Journal of Discrete Mathematical Sciences & Cryptography
ISSN 0972-0529 (Print), ISSN 2169-0065 (Online)
Vol. 26 (2023), No. 8, pp. 2217–2228
DOI : 10.1080/09720529.2022.2036405

A new generalization of Fibonacci and Lucas type sedenions

Can Kızılateş *
Selihan Kirlak †
Department of Mathematics
Zonguldak Bulent Ecevit University
Zonguldak 67100
Turkey

Abstract

In this paper, by using the q-integer, we introduce a new generalization of Fibonacci and Lucas type sedenions called q-Fibonacci and q-Lucas sedenions. We state that the special cases of these types of sedenions give the various sedenions whose components are defined by second order integer sequences. We also present some fundamental properties of these types of sedenions.

Subject Classification: *(2010) 11B39, 11R52, 05A15, 05A30.*

Keywords: *Sedenion algebra, Horadam number, q-integer, Binet-Like formula, Exponential generating function.*

1. Introduction

The set of sedenions, denoted by $\mathbb{S}$, are 16-dimensional algebra. Sedenions are noncommutative and nonassociative algebra over the set of real numbers, obtained by applying the Cayley--Dickson construction to the octonions. A sedenion is defined by

$$p=\sum_{i=0}^{15} q_i \mathbf{e}_i ,$$

where $q_0, q_1, ..., q_{15} \in \mathbb{R}$ and $\mathbf{e}_0, \mathbf{e}_1, ..., \mathbf{e}_{15}$ are called unit sedenion such that $\mathbf{e}_0$ is the unit element and $\mathbf{e}_1, \mathbf{e}_2, ..., \mathbf{e}_{15}$ are imaginaries satisfying, for $i, j, k = 1, 2, \ldots, 15$ the following multiplication rules:

* *E-mail:* **cankizilates@gmail.com (Corresponding Author)**
† *E-mail:* **selihankirlak@gmail.com**

$$\mathbf{e}_0\mathbf{e}_i = \mathbf{e}_i\mathbf{e}_0 = \mathbf{e}_i, \ (\mathbf{e}_i)^2 = -\mathbf{e}_0, \tag{1.1}$$

$$\mathbf{e}_i\mathbf{e}_j = -\mathbf{e}_j\mathbf{e}_i, \ i \neq j, \tag{1.2}$$

$$\mathbf{e}_i(\mathbf{e}_j\mathbf{e}_k) = -(\mathbf{e}_i\mathbf{e}_j)\mathbf{e}_k, \ i \neq j, \ \mathbf{e}_i\mathbf{e}_j \neq \pm\mathbf{e}_k. \tag{1.3}$$

The addition of sedenions is defined as componentwise and for $p_1, p_2 \in \mathbb{S}$, the multiplication is defined as follows:

$$\begin{aligned} p_1p_2 &= \left(\sum_{i=0}^{15} a_i\mathbf{e}_i\right)\left(\sum_{j=0}^{15} b_j\mathbf{e}_j\right) \\ &= \sum_{i,j=0}^{15} a_ib_j(\mathbf{e}_i\mathbf{e}_j), \end{aligned}$$

where $\mathbf{e}_i\mathbf{e}_j$ satisfies the identities (1.1), (1.2) and (1.3).

For $a,b,p,q \in \mathbb{R}$, the Horadam numbers $h_n = h_n(a,b;p,q)$ are defined by the following recurrence relation

$$h_n = ph_{n-1} + qh_{n-2}, \qquad h_0 = a, \qquad h_1 = b.$$

Some special cases of the Horadam sequence $h_n(a,b;p,q)$ are as follows:

(1) $h_n(0,1;1,1) = F_n$, the Fibonacci sequence;

(2) $h_n(2,1;1,1) = L_n$, the Lucas sequence;

(3) $h_n(0,1;2,1) = P_n$, the Pell sequence;

(4) $h_n(2,2;2,1) = PL_n$, the Pell-Lucas sequence;

(5) $h_n(0,1;k,1) = F_{k,n}$, the *k*-Fibonacci sequence;

(6) $h_n(2,1;k,1) = L_{k,n}$, the *k*-Fibonacci sequence;

(7) $h_n(0,1;1,2) = J_n$, the Jacobsthal sequence;

(8) $h_n(2,1;1,2) = j_n$, the Jacobsthal-Lucas sequence;

(9) $h_n(0,1;2,k) = P_{k,n}$, the *k*-Pell sequence;

(10) $h_n(2,2;2,k) = PL_{k,n}$, the *k*-Pell-Lucas sequence;

The Binet formula of the Horadam number is given by, for $n \geq 0$,

$$h_n = \frac{A\alpha^n - B\beta^n}{\alpha - \beta},$$

where $A = b - a\beta$, $B = b - a\alpha$, α and β are the roots of the $x^2 - px - q = 0$. We note that the above numbers have been studied in the literature widely and extensively (see, for example, [4, 5, 11-15, 20]). Many researchers have given important properties for special type numbers and polynomials using the generating functions. For example, in [23] Shablya, Kruchinin and Shelupanov examined the properties of coefficients of a composition of ordinary generating functions. Using the notion of a composita, they obtained some new properties for this composition. Finally as an application, they showed how to get primality criteria for the Mersenne numbers, the Lucas numbers, the Pell-Lucas numbers, the Jacobsthal-Lucas numbers, and other. Ozdemir and Simsek [21] defined the two variables of Fibonacci-type polynomials by the following generating function

$$H(t;x,y;k,m,n) = \sum_{n=0}^{\infty} \mathcal{G}_j(x,y;k,m,n)t^j = \frac{1}{1 - x^k t - y^m t^{m+n}}, \tag{1.4}$$

where $k, m, n \in \mathbb{N}_0$. They obtained some important properties of these polynomials. Moreover, they gave relationships between the Fibonacci, Jacobsthal, Chebyshev polynomials and the other well known polynomials. For more information related to special numbers and polynomials, we refer to [19, 21, 22].

Now, we talk about some notations related to q-calculus. For $x \in \mathbb{N}_0$, the q–integer $[x]_q$ is defined as follows:

$$[x]_q = \frac{1-q^x}{1-q} = 1 + q + q^2 + \dots + q^{x-1}. \tag{1.5}$$

From (1.5), for all $x, y \in \mathbb{Z}$, we get

$$[x+y]_q = [x]_q + q^x [y]_q.$$

Many of the papers on quantum (q-) calculus which has been provided by the mathematicians in the past for the purpose of application to the branches of mathematics such as combinatorics, differential equations as well as in other areas in physics, probability theory and so on. For further information, we specially refer to books in [2, 16].

Several generalizations of the well-known sedenions such as Fibonacci sedenions, Lucas sedenions, k-Pell and k-Pell-Lucas sedenions, Jacobshtal and Jacobshtal-Lucas sedenions, and so on have been studied by several researchers. For example, in [3], the authors defined the Fibonaccci and Lucas sedenions

$$\hat{F}_n = \sum_{s=0}^{15} F_{n+s}\mathbf{e}_s,$$

and

$$\hat{L}_n = \sum_{s=0}^{15} L_{n+s}\mathbf{e}_s.$$

Then they also obtained the generating functions, Binet-Like formulas and some interesting identities related to Fibonacci and Lucas sedenions. For more detailed information, please refer to the closely related to the paper [3, 6, 8. 10, 17].

Akkus and Kizilaslan [17], defined a more general quaternion sequence by receiving components from complex sequences. Then, they gave some properties and identities related to these quaternions. In [18] Kızılateş defined the another generalization of hybrid numbers which called the q-Fibonacci hybrid numbers and q-Lucas hybrid numbers. Moreover, the author gave some important algebraic properties of these numbers.

In this paper, by the help of the q-integers, we define here a new family of sedenions called q-Fibonacci sedenions and q-Lucas sedenions. We will obtain some algebraic properties of q-Fibonacci sedenions and q-Lucas sedenions.

2. Main Results

In this part of the our paper, we will introduce the q-Fibonacci sedenions and the q-Lucas sedenions. Then we also give some algebraic properties of these sedenions. Throughout the paper, we take $n \in \mathbb{N}$ and $1-q \neq 0$ is a real number.

Definition 2.1 : The q-Fibonacci sedenions and the q-Lucas sedenions are defined by the following:

$$\mathbb{SF}_n(\alpha;q) = \sum_{s=0}^{15} \alpha^{n+s-1}[n+s]_q \mathbf{e}_s, \tag{2.1}$$

and

$$\mathbb{SL}_n(\alpha;q) = \sum_{s=0}^{15} \alpha^{n+s} \frac{[2(n+s)]_q}{[n+s]_q} \mathbf{e}_s. \tag{2.2}$$

Table 1

Special cases for *q*-Fibonacci Sedenions and *q*-Fibonacci sedenions

α	q	q-Fibonacci Sedenions	q-Lucas sedenions
$\frac{1+\sqrt{5}}{2}$	$\frac{-1}{\alpha^2}$	Fibonacci sedenion; SF_n	Lucas sedenion; SL_n
$1+\sqrt{2}$	$\frac{-1}{\alpha^2}$	Pell sedenion; SP_n	Pell-Lucas sedenion; SPL_n
$\frac{k+\sqrt{k^2+4}}{2}$	$\frac{-1}{\alpha^2}$	k-Fibonacci sedenion; $SF_{k,n}$	k-Lucas sedenion; $SL_{k,n}$
2	$\frac{-1}{\alpha^2}$	Jacobsthal sedenion; SJ_n	Jacobsthal-Lucas sedenion; Sj_n
$1+\sqrt{1+k}$	$\frac{-k}{\alpha^2}$	k-Pell sedenion; $SP_{k,n}$	k-Pell-Lucas sedenion; $SPL_{k,n}$

Some special cases of q-Fibonacci sedenions $\mathbb{SF}_n(\alpha;q)$ or shortly $\mathbb{SF}_n$ and q-Lucas sedenions $\mathbb{SL}_n(\alpha;q)$ or shortly $\mathbb{SL}_n$ are given as in Table 1.

Theorem 2.2 : *The Binet formulas for the q-Fibonacci sedenions* $\mathbb{SF}_n$ *and the q-Lucas sedenions* $\mathbb{SL}_n$ *are*

$$\mathbb{SF}_n = \frac{\alpha^n \tilde{\alpha} - (\alpha q)^n \tilde{\beta}}{\alpha(1-q)}, \tag{2.3}$$

and

$$\mathbb{SL}_n = \alpha^n \tilde{\alpha} + (\alpha q)^n \tilde{\beta}, \tag{2.4}$$

where $\tilde{\alpha} = \sum_{s=0}^{15} \alpha^s \mathbf{e}_s$ and $\tilde{\beta} = \sum_{s=0}^{15} \beta^s \mathbf{e}_s$.

Proof : Owing to (2.1) and (1.5), we find that

$$\begin{aligned}
\mathbb{SF}_n &= \sum_{s=0}^{15} \alpha^{n+s-1} (n+s)_q \mathbf{e}_s \\
&= \alpha^{n-1}(n)_q \mathbf{e}_0 + \alpha^n (n+1)_q \mathbf{e}_1 + \ldots + \alpha^{n+14}(n+15)_q \mathbf{e}_{15} \\
&= \alpha^{n-1}\left(\frac{1-q^n}{1-q}\right)\mathbf{e}_0 + \alpha^n\left(\frac{1-q^{n+1}}{1-q}\right)\mathbf{e}_1 + \ldots + \alpha^{n+14}\left(\frac{1-q^{n+15}}{1-q}\right)\mathbf{e}_{15}
\end{aligned}$$

$$=\frac{1}{\alpha(1-q)}((\alpha^n-(\alpha q)^n)\mathbf{e}_0+(\alpha^{n+1}-(\alpha q)^{n+1})\mathbf{e}_1+\ldots+(\alpha^{n+15}-(\alpha q)^{n+15})\mathbf{e}_{15})$$
$$=\frac{1}{\alpha(1-q)}(\alpha^n(\mathbf{e}_0+\alpha\mathbf{e}_1+\ldots+\alpha^{15}\mathbf{e}_{15})-(\alpha q)^n(\mathbf{e}_0+(\alpha q)\mathbf{e}_1+\ldots+(\alpha q)^{15}\mathbf{e}_{15})$$
$$=\frac{\alpha^n\tilde{\alpha}-(\alpha q)^n\tilde{\beta}}{\alpha(1-q)}.$$

Similarly, equality (2.4) can be obtained. □

Theorem 2.3 : The exponential generating functions for the q-Fibonacci sedenions and q-Lucas sedenions are

$$\sum_{n=0}^{\infty}\mathbb{SF}_n\frac{x^n}{n!}=\frac{\tilde{\alpha}e^{\alpha x}-\tilde{\beta}e^{\alpha q x}}{\alpha(1-q)}, \tag{2.5}$$

and

$$\sum_{n=0}^{\infty}\mathbb{SL}_n\frac{x^n}{n!}=\tilde{\alpha}e^{\alpha x}+\tilde{\beta}e^{\alpha q x}. \tag{2.6}$$

Proof : From the Binet formula for the q-Fibonacci sedenions, we obtain

$$\sum_{n=0}^{\infty}\mathbb{SF}_n\frac{x^n}{n!}=\sum_{n=0}^{\infty}\left(\frac{\alpha^n\tilde{\alpha}-(\alpha q)^n\tilde{\beta}}{\alpha(1-q)}\right)\frac{x^n}{n!}$$
$$=\frac{\tilde{\alpha}}{\alpha(1-q)}\sum_{n=0}^{\infty}\frac{(\alpha x)^n}{n!}-\frac{\tilde{\beta}}{\alpha(1-q)}\sum_{n=0}^{\infty}\frac{(\alpha q x)^n}{n!}$$
$$=\frac{\tilde{\alpha}e^{\alpha x}-\tilde{\beta}e^{\alpha q x}}{\alpha(1-q)}.$$

Equality (2.6) can be similarly derived. □

Theorem 2.4 : For $n,k\geq 0$, we have

$$\sum_{i=0}^{n}\binom{n}{i}(-\alpha^2 q)^{n-i}\mathbb{SF}_{2i+k}=\begin{cases}\Delta^{\frac{n}{2}}\mathbb{SF}_{n+k} & \text{if } n \text{ is even,}\\ \Delta^{\frac{n-1}{2}}\mathbb{SL}_{n+k} & \text{if } n \text{ is odd,}\end{cases} \tag{2.7}$$

$$\sum_{i=0}^{n}\binom{n}{i}(-\alpha^2 q)^{n-i}\mathbb{SL}_{2i+k}=\begin{cases}\Delta^{\frac{n}{2}}\mathbb{SL}_{n+k} & \text{if } n \text{ is even,}\\ \Delta^{\frac{n+1}{2}}\mathbb{SF}_{n+k} & \text{if } n \text{ is odd,}\end{cases} \tag{2.8}$$

$$\sum_{i=0}^{n}\binom{n}{i}(-1)^{i}(-\alpha^{2}q)^{n-i}\,\mathbb{SF}_{2i+k}=\left(-\alpha[2]_{q}\right)^{n}\mathbb{SF}_{n+k}, \tag{2.9}$$

$$\sum_{i=0}^{n}\binom{n}{i}\left(-1\right)^{i}(-\alpha^{2}q)^{n-i}\,\mathbb{SL}_{2i+k}=(-\alpha[2]_{q})^{n}\,\mathbb{SL}_{n+k}. \tag{2.10}$$

where $\Delta=(\alpha-\alpha q)^{2}$.

Proof : Let's first prove the equality (2.7). Similarly (2.8), (2.9) and (2.10) can be obtained. From Binet formula (2.5), we find that

$$\begin{aligned}\sum_{i=0}^{n}\binom{n}{i}(-\alpha^{2}q)^{n-i}\,\mathbb{SF}_{2i+k}&=\sum_{i=0}^{n}\binom{n}{i}(-\alpha^{2}q)^{n-i}\,\frac{\alpha^{2i+k}\tilde{\alpha}-(\alpha q)^{2i+k}\tilde{\beta}}{\alpha(1-q)}\\&=\frac{1}{\alpha-\alpha q}(\alpha^{2}-\alpha^{2}q)^{n}\alpha^{k}\tilde{\alpha}-(\alpha^{2}q^{2}-\alpha^{2}q)^{n}(\alpha q)^{k}\tilde{\beta}\\&=\frac{(\alpha\sqrt{\Delta})^{n}\alpha^{k}\tilde{\alpha}-(-\alpha q\sqrt{\Delta})^{n}(\alpha q)^{k}\tilde{\beta}}{\alpha-\alpha q}.\end{aligned} \tag{2.11}$$

If n is even in (2.11), we have

$$\begin{aligned}\sum_{i=0}^{n}\binom{n}{i}(-\alpha^{2}q)^{n-i}\,\mathbb{SF}_{2i+k}&=\frac{(\alpha\sqrt{\Delta})^{n}\alpha^{k}\tilde{\alpha}-(\alpha q\sqrt{\Delta})^{n}(\alpha q)^{k}\tilde{\beta}}{\alpha-\alpha q}\\&=\sqrt{\Delta}^{\,n}\,\frac{\alpha^{n+k}\tilde{\alpha}-(\alpha q)^{n+k}\tilde{\beta}}{\alpha(1-q)}\\&=\Delta^{\frac{n}{2}}\mathbb{SF}_{n+k}.\end{aligned}$$

If n is odd in (2.12), we obtain

$$\begin{aligned}\sum_{i=0}^{n}\binom{n}{i}(-\alpha^{2}q)^{n-i}\,\mathbb{SF}_{2i+k}&=\frac{(\alpha\sqrt{\Delta})^{n}\alpha^{k}\tilde{\alpha}+(\alpha q\sqrt{\Delta})^{n}(\alpha q)^{k}\tilde{\beta}}{\alpha-\alpha q}\\&=\sqrt{\Delta}^{\,n-1}(\alpha^{n+k}\tilde{\alpha}+(\alpha q)^{n+k}\tilde{\beta})\\&=\Delta^{\frac{n-1}{2}}\,\mathbb{SL}_{n+k}.\end{aligned}$$

Thus the proof is finished. ☐

Theorem 2.5 : *For* $n\geq 0$, *we get*

$$\sum_{i=0}^{n}\binom{n}{i}(\alpha+\alpha q)^{i}(-\alpha^{2}q)^{n-i}\mathbb{SF}_{i}=\mathbb{SF}_{2n}, \tag{2.12}$$

$$\sum_{i=0}^{n}\binom{n}{i}(\alpha+\alpha q)^{i}(-\alpha^{2}q)^{n-i}\mathbb{SL}_{i}=\mathbb{SL}_{2n}. \tag{2.13}$$

Proof : Using the Binet formula for the the q-Fibonacci sedenions (2.3), we have

$$\begin{aligned}
&\sum_{i=0}^{n}\binom{n}{i}(\alpha+\alpha q)^{i}(-\alpha^{2}q)^{n-i}\mathbb{SF}_{i}\\
&=\sum_{i=0}^{n}\binom{n}{i}(\alpha+\alpha q)^{i}(-\alpha^{2}q)^{n-i}\frac{\alpha^{i}\tilde{\alpha}-(\alpha q)^{i}\tilde{\beta}}{\alpha(1-q)}\\
&=\sum_{i=0}^{n}\binom{n}{i}\frac{(\alpha+\alpha q)^{i}(-\alpha^{2}q)^{n-i}\alpha^{i}\tilde{\alpha}}{\alpha(1-q)}-\sum_{i=0}^{n}\binom{n}{i}\frac{(\alpha+\alpha q)^{i}(-\alpha^{2}q)^{n-i}(\alpha q)^{i}\tilde{\beta}}{\alpha(1-q)}\\
&=\frac{(\alpha^{2}(1+q)-\alpha^{2}q)^{n}\tilde{\alpha}}{\alpha(1-q)}-\frac{(\alpha^{2}q(1+q)-\alpha^{2}q)^{n}\tilde{\beta}}{\alpha(1-q)}\\
&=\mathbb{SF}_{2n}.
\end{aligned}$$

Equality (2.13) can be similarly derived. □

Theorem 2.6 : *(Catalan's Identity). For positive integers n and r, with $n\geq r$, then the following identities are true:*

$$\mathbb{SF}_{n+r}\mathbb{SF}_{n-r}-\mathbb{SF}_{n}^{2}=\frac{\alpha^{2n-2}q^{n}(1-q^{r})(\tilde{\beta}\tilde{\alpha}-q^{-r}\tilde{\alpha}\tilde{\beta})}{(1-q)^{2}}, \tag{2.14}$$

$$\mathbb{SL}_{n+r}\mathbb{SL}_{n-r}-\mathbb{SL}_{n}^{2}=\alpha^{2n}q^{n-r}(1-q^{r})(\tilde{\alpha}\tilde{\beta}-q^{r}\tilde{\beta}\tilde{\alpha}). \tag{2.15}$$

Proof : From the Binet formula of the q-Fibonacci sedenions, we have the LHS of the equality (2.14),

$$\begin{aligned}
&\mathbb{SF}_{n+r}\mathbb{SF}_{n-r}-\mathbb{SF}_{n}^{2}\\
&=\left(\frac{\alpha^{n+r}\tilde{\alpha}-(\alpha q)^{n+r}\tilde{\beta}}{\alpha(1-q)}\right)\left(\frac{\alpha^{n-r}\tilde{\alpha}-(\alpha q)^{n-r}\tilde{\beta}}{\alpha(1-q)}\right)-\left(\frac{\alpha^{n}\tilde{\alpha}-(\alpha q)^{n}\tilde{\beta}}{\alpha(1-q)}\right)^{2}.
\end{aligned}$$

After some calculations, we get

$$\mathbb{SF}_{n+r}\mathbb{SF}_{n-r}-\mathbb{SF}_n^2=\frac{\alpha^{2n-2}q^n(1-q^r)(\tilde{\beta}\tilde{\alpha}-q^{-r}\tilde{\alpha}\tilde{\beta})}{(1-q)^2}.$$

The result (2.15) can be similarly obtained.

Theorem 2.7 : *(Cassini's Identity). For $n\geq 1$, the following equalities hold:*

$$\mathbb{SF}_{n+1}\mathbb{SF}_{n-1}-\mathbb{SF}_n^2=\frac{\alpha^{2n-2}q^n(\tilde{\beta}\tilde{\alpha}-q^{-1}\tilde{\alpha}\tilde{\beta})}{(1-q)}, \tag{2.16}$$

$$\mathbb{SL}_{n+1}\mathbb{SL}_{n-1}-\mathbb{SL}_n^2=\alpha^{2n}q^{n-1}(1-q)(\tilde{\alpha}\tilde{\beta}-q\tilde{\beta}\tilde{\alpha}). \tag{2.17}$$

Proof : If we take $r=1$, in (2.14) and (2.15), we obtain the assertions of the theorem. □

Theorem 2.8 : *(d'Ocagne's Identity). Suppose that n is a non-negative integer number and m natural number. If $m>n+1$, then the expression of the d'Ocagne's identities are given by the following:*

$$\mathbb{SF}_m\mathbb{SF}_{n+1}-\mathbb{SF}_{m+1}\mathbb{SF}_n=\frac{\alpha^{m+n-1}(q^n\tilde{\alpha}\tilde{\beta}-q^m\tilde{\beta}\tilde{\alpha})}{(1-q)}, \tag{2.18}$$

$$\mathbb{SL}_m\mathbb{SL}_{n+1}-\mathbb{SL}_{m+1}\mathbb{SL}_n=\alpha^{m+n+1}(q-1)(q^n\tilde{\alpha}\tilde{\beta}-q^m\tilde{\beta}\tilde{\alpha}). \tag{2.19}$$

Proof : Using the Binet formula of the q-Fibonacci sedenions, we obtain

$$\begin{aligned}\mathbb{SF}_m\mathbb{SF}_{n+1}-\mathbb{SF}_{m+1}\mathbb{SF}_n&=\frac{\alpha^m\tilde{\alpha}-(\alpha q)^m\tilde{\beta}}{\alpha(1-q)}\frac{\alpha^{n+1}\tilde{\alpha}-(\alpha q)^{n+1}\tilde{\beta}}{\alpha(1-q)}\\&\quad-\frac{\alpha^{m+1}\tilde{\alpha}-(\alpha q)^{m+1}\tilde{\beta}}{\alpha(1-q)}\frac{\alpha^n\tilde{\alpha}-(\alpha q)^n\tilde{\beta}}{\alpha(1-q)}.\end{aligned}$$

Thus

$$\mathbb{SF}_m\mathbb{SF}_{n+1}-\mathbb{SF}_{m+1}\mathbb{SF}_n=\frac{\alpha^{m+n-1}(q^n\tilde{\alpha}\tilde{\beta}-q^m\tilde{\beta}\tilde{\alpha})}{(1-q)}.$$

Equality (2.19) can be similarly derived. □

Remark 2.9 : This paper is a slightly corrected and revised version of the electronic preprint [24].

3. Conclusion and Discussion

In our recent study, we have introduced and dealt with q-Fibonacci sedenions and q-Lucas sedenions which are defined by means of the q-integer. Utilizing the Binet-like formulas and some properties of q-Fibonacci sedenions and q-Lucas sedenions, we have derived exponential generating functions, summation formulas, Cassini's identities, Catalan's identities, and d'Ocagne's identities. Depending on the special values of α and q, all the results given in Section 2 are true for all Fibonacci-type sedenions and Lucas-type sedenions stated in Table 1. On the other hand, a trigintaduonion is defined by

$$t = a_0 + \sum_{i=0}^{31} a_i \mathbf{e}_i,$$

where $\{a_i\}$, $i = 0, 1, \ldots, 31$ are real numbers and $\{\mathbf{e}_i\}$, $i = 0, 1, \ldots, 31$ are imaginary units. Detailed information about the trigintaduonion have been presented in the literature (for example, see [7]). Indeed, for the interested readers of this work, results presented here have the potential to motivate further researches of the subject of the q-Fibonacci Trigintaduonions and q-Lucas Trigintaduonions.

References

[1] Akkus I., Kizilaslan G. Quaternions: Quantum calculus approach with applications, *Kuwait Journal of Science,* 46 (4), (2019) 1-13.

[2] Andrews GE, Askey R, Roy R. Special functions. Cambridge Univ. Press, New York, (1999).

[3] Bilgici G., Tokeser U., Unal Z. Fibonacci and Lucas Sedenions, *J. Integer Seq.,* Vol. 20, Article 17.1.8, (2017) 1-11.

[4] Catarino P. On some identities and generating functions for *k*-Pell numbers, *Int. J. Math. Anal.,* 7 (38), (2013) 1877-1884.

[5] Catarino P., Vasco P. On Some Identities and Generating Functions for k-Pell-Lucas Sequence, *Appl. Math. Sci.,* 7 (98), (2013) 4867-4873.

[6] Catarino P. *k*-Pell, k-Pell-Lucas and modified *k*-Pell sedenions. *Asian-Eur. J. Math.,* 12 (1), (2019) 1-10.

[7] Cawagas R. E., Carrascal A. S., Bautista L. A., Maria J. P. Sta., Urrutia J. D., Nobles, B. The Basic Subalgebra Structure Of The Cayley-Dickson

Algebra Of Dimension 32 (Trigintaduonions), arXiv:0907.2047v3, (2009).

[8] Cimen C., Ipek A. On Jacobsthal and Jacobsthal-Lucas Sedenios and Several Identities Involving These Numbers. Math. Aeterna, 7, (2017) 447-454.

[9] Cimen C.B., Ipek A. On Jacobsthal and Jacobsthal--Lucas Octonions, Mediterr. J. Math., 14(37), (2017) 1-13.

[10] Çimen C., Ipek A. Çimen E. On Horadam Sedenions, *Journal of Science and Arts*, 4, (49), (2019) 887--898.

[11] Falcon S., Plaza A. On the Fibonacci k-Numbers, Chaos, Solitons Fractals, 32 (5), (2007) 1615-1624.

[12] Falcon S. On the k-Lucas numbers, *Int. J. Contemp. Math. Sci*., 6 (21), (2011) 1039-1050.

[13] Horadam A.F. Basic properties of a certain generalized sequence of numbers, Fibonacci Quart., 3, (1965) 161-176.

[14] Horadam A.F. Generating functions for powers of a certain generalized sequence of numbers, *Duke Math. J.*, 32, (1965) 437-446.

[15] Horzum T. Kocer E.G. On Some Properties of Horadam Polynomials, *Int. Math. Forum*, 4, 25, (2009) 1243-1252.

[16] Kac V, Cheung P. Quantum Calculus Universitext, Springer-Verlag, New York, (2002).

[17] Kızılateş C., Polatl E. New Families of Fibonacci and Lucas Octonions With Q-Integer Components, *Indian J. Pure Appl. Math.*, 52, (2021) 231-240.

[18] Kızılateş C. A new generalization of Fibonacci hybrid and Lucas hybrid numbers, Chaos Solitons Fractals, 130, (2020) 1-5.

[19] Kızılateş C., Cekim B., Tuglu N., Kim T. New families of three-variable polynomials coupled with well-known polynomials and numbers. Symmetry, 11(264), (2019) 1-13.

[20] Kocer E.G., Tuglu, N., Stakhov A. On the m-extension of the Fibonacci and Lucas p-numbers, Chaos Solitons Fractals, 40 (4), (2009) 1890-1906.

[21] Ozdemir G., Simsek Y. Generating functions for two-variable polynomials related to a family of Fibonacci type polynomials and numbers. Filomat, 30(4), (2016) 969-975.

[22] Ozdemir G., Simsek Y., Milovanović G. V. Generating functions for special polynomials and numbers including Apostol-type and Humbert-type polynomials. *Mediterr. J. Math.*, 14(117), (2017) 1-17.

[23] Shablya Y. V., Dmitry V. K., Shelupanov A. A. New properties of a composition of ordinary generating functions for primes, *J. Discrete Math. Sci. Cryptography*, 24(4), (2021) 917-930.

[24] Kızılateş C., Kırlak S. A New Generalization of Fibonacci and Lucas Type Sedenions. Preprints, (2020).

Received May, 2021

Revised October, 2021