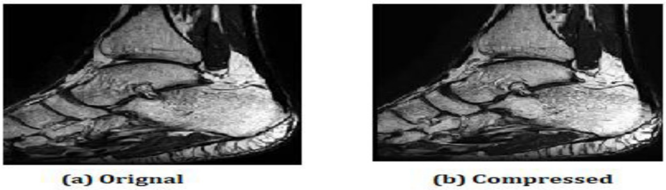

Figure 8

Original and Compressed image of MRI

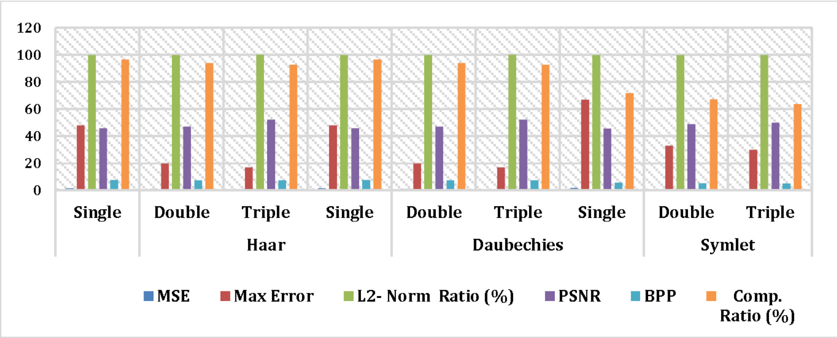

Figure 9

Performance indices of multiple compressions

It is clear that for each technique, the PSNR value rises as compression occurs more frequently. A PSNR number over 40 often denotes a picture of good quality. Figure 7 shows the performance metrics for sporadic images, though. All of these findings suggest that additional compression of this image could not produce good results. When accepting lossy data is allowed, this characteristic can be used to prioritize storage and transmission efficiency.

Our main goal is to effectively compress medical images so that they may be kept in a small amount of space and transmitted at a high speed. We take an MRI database into consideration as an illustration. The compression operation was performed on one of the many frames that make up these photos, and the result is shown in Figure 9. Lossy compression techniques are used to produce a significant compression ratio while maintaining the visual quality of the reconstructed image (Figure 7). shows the quantitative analysis of all the algorithms that were highlighted. It is clear that across a range of frequencies, the compression ratio (CR) of all approaches is largely stable. The evaluation of several wavelets' performance in Figure 8 makes it easier to choose the best mother wavelet for various picture compression tasks.

8. Conclusion

In order to facilitate a comparative analysis of the efficiency and reliability of various compression techniques, true-colour images (representative medical images) for illustration purposes as well as grayscale images (MRI images) from the medical domain were thoroughly investigated in this research paper. For compression, a variety of wavelet-and wavelet-packet-based techniques were used. Bits Per Pixel (BPP), Compression Ratio (CR), MSE, and PSNR were computed and compared for evaluation. The results of this comparison and investigative study highlight the potential benefits of using double or triple compression frequencies, which can improve performance in terms of image transmission, retrieval, and storage. However, it's crucial to remember that increasing compression can cause information loss and image degradation. The potential for this idea to revolutionize tasks like storing enormous amounts of medical data, quickly transmitting information to distant specialists, and enabling remote patients to access crucial data in emergency situations is, however, enormous.

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