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On β*-supra topological spaces

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Abstract

In this paper, a new type of supra closed sets is introduced which we called supra β^*-closed sets in a supra topological space. A new set of separation axioms is defined, and its many properties are examined. The relationships between supra β^*-T_i–spaces $(i = 0, 1, 2)$ are studied and shown with instances. Additionally, new varieties of supra β^*-continuous maps have been taken into consideration based on the supra β^*-open sets theory.

Subject Classification: *54A05, 18B30.*

Keywords: *Supra space, Supra open, Supra g- open set, Supra continuous maps, Mildly closed set.*

1. Introduction

In 1983, A. S. Mashhour [1] investigated S-continuous functions and introduced the supra topological spaces. After Mashhour many researchers have worked and generalized many concepts related to supra topological spaces [for instance see[2,3,4] Recall that a subcollection τ^* of P(X̡) is called a supra topology on X̡ if X̡ belongs to τ^* and τ^* is closed under arbitrary union. Each component of τ^* is known as a supra open, whereas its complement is known as a supra closed. [5].

In what follows we mean by a supra space (X̡, τ^*) (resp. int$^{\tau^*}$(A̡),cl$^{\tau^*}$(A̡)) a supra topological space (resp. supra int (A̡), supra cl(A̡)), where A̡ is a subset of X̡. A subset A̡ of a supra.space (X̡, τ^*) is termed supra generalized closed in brief

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(supra g-closed) if $cl^{\tau^*}(A) \subset W$ wherever $A \subset W$ and W is supra open [6]. Supra g -open is the complement of supra g-closed.

As a generalization to a g-closed set in ordinary topology, J. K. Park and J. H. Park [7] presented the following for the idea of mildly closed sets : A subset A of the topological space (X, τ) is termed a mildly closed set if $cl(int(A)) \subset U$, wherever $A \subset U$ and U is g- open. In this paper, we have developed the concept of mildly closed sets via supra spaces and we named it supra β^*-closed sets. Some important properties were proved. Further, we introduced the supra β^*-separation axioms in particular we studied supra β^*-T_i (i= 0,1,2) and their relationships as well as many examples to illustrate were given.

Finally, the symbols SgO(X)and SgC(X) means all supra g-open and supra g-closed subsets in supra space (X, τ^*) respectively.

2 - supra β^*-closed sets.

This section introduces the concept of supra β^*-closed sets . We begin with the following definition.

Definition 2.1: Let A be a subset of a supra space (X,τ^*) then A is called supra β^*-closed , if $(cl^{\tau^*}(int^{\tau^*}(A)) \subset W$, wherever W is supra-g-open and $A \subset W$. supra β^*-open is the complement of supra β^*-closed. $S\beta^*O(X)$ (resp. $S\beta^*C(X)$) will be used to denote supra β^*-open in X (rep. supra β^*-closed).

Example 2.2: Let $X = \{ w_1 , w_2, w_3 \}$ and let $\tau^* = \{ X , \emptyset , \{ w_1\} ,\{ w_1, w_2\}, \{ w_2, w_3\} \}$ be an X supra topology , so $S\beta^*C = \{ \emptyset , \{ w_2, w_3\} , \{ w_3\} , \{ w_1\} , \{ w_1, w_3\}, X , \{ w_2\} \}$,hence $S\beta^*O = \{ \{ w_2\}, \{ w_3\} , \{w_1, w_2\} , X ,\{w_2 ,w_3\} , \emptyset ,\{w_1,w_3\} \}$.

Remark 2.3:

(1) Every set that is supra g-closed is also supra β^*-closed.

(2) Every set that is supra closed is also supra β^*-closed.

Proof: (1) Let $A \in SgC(X)$ and U be a supra g-open that contains A in X , then $cl^{\tau^*}(A) \subset U$. Hence $cl^{\tau^*}(int^{\tau^*}(A)) \subset cl^{\tau^*}(A) \subset U$, implies A is supra β^*-closed.
(2) Follows immediately from (1) since any supra closed is also a supra g-closed [8]. The opposite of the preceding statement is not correct, as shown by the example that follows.

Example 2.4: Assume $X = \{w_1, w_2,w_3\}$ and $\tau^* = \{ X , \emptyset , \{ w_1\} , \{ w_1, w_2\} , \{ w_2, w_3\} \}$ be an X supra topology . Now that supra g-closed $=\{ \emptyset , X , \{ w_2, w_3\} , \{ w_3\} , \{ w_1\} , \{ w_1, w_3\} \}$ and supra g-open $=\{ X , \emptyset , \{ w_1 \} , \{ w_1, \omega_2\} , \{ w_2, w_3\} ,\{ w_2\} \}$ .The collection of all supra β^*-closed is the family $\{ \emptyset , X , \{ w_2, w_3\} , \{ w_3\} , \{ w_1\} , \{ w_1,w_3\} ,\{ w_2 \} \}$,implies $S\beta^*O =\{ \{ w_2\} , \{\omega_1\} ,\emptyset , \{ w_1, w_2\} , \{ w_2, w_3\}, X ,\{ w_1, w_3\} \}$. In this example, it is clear that the collections of supra open,SgO (X)and $S\beta^*O(X)$ are not identical .

The following proposition explain that the family of supra β^*-closed sets satisfies a useful property.

Proposition 2.5: Intersection of any collection supra β^*-closed sets is a supra β^*-closed.

Proof: Assume that a family of supra β^*-closed sets is$\{ E_i; i\in I \}$, we want to show $\cap_{i\in I} E_i$ is supra β^*-closed. Now $\cap_{i\in I} E_i \subset E_i$, so $cl^{\tau^*}($ int $\cap_{i\in I} E_i))\subset cl^{\tau^*}(int^{\tau^*}(E_i))$ [9] and sinceE_i is supra β^*-closed for all i , implies for each $u_i \in$ SgO(X) such that $E_i \subset u_i$, $cl^{\tau^*}(int^{\tau^*}(u_i))\subset u_i$. Hence for each supra g-open containing $\cap_{i\in I} E_i$ must contain $cl^{\tau^*}(int^{\tau^*}(\cap_{i\in I} E_i)$ which complete the proof.

Theorem 2.6: *A subset A of supra.space (X,τ^*) is supra β^*-closed if $cl^{\tau^*}(int^{\tau^*}(A))$ - A contains no non-empty supra closed.*

Proof: Assume that $W \subseteq cl^{\tau^*}(int^{\tau^*}(A)) - A$ where W is nonempty supra closed , implies $W \subseteq cl^{\tau^*}(int^{\tau^*}(A)) \cap A^c$, which leads to $A \subseteq W^c$. Hence W^c is supra open as well as supra g-open [8] that contains A.But A is supra β^*-closed ,so we have $cl^{\tau^*}(int^{\tau^*}(A)) \subseteq W^c$ implies $W\subseteq cl^{\tau^*}(int^{\tau^*}(A)) \cap (cl^{\tau^*}(int^{\tau^*}(A)))^c = \emptyset$ which is impossible . As a result, $cl^{\tau^*}(int^{\tau^*}(A))$ -A includes no nonempty supra closed set.

Many supra g-closed subsets in supra.space satisfy $cl^{\tau^*}(A\cup B)=cl^{\tau^*}(A)\cup cl^{\tau^*}(B)$ as illustrated in the following example.

Example 2.7: Soppose $X= \{w_1,w_2,w_3,w_4\}$ with supra topology $\tau^*= \{ X, \emptyset, \{w_1,w_2\}, \{w_3,w_4\}, \{w_2,w_3,w_4\} \}$. If $A = \{w_1,w_2\}$, $B = \{w_3, w_4\}$. It is simple to demonstrate that A and B are both supra g-closed and $cl^{\tau^*}(A \cup B) = cl^{\tau^*}(A) \cup cl^{\tau^*}(B)$.

We mean by a supra g-open subset A has a property Γ if $A\cap G$ is supra g-open for each supra open set G in a supra.space (X,τ^*). We have the following theorem.

Theorem 2.8: *Consider the following: $B \subseteq A \subseteq X$, B is supra β^*closed set relative to A and A is supra β^*closed subset of a supra space (X,τ^*) which has property Γ. Then B is supra β^*closed set relative to X.*

Proof: Assume $B \subseteq G$ where G be a supra g-open in X. Now we have $B \subseteq A \subseteq X$ so $A \subseteq B$ also $B \subseteq G$ and hence $B \subseteq A \cap G$ .Where B is supra β^*-closed set relative to A, hence $cl^{\tau^*}(int^{\tau^*}(B)) \subseteq A \cap G$ which leads to $A \cap cl^{\tau^*}(int^{\tau^*}(B)) \subseteq A \cap G$ implies $A \cap cl^{\tau^*}(int^{\tau^*}(B)) \subseteq G$ .Thus $A\cap cl^{\tau^*}(int^{\tau^*}(B)) \cup [cl^{\tau^*}(int^{\tau^*}(B))]^c \subseteq G \cup [cl^{\tau^*}(int^{\tau^*}(B))]^c$,implies $A \subseteq G \cup [cl^{\tau^*}(int^{\tau^*}(B))]^c$. Therefore A is supra β^*-closed in X . We now have $cl^{\tau^*}(int^{\tau^*}(A)) \subseteq G \cup [cl^{\tau^*}(int^{\tau^*}(B))]^c$. Since $B \subseteq A \subseteq [cl^{\tau^*}(int^{\tau^*}(B))]^c$ .Therefor $cl^{\tau^*}(int^{\tau^*}(B))\subseteq G$,since $cl^{\tau^*}(int^{\tau^*}(B))$ does not contain $[cl^{\tau^*}(int^{\tau^*}(B))]^c$ .Hence B is supra β^*closed set relative to X.

3. Supra β^*-Separations Axioms:-

We introduce new classes of supra β^*-separation axioms and investigate their characterizations in this section.

Definition 3.1: Asupra space (X, τ^*) is also known as supra β^*- T_0 ($S\beta^*$- T_0)if for all $\omega_1 \neq \omega_2 \in X$. If $U \in S\beta^*O(X)$, then ω_1 U and $\omega_2 \notin U$.

Example 3.2: Let $X=\{w_1, w_2, w_3, w_4\}$ with supra topology $\tau^*=\{X, \emptyset, \{w_1,w_2\}, \{w_2, w_3, w_4\}\}$, so $S\beta^*C=\{X, \emptyset, \{w_3, w_4\}, \{w_1\}, \{w_3\}, \{w_4\}, \{w_2\}, \{w_1, w_3, w_4\}, \{w_1, w_3\}, \{w_1,w_4\}, \{w_2,w_4\}, \{w_2,w_3\}\}$, hence $S\beta^*O=\{X, \emptyset, \{w_1,w_2\}, \{w_2,w_3, w_4\}, \{w_1, w_2,w_4\}, \{w_1,w_3, w_4\}, \{w_1,w_2,w_3\}, \{w_2,w_4\}, \{w_2,w_3\}, \{w_1, w_3\}, \{w_1,w_4\}\}$. It is not difficult to show that (X, τ^*) is not supra T_0-space, while it is $S\beta^*$- T_0-space .

Remark 3.3: All supra T_0-spaces are supra β^*-T_0-spaces , the opposite of the preceding is not correct, as shown by the example that follows .

Example 3.4: Let $X=\{w_1,w_2,w_3\}$ with supra topology $\tau^*=\{X, \emptyset, \{w_1\}, \{w_1, w_2\}, \{w_2, w_3\}\}$, $\tau^{*c}=\{\emptyset, X, \{w_2, w_3\}, \{w_3\}, \{w_1\}\}$. Thus $S\beta^*C=\{\emptyset, X, \{w_2, w_3\}, \{w_3\}, \{w_1\}, \{w_1, w_3\}, \{w_3\}\}$,and $S\beta^*O=\{X, \emptyset, \{w_1\}, \{w_1, w_2\}, \{w_2, w_3\}, \{w_2\}, \{w_1, w_3\}\}$.

Definition 3.5: Considering (X, τ), (X^*, τ^*) two supra spaces, then a map $f:(X,\tau)\longrightarrow(X^*,\tau^*)$ is called

(1) Supra β^*-open map if each supra β^*-open in X image is supra β^*-open in X^*

(2) Supra β^*-Continuous: if $f^{-1}(A) \in S\beta^*O(X^*)$, then A is a supra β^*-open set in$X$.

Theorem 3.6: *Consider the following: $f:(X,\tau)\longrightarrow(X^*,\tau^*)$ be a bijective, supra β^*-open map, if (X,τ) is $S\beta^*$- T_0, then (X^*,τ^*) is $S\beta^*$- T_0.*

Proof: Assume (X,τ) be a $S\beta^*$- T_0. Next, we must demonstrate that (X^*,τ^*) is $S\beta^*$- T_0. Suppose $\omega_1^* \neq \omega_2^* \in X^*$ and because f be a bijective map , then there are $\omega_1 \neq \omega_1 \in X$ implies that $\omega_1^*=f(\omega_1)$, $\omega_2^*=f(\omega_2)$ and since (X, τ) is $S\beta^*$- T_0. there exists a supra β^*open set G in X implies that $\omega_1 \in G$, $\omega_2 \notin G$. Hence $f(G) \subseteq X^*$ is a supra β^*-open set, which contains ω_1^*. It's obvious $\omega_2^* \notin f(G)$, (X^*,τ^*) is thus $S\beta^*$- T_0.

Theorem 3.7: *Let (X,τ) and (X^*,τ^*) be two supra.spaces, where (X^*,τ^*) is $S\beta^*$- T_0, let $f:(X,\tau) \longrightarrow (X^*,\tau^*)$ is a supra β^*-continuous bijective map, then (X, τ) is $S\beta^*$- T_0.*

Proof: Assume $\omega_1 \neq \omega_2 \in X$ and ever since f be a bijective map then there exists $\omega_1^* \neq \omega_2^* \in X^*$, implies $\omega_1^*=f(\omega_1)$, $\omega_2^*=f(\omega_2)$. Since (X^*,τ^*) is $S\beta^*$- T_0 , since G

is a supra β^*-open set in X^* exists such that $\omega_1^* \in G$, $\omega_2^* \notin G$. Now f is a supra β^*-continuous map, so $f^{-1}(G)$ is a supra β^*-open set in X that contains ω_1 but does not contain ω_2, which completes the proof.

Definition 3.8: A supra.space (X, τ^*) is also known as supra β^*-T_1-space $(S\beta^*$-$T_1)$ if for every $\omega_1 \neq \omega_2 \in X$, there exist $G, H \in S\beta^*O$ implies $\omega_1 \in G - H$ and $\omega_2 \in H - G$.

Example 3.9: Let $X = \{w_1, w_2, w_3, w_4\}$ with supra topology $\tau^* = \{X, \emptyset, \{w_1, w_2\}, \{w_2, w_3, w_4\}\}$, so $S\beta^*O = \{X, \emptyset, \{w_1, w_2\}, \{w_2, w_3, w_4\}, \{w_1, w_2, w_4\}, \{w_1, w_3, w_4\}, \{w_1, w_2, w_3,\}, \{w_2, w_3\}, \{w_2, w_3\}, \{w_1, w_3\}, \{w_1 w_4\}\}$, and $S\beta^*C = \{\{w_2\ w_4,\}, X, \{w_2\}, \emptyset, \{w_3, w_4\}, \{w_1\}, \{w_3\}, \{w_4\}, \{w_1, w_3, w_4\}, \{w_1, w_3\}, \{w_1, w_4\}, \{w_2, w_3\}\}$. its clear that (X, τ^*) is not supra T_1-space,white it is $S\beta^*$-T_1-space. The following theorem will be given a characterization for a supra space to be $S\beta^*$- T_1.

Theorem 3.10: *A supra.space (X, τ^*) is $S\beta^*$- T_1 if and only if $\{w\}$ is supra β^*-closed set, for all $w \in X$.*

Proof: Assume (X, τ^*) is a supra.space . We want to show $\{w\}^c$ is supra β^*-open in X. Assume $\omega \in \{w\}^c$ and by (3.8) there exists a supra β^*-neighborhood of a, G_ω does not include w. Hence $\omega \in G_\omega \subseteq \{w\}^c$, thus $\{w\}^c = \{G_\omega, \omega \in \{w\}^c\}$ implies $\{w\}^c$ is the union every supra β^*-open sets so by (2.5) $\{w\}^c$ is supra β^*-open. Hence $\{w\}$ is supra β^*-closed. Assume, on the other hand, $\{w\}$ is supra β^*-closed for all $w \in X$ and let $\omega_1 \neq \omega_2 \in X$, hence $\omega_1 \in \{\omega_2\}^c$, $\omega_2 \in \{\omega_1\}^c \in S\beta^*O$, implies (X, τ^*) is $S\beta^*$- T_1.

Theorem 3.11: *Let $f: (X, \tau) \longrightarrow (X^*, \tau^*)$ be a bijective, supra β^*-open map .Then (X^*, τ^*) is $S\beta^*$- T_1 if (X, τ) is $S\beta^*$- T_1.*

Proof: Assume (X, τ) is $S\beta^*$-T_1 . We must now demonstrate (X^*, τ^*) is $S\beta^*$- T_1. Let $\omega_1^*, \omega_2^* \in X$ where $\omega_1^* \neq \omega_2^*$ and since f is bijective map , let $\omega_1 \neq \omega_2 \in X$, implies $\omega_1^* = f(\omega_1)$,$\omega_2^* = f(\omega_2)$ .But (X, τ) is $S\beta^*$- T_1, there exist $G, H \in S\beta^*O$, such that $\omega_1 \in G$, $\omega_1 \notin H$ and $\omega_2 \notin G$,$\omega_2 \in H$,then we have $f(G), f(H) \in S\beta^*O$, implies $\omega_1^* \in f(G)$, $\omega_1^* \notin f(H)$ and $\omega_2^* \notin f(G)$, $\omega_2^* \in f(H)$,so we have done. □

It is known [for instance see[10]] that every finite T_1- space is discrete space but in $S\beta^*$- T_1. It's not always accurate, as seen by the example that follows.

Example 3.12: Suppose $X = \{\omega_1, \omega_2, \omega_3\}$ and let $\tau^* = \{\ ,\{\omega_1\}, X, \{\omega_2\}, \{\omega_1, \omega_2\}, \emptyset, \{\omega_2, \omega_3\}\}$, $\tau^{*c} = \{\{\omega_1\}, X, \{\omega_3\}, \{\omega_2\}, \emptyset, \{\omega_2, \omega_3\}, \{\omega_1, \omega_3\}\}$. It is not difficult to compute $S\beta^*C = \{\{\omega_1\}, X, \{\omega_3\}, \emptyset, \{\omega_2\}, \{\omega_2, \omega_3\}, \{\omega_1, \omega_3\}\}$. Hence $S\beta^*O = \{X, \emptyset, \{\omega_1\}, \{\omega_2\}, \{\omega_1, \omega_2\}, \{\omega_2, \omega_3\}\}$, clear that this space is $S\beta^*$- T_1 but it is not discrete space.

Remark 3.13: Every Sβ^*- T_1-*space* is Sβ^*- T_0-*space*, however, the opposite is untrue. The following example shows this fact.

Example 3.14: Let Ҳ = {ω_1, ω_2}, τ^* = { Ҳ, Ø, {ω_1}}, τ^{*c}= {Ҳ, Ø, {ω_2}}. Hence supra β^*-open = {Ҳ , Ø, { ω_1}}.

Definition 3.15: A supra.space (Ҳ,τ^*) is also known as supra β^*-T_2-space (Sβ^*-T_2)if for each $\omega_1 \neq \omega_2$ in Ҳ, there exist Ģ, Ḩ∈ Sβ^*O(Ҳ) such that x ∈ Ģ, y∈ Ḩ and Ģ ∩ Ḩ =∅ .

Theorem 3.16: *Let f :(Ҳ, τ)⟶(Ҳ *,τ^*) be a bijective supra β^*-open map. If (Ҳ, τ) is Sβ^*- T_2, then (Ҳ *,τ^*) is Sβ^*- T_2.*

Proof: Assume (Ҳ,τ) is Sβ^*- T_2. We must prove this (Ҳ*,τ^*) is Sβ^*- T_1. Let ω_1^*, ω_2^*∈Ҳ* where $\omega_1^* \neq \omega_2^*$ and *by* assumption f is a bijective map . There exists $\omega_1 \neq \omega_2$∈Ҳ, implies that $\omega_1^*= f(\omega_1)$,$\omega_2^*= f(\omega_2)$ and by the Sβ^*- T_2 property of (Ҳ, τ) , we have Ģ, Ḩ ⊆ Ҳ two disjoint supraβ^*-open such that ω_1∈Ģ,ω_2∈ Ḩ. Since f is a supra β^*-open map implies f(Ģ), f(Ḩ)∈ Sβ^*O(Ҳ). Thus ω_1^*∈f(Ģ), ω_2^*∈ f(Ḩ) and f(Ģ)∩f(Ḩ)=f(Ģ ∩ Ḩ)= ∅ [11] and we have done.

Remark 3.17: Every Sβ^*- T_2 is Sβ^*- T_1. The opposite, however, is untrue., as seen by the example that follows.

Example 3.18: Let Ҳ={ ω_1, ω_2, ω_3} with supra topology τ^*={ Ҳ , Ø,{ ω_1,ω_2}, {ω_2,ω_3}}, Hence, τ^{*c} = {Ҳ ,Ø ,{ω_1},{ω_3}}. Thus Sβ^*C={Ҳ, Ø, {ω_1},{ω_3}, {ω_1,ω_3},{ω_2}}, implies Sβ^*O={Ҳ , {ω_1 , ω_2},{ω_2, ω_3}, Ø,{ω_1,ω_3},{ω_2}}. Note that ω_1, ω_2 ∈ Ҳ cannot separate by disjoint supra β^*-open.

Remark 3.19

(1) Every subspace within a supra T_0-space is a supra β^*-T_0-space.
(2) Every subspace within a supra T_1-space is a supra β^*-T_1-space.
(3) Every subspace within a supra T_2-space is a supra β^*-T_2-space.

Proof: (1) Assume У be a supra subspace of a supra T_0-space (Ҳ,τ^*) and let ω_1,ω_2∈ У, hence ω_1,ω_2∈ Ҳ. Now since (Ҳ, τ^*) is supra T_0-space, therefore, a super open set exists Ģ ⊆ Ҳ implies that ω_1∈ Ģ, ω_2∉ Ģ .Thus Ģ$_y$=У∩ Ģ is supra open set in У [12] and clear that ω_1∈ Ģ$_y$,ω_2∉ Ģ$_y$, implies Ģ$_y$ is supra open in У and hence supra β^*-open (2.3), which contain $\omega_1$and not contain ω_2, implies (У,τ_y^*) is supra β^*-T_0-subspace. The proof of (2), and (3) is similar. □

5. Conclusion

Some separation axioms appeared in a new generalization by using of supra β^*-open sets in supra.spaces . In addition, many properties and relations of supra β^*-T$_i$-space ($i = 0,1,2$) have been investigated and illustrated by examples.

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