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Totally antimagic total labeling of ladders, prisms and generalised Petersen graphs

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Abstract

A total labeling is called *edge-antimagic total (vertex-antimagic total)* if all edge-weights (vertex-weights) are pairwise distinct. If a labeling is simultaneously edge-antimagic total and vertex-antimagic total, it is called a *totally antimagic total labeling*. A graph that admits totally antimagic total labeling is called a *totally antimagic total graph*.

In this paper, we prove that ladders, prisms and generalised Pertersen graphs are totally antimagic total graphs. We also show that the chain graph of totally antimagic total graphs is a totally antimagic total graphs.

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1. Introduction

We consider finite, undirected graphs without loops and multiple edges. For a graph G, $V(G)$ and $E(G)$ denotes the vertex-set and the edge-set respectively. A (p, q) graph G is a graph such that $|V(G)| = p$ and

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$|E(G)| = q$. We refer the reader to [10] and [11] for all other terms and notation not provided in this paper.

Let *f* be a vertex labeling of a graph *G*, we define the edge-weight of $uv \in E(G)$ to be $wt_f(uv) = f(u) + f(v)$. If *f* is a total labeling, then the edge-weight of *uv* is $wt_f(uv) = f(u) + f(v) + f(uv)$. The vertex-weight of a vertex $v,\ v \in E(G)$ is defined by

$$wt_f(v) = \sum_{u \in N(v)} f(uv) + f(v)$$

where $N(v)$ is the set of the neighbors of V. If the vertices are labeled with the smallest posssible numbers i.e. $f(V(G)) = \{1, 2, 3, \ldots, p\}$, then the total labeling is called *super*.

A labeling *f* is called *edge-antimagic total(vertex-antimagic total)*, for short EAT(VAT), if all edge-weights (vertex-weights) are pairwise distinct. A graph that admits EAT (VAT) labeling is called an EAT (VAT) graph. If the edge-weights (vertex-weights) are all the same, then the total labeling is called *edge-magic total (vertex-magic total)*. For an edge labeling, a *vertex-antimagic edge* (VAE) labeling is a labeling whereby a vertex-weight is the sum of the labels of all edges incident with the vertex.

In 1990, Harsfield and Ringel [6] introduced the concept of an antimagic labeling of graphs. If a VAE labeling satisfies the condition that the set of all the vertex-weights is $\{a, a+d, \ldots, a+(p-1)d\}$ where $a > 0$ and $d \geq 1$ are two fixed integers, then the labeling is called an *(a,d)-VAE labeling*. For further results on graph labeling see [2], [5] and [9]. If the labeling is simultaneously vertex-antimagic total and edge-antimagic total, then it refers to as *totally-antimagic total* (TAT) labeling and a graph that admits such labeling is a *totally-antimagic total* (TAT) graph. The definition of totally antimagic total labeling is a natural extension of the concept of totally magic labeling. In [1], Baca *et al.* finds totally-antimagic total labeling of some classes of graphs. They proved that paths, cycles, stars, double-stars and wheels are totally antimagic total. The edge-distance-balanced property of the generalized Petersen graphs was given in [3, 4]. Also, antimagic labelings of some cycle related graphs, Formulas and algorithms of antimagic labelings of some helm related graphs were given in [7, 8]. In this paper, we give the totally antimagic total labeling of ladders, prisms and generalised Petersen graphs. We also show that the chain graph obtained by concatenation of totally antimagic total graphs is totally antimagic total graph.

First, we provide some definitions which are related to the present work.

Definition 1.1: Ladder is a graph obtained by the cartesian product of path P_n and path P_2 denoted by L_n, i.e. $L_n \simeq P_n \times P_2$ where $V(L_n) = \{u_i v_i : 1 \le i \le n\}$ and $E(L_n) = \{u_i u_{i+1}, v_i v_{i+1} : 1 \le i \le n-1\} \cup \{u_i v_i : 1 \le i \le n\}$.

Definition 1.2 : A labeling g is ordered (sharp ordered) if $wt_g(u) \le wt_g(v)$ $(wt_g(u) < wt_g(v))$ holds for every pair of vertices $u, v \in G$ such that $g(u) < g(v)$. A graph that admits a sharp ordered labeling is called a (sharp) ordered graph. Also, if the vertex set can be partitioned into n sets such that each set is sharp ordered, then the graph is referred to as weak ordered graph.

Definition 1.3 : The prism graph can be defined as the cartesian product $C_n \times P_2$ of a cycle on n vertices with a path of length 2. The vertex set is $V(C_n \times P_2) = \{u_i v_i : 1 \le i \le n\}$ and the edge set is $E(C_n \times P_2) = \{u_i u_{i+1}, v_i v_{i+1} : 1 \le i \le n\} \cup \{u_i v_i : 1 \le i \le n\}$ where i is calculated modulo n. The orders of the vertex set and the edge set are $2n$ and $3n$ respectively.

Definition 1.4 : The generalised Pertersen graph $P(n, m), n \ge 3$ and $1 \le m \le \lfloor \frac{n-1}{2} \rfloor$ consists of an outer n-cycle $u_i, u_2, \ldots, u_n$, a set of n spokes $u_i v_i, 1 \le i \le n$ and n edges $v_i v_{i+m}$, $1 \le i \le n$ with indices taken modulo n.

2. Totally antimagic total graphs

In this section, we prove that Ladders, prism graphs and generalised Petersen graphs are totally antimagic total graphs.

Theorem 2.1 : *The Ladder graph L_n, $n \ge 2$ is a weak ordered super TAT.*

Proof : We denotes the vertices of L_n by the following symbols $V(L_n) = \{u_i v_i : 1 \le i \le n\}$ such that $E(L_n) = \{u_i u_{i+1}, v_i v_{i+1} : 1 \le i \le n-1\}$ $\cup \{u_i v_i : 1 \le i \le n\}$. Consider the labeling g of L_n as follows:

$$g(u_i) = \begin{cases} 2i-1, & 1 \le i \le \lceil \frac{n}{2} \rceil \\ 2(n-i+1), & \lceil \frac{n}{2} \rceil + 1 \le i \le n \end{cases}$$

and

$$g(v_i) = \begin{cases} n+2i-1, & 1 \le i \le \lceil \frac{n}{2} \rceil \\ 3n+2(1-i), & \lceil \frac{n}{2} \rceil + 1 \le i \le n \end{cases}$$

Also, the labeling of the edges is as follows:

$$g(u_i v_i) = \begin{cases} 2(n+i)-1, & 1 \le i \le \lceil \frac{n}{2} \rceil \\ 2(2n-i+1), & \lceil \frac{n}{2} \rceil + 1 \le i \le n-1 \end{cases}$$

$$g(u_i u_{i+1}) = \begin{cases} 3n+2i-1, & 1 \le i \le \lfloor \frac{n}{2} \rfloor \\ 5n-2i, & \lfloor \frac{n}{2} \rfloor + 1 \le i \le n-1 \end{cases}$$

$$g(v_i v_{i+1}) = \begin{cases} 2(2n+i-1), & 1 \le i \le \lfloor \frac{n}{2} \rfloor \\ 2(3n-i)-1, & \lfloor \frac{n}{2} \rfloor + 1 \le i \le n-1 \end{cases}$$

The above labeling is supper. Next, we show that the vertex-weights are pairwise distinct.

The vertex-weights of $V(L_n)$ is

$$wt_g(u_i) = g(u_i) + \sum_{u \in N(u)} g(u_i u)$$

where vertex u is any vertex adjacent to vertex u_i. For $i=1$ and $i=n$, we have the weights

$$wt_g(u_1) = 3+5n$$

and

$$wt_g(u_n) = 6+5n.$$

Also, for $i = 2, \ldots, n-1$, we have the weight

$$wt_g(u_i) = \begin{cases} 2(4i+4n-3), & 2 \le i \le \lfloor \frac{n}{2} \rfloor \\ 12n-3, & i = \lfloor \frac{n}{2} \rfloor + 1 \\ 2(4(2n-i)+3), & \lfloor \frac{n}{2} \rfloor + 2 \le i \le n-1 \end{cases}$$

Furthermore,

$$wt_g(v_i) = \begin{cases} W_1, \\ W_2 \end{cases}$$

Where

$$W_1 = 2(2i-1) + k(2n+i-1) - (t+3n)$$

with the following conditions:

for $i = 1, k = 1$ and $t = 0$,

for $2 \le i \le \lfloor \frac{n}{2} \rfloor, k = t = 2.$.

While

$$W_2 = 2(2(1-i) + k(3n-i)) + (t-k) + 7n$$

with the following conditions:

$k = t = 2$ whenever $\lfloor \frac{n}{2} \rfloor + 2 \le i \le n-1$,

$k = 1$ and $t = 2$ for $i = n$,

for n even and $i = \frac{n}{2} + 1$, $k = 2$ and $t = 1$,

for n odd and $i = \lfloor \frac{n}{2} \rfloor + 1$, $k = 2$ and $t = -3$.

In view of the above labeling, the weights of all the vertices are different which means that the labeling is vertex-antimagic total.

Now, we show that the edge-weights are pairwise distinct. The edge-weight of the edges under labeling g is as follows:

$$wt_g(u_i u_{i+1}) = \begin{cases} W_3, \\ W_4 \end{cases}$$

Where

$$W_3 = 3(2i+n-1) + t$$

with the following conditions:

$t = 2$ for $1 \le i \le \lceil \frac{n}{2} \rceil - 1$,

$t = 1$ for n even and $i = \frac{n}{2}$,

$t = -2$ for n odd and $i = \lceil \frac{n}{2} \rceil$.

While

$$W_4 = 3(3n-2i) + 2$$

for $\lceil \frac{n}{2} \rceil + 1 \le i \le n-1$. Also,

$$wt_g(v_i v_{i+1}) = \begin{cases} W_5, \\ W_6 \end{cases}$$

where

$$W_5 = 2(3(n+i)-2)+t$$

with the following conditions:

$t = 2$ whenever $1 \le i \le \lceil \frac{n}{2} \rceil - 1$,

$t = 1$ whenever n is even and $i = \frac{n}{2}$,

$t = -2$ whenever $i = \lceil \frac{n}{2} \rceil$ and n is odd. Also,

$$W_6 = 3(2(2n-i)+1)-2, \lceil \tfrac{n}{2} \rceil + 1 \le i \le n-1$$

In view of the above labeling, the edge-weights are pair-wise distinct, which implies that the labeling is edge-antimagic labeling, i.e. ladders is an edge-antimagic total graph.

If we partition the vertex set into two, i.e. u_i and v_i, $1 \le i \le n$, each vertex set is sharp ordered which implies that the graph is a weak ordered graph. Therefore Ladders is a super weak ordered TAT graph since it is both edge-antimagic total graph and vertex-antimagic total graph. □

The following theorem shows that Prism graph is a TAT graph.

Theorem 2.2 : *The Prism graph* $C_n \times P_2$ *is a super TAT graph for every* $n > 2$.

Proof : Denote the vertices of prism graph $C_n \times P_2$ by $\{u_i v_i : 1 \le i \le n\}$ and the edges by $\{u_i u_{i+1}, v_i v_{i+1} : 1 \le i \le n\} \cup \{u_i v_i : 1 \le i \le n\}$. Let g be the labeling on $C_n \times P_2$. Define the labeling g on the vertices as follows: Whenever $i = 1$, $g(u_1) = 1$ and $g(v_1) = n+1$. Also,

$$g(u_i) = \begin{cases} 2(i-1), & 2 \le i \le \lfloor \frac{n}{2} \rfloor + 1 \\ 2(n-i)+3, & \lfloor \frac{n}{2} \rfloor + 2 \le i \le n \end{cases}$$

$$g(v_i) = \begin{cases} n+2(i-1), & 2 \le i \le \lfloor \frac{n}{2} \rfloor + 1 \\ 3(n+1)-2i, & \lfloor \frac{n}{2} \rfloor + 2 \le i \le n \end{cases}$$

From the labeling above, we have the labeling g to be super.

Moreover, define the labeling g on the edge set as follows:

$$g(u_i u_{i+1}) = \begin{cases} 2(n+i)-1, & 1 \le i \le \lceil \frac{n}{2} \rceil \\ 2(2n+1-i, & \lceil \frac{n}{2} \rceil + 1 \le i \le n \end{cases}$$

$$g(v_i v_{i+1}) = \begin{cases} 2(2n+i)-1, & 1 \le i \le \lceil \frac{n}{2} \rceil \\ 2(3n-i+1), & \lceil \frac{n}{2} \rceil + 1 \le i \le n \end{cases}$$

$$g(u_i v_i) = \begin{cases} 2(2n-1)+3, & 2 \le i \le \lfloor \frac{n}{2} \rfloor + 1 \\ 2(n-1+i), & \lfloor \frac{n}{2} \rfloor + 2 \le i \le n \end{cases}$$

For $i = 1,$ we have $g(u_1 v_1) = 4n.$

Next, we consider the vertex-weights and show that they are pairwise distinct. The vertex-weights of $C_n \times P_2$ is as follows:

$$wt_g(u_i) = \begin{cases} W_7 \\ W_8 \end{cases}$$

where

$$W_7 = 4(2n+i) + (t-1)$$

with the following conditions:

whenever $i = 1,$ $t = 1,$

whenever $2 \le i \le \lceil \frac{n}{2} \rceil,$ $t = -2.$

and

$$W_8 = 4(3n-i) + (t+5)$$

with the following conditions:

when $i = \frac{n}{2} + 1, n$ even, $t = -1,$

when $i = \lceil \frac{n}{2} \rceil + 1, n$ odd, $t = 1,$

when $\lceil \frac{n}{2} \rceil + 2 \le i \le n,$ $t = 2.$

Also,

$$wt_g(v_i) = \begin{cases} W_9 \\ W_{10} \end{cases}$$

Where

$$W_9 = 13n + 4i + (t-1)$$

with the following conditions:

$t = 1$ whenever $i = 1$ and $t = -2$ whenever $2 \le i \le \lceil \frac{n}{2} \rceil$.

Also,

$$W_{10} = 17n - 4i + (t+5)$$

with the following conditions:

$t = -1$ whenever $i = \frac{n}{2} + 1$ and n even,

$t = 1$ for $i = \lceil \frac{n}{2} \rceil + 1$ and n odd,

$t = 2$ whenever $\lceil \frac{n}{2} \rceil + 2 \le i \le n$.

In view of the above, the set $\{W\}_{i=7}^{i=10}$ are pairwise distinct which shows that the labeling g is a vertex-antimagic total.

For the edge-weights under the labeling g, we have the following:

$$wt_g(u_i u_{i+1}) = \begin{cases} W_{11} \\ W_{12} \end{cases}$$

Where

$$W_{11} = 2(n + 3i) + (t-5)$$

satisfying the following:

$t = 3$ for $i = 1$,

$t = 2$ for $2 \le i \le \lfloor \frac{n}{2} \rfloor$,

$t = 1$ for $i = \lceil \frac{n}{2} \rceil$, n odd.

Also,

$$W_{12} = 2(4(n+1) - 3i) + t$$

satisfying the following conditions:

$t = -3$ whenever $i = \frac{n}{2} + 1, n$ even,

$t = -2$ whenever $\lfloor \frac{n}{2} \rfloor + 2 \le i \le n$.

$$wt_g(v_i v_{i+1}) = \begin{cases} W_{13} \\ W_{14} \end{cases}$$

Where

$$W_{13} = 6(n+i) + (t-5)$$

with the following conditions:

for $i = 1$, $t = 3$,

for $2 \le i \le \lfloor \frac{n}{2} \rfloor$, $t = 3$,

for $i = \lceil \frac{n}{2} \rceil$ and n odd, $t = 1$. Furthermore,

$$W_{14} = 2(3(2n-i)+4)+t$$

satisfying the followings:

$t = -3$ whenever $i = \frac{n}{2} + 1$ and n even,

$t = -2$ whenever $\lfloor \frac{n}{2} \rfloor + 2 \le i \le n$.

Moreover,

$$wt_g(u_i v_i) = \begin{cases} 5n + 2i + t, & t = 0 \text{ for } i = 1 \text{ and } t = -1 \text{ for } 2 \le i \le \lfloor \frac{n}{2} \rfloor + 1 \\ 7n + 2(2-i), & \lfloor \frac{n}{2} \rfloor + 2 \le i \le n \end{cases}$$

The edge-weights of the edges in $C_n \times P_2$ under the labeling g are all different which implies that the labeling is edge-antimagic total. Therefore the labeling g is TAT labeling since it is both vertex-antimagic total and edge-antimagic total. Thus the prism graphs $C_n \times P_2$ is a totally antimagic total graph. □

Next, we give the TAT labeling of generalised Petersen graph.

Theorem 2.3 : *The generalised Petersen graph* $P(n,m), n \geq 3$ *and* $1 \leq m \leq \lfloor \frac{n-1}{2} \rfloor$ *is a super TAT graph.*

Proof : Denotes the vertices of the graph $P(n,m)$ by the symbols $u_i v_i, 1 \leq i \leq n$. Let g be a labeling on the graph $P(n,m)$ defined in the following way:

$$g(u_i) = i, 1 \leq i \leq n$$

$$g(v_i) = \begin{cases} n+1, & 1 \leq i \leq n-1 \\ 2(3n-i+1), & j = n \end{cases}$$

Also, define labeling g on the edges as follows:

$$g(u_i u_{i+1}) = 3n-(i-1), 1 \leq i \leq n$$

where i is calculated modulo n.

$$g(u_i v_i) = \begin{cases} 3n+1, & i = 1 \\ 4n-(i-2), & 2 \leq i \leq n \end{cases}$$

$$g(v_i v_{i+m}) = 5n-(i-1), 1 \leq i \leq n$$

with indices $i+m$ taken modulo n.

It is easy to see that the labeling g is super. Also, the vertex-weights under the labeling g is as follows:

$$wt_g(u_i) = \begin{cases} 10n+2-i, & i = 1 \\ 5(2n+1)-2i, & 2 \leq i \leq n \end{cases}$$

$$g(v_i v_{i+1}) = \begin{cases} 14n+2-m, & i = 1 \\ 15n-2i+5-t, & \\ \text{and } t = -m \text{ for } \lfloor \frac{n}{2} \rfloor + 1 \leq i \leq n-1 & t = m+1 \text{ for } 2 \leq i \leq \lfloor \frac{n}{2} \rfloor \\ 12n+m+5, & i = n \end{cases}$$

In view of the above labeling, the vertex-weights are pairwise distinct which implies that the labeling g is super vertex-antimagic total.

Next we consider the edge-weights of the graph $P(n,m)$ as follows:

$$wt_g(u_i u_{i+1}) = \begin{cases} 3n+i+2, & 1 \leq i \leq n-1 \\ 3n+2, & i = n \end{cases}$$

$$wt_g(v_i v_{i+m}) = 7n + 3 - t$$

with the following conditions:

$t = -(k + j)$ whenever $1 \le i \le \lfloor \frac{n}{2} \rfloor$,

$t = k - j + 1$ whenever $\lfloor \frac{n}{2} \rfloor + 1 \le i \le n - 1$,

$t = k + 1$ whenever $i = n$. Also,

$$wt_g(u_i v_i) = \begin{cases} 4(n+1), & i = 1 \\ 5n + 3 + i, & 2 \le i \le n - 1 \\ 5n + 3, & i = n \end{cases}$$

This means that all the edges in $P(n, m)$ have different edge-weights which implies that the graph $P(n, m)$ is an edge-antimagic total graph. Therefore the generalised Petersen graph $P(n, m)$ is a super totally antimagic total graph since it admits both vertex-antimagic total labeling and edge-antimagic total labeling. $\square$

3. Totally antimagic total labeling of chain graphs

In this section, we prove that the chain graphs of totally antimagic total graphs is totally antimagic total graph. Suppose now that the graphs $B_1, B_2, ..., B_m$ are blocks and that for any $i \in \{1, 2, ..., m\}$, B_i and B_{i+1} have a vertex in common in such a way that the block-cut point is a path. The graph G obtained by the concatenation will be called a *chain* graph.

Theorem 3.1 : *The chain graph G obtained by concatenation of totally antimagic total graphs is a TAT graph.*

Proof : Let G denotes that chain graph with blocks $G_i, 1 \le i \le m$. Let $g_i, 1 \le i \le m$ be a TAT labeling of G_i. Also, let $p = |V(G_i)|$ and

$$g_i : V(G_i) \cup E(G_i) \to \{1, 2, ..., |V(G_i)| + |E(G_i)|\}$$

such that $wt_{g_i}(v) \ne wt_{g_i}(u)$ for all $u, v \in V(G_i), u \ne v$ and $wt_{g_i}(e) \ne wt_{g_i}(h)$ for all $e, h \in E(G_i), e \ne h$.

Consider the cut-vertex between G_i and G_{i+1} as the concatenation of the vertex labeled with $1 \in V(G_i)$ and vertex labeled with $p \in V(G_{i+1})$ denoted by r_i. Without loss of generality we may assume that $r_{i+1} > r_i$. Define a labeling for G such that

$$f(x)=\begin{cases} g_1(x), & x\in V(G_1)\\ g_i(x)+\sum_{j=1}^{i-1}|V(G_j)|+\sum_{j=1}^{i-1}|E(G_j)|-(i-2), & x\in V(G_i), i=1,2,\dots,m\end{cases}$$

and

It is easy to see that f is a total antimagic total labeling of G. For the edge-weights under the labeling f, we obtain

$$wt_f(e)=\begin{cases} wt_{g_1}(e), & e\in E(G_1)\\ wt_{g_i}(e)+3(\sum_{j=1}^{i-1}|V(G_j)|+\sum_{j=1}^{i-1}|E(G_j)|)-1, & e\in E(G_i), i=1,2,\dots,m\end{cases}$$

Also, the edge-weights for the edges in G_{i+1} incidents with the cut-vertex r_i, under the labeling f is as follows:

$$wt_f(e)=\begin{cases} wt_{g_1}(e), & e\in E(G_1)\\ wtg_i(e)+2(\sum_{j=1}^{i-1}|V(G_j)|+\sum_{j=1}^{i-1}|E(G_j)|)-p+r_i-r_1, & e\in E(G_i), i=1,2,\dots,m\end{cases}$$

As $g_i, i=1,2,\dots,m$ is edge-antimagic labeling, the edge-weights of all edges in G under the labeling f are pairwise distinct.

The maximum edge-weight of an edge $e\in E(G_i)$, $1\le i\le m$ is

$$wt_f^{max}(e)\le 3\left(\sum_{j=1}^{i}|V(G_j)|+\sum_{j=1}^{i-1}|E(G_j)|\right)-|V(G_1)|+|E(G_1)|, e\in E(G_i), i=1,2,\dots,m.$$

Thus f is an edge-antimagic labeling of G.

For the vertex-weight under labeling f, we get

$$wt_f(v)=\begin{cases} wt_{g_1}(v), & v\in V(G_1)\\ wtg_i(v)+(deg(v)_+1)(\sum_{j=1}^{i-1}|V(G_j)|+\sum_{j=1}^{i-1}|E(G_j)|-1)+1, & e\in E(G_i), i=1,2,\dots,m\end{cases}$$

For the cut-vertices r_i, the weights under labeling f, for cycle-like structure, we get

$$wt_f(r_i)=wt_{g_1}(v_p)+wt_{g_1}(v_1)+deg(v)\left(\sum_{j=1}^{i-1}|V(G_j)|+|E(G_j)|\right)+$$

$$(deg(v)+1)\left(\sum_{j=1}^{i-2}|V(G_j)+E(G_j)|\right)-deg(r_i)-p.$$

where v_p and v_1 are the vertices in G_1 labeled with *p* and 1 respectively.

Also, the cut-vertices weights under labeling *f* for path-like structure is as follows:

$$wt_f(r_i) = wt_{g_1}(v_p) + wt_{g_1}(v_1) + (deg(v)+1)(2(\sum_{j=1}^{i-1} |V(G_j)| + |E(G_j)|$$

$$+ |V(G_{j-1})| + |E(G_{j-1})|) - deg(r_i) - p$$

In view of the above labeling, the chain graph *G* is a TAT graph. □

Corollary 3.2 : *The tree graph is a TAT graph.*

Proof : The chain graph formed from the concatenation of paths gives a tree and it follows from theorem 3.1 that a tree is a TAT graph. □

Conjecture: All trees are TAT graph.

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