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On gamma T_pure submodules

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Abstract

A gamma T_ pure sub-module also the intersection property for gamma T_pure sub-modules have been studied in this action. Different descriptions and discuss some ownership, as Γ-module Z owns the T_Γ_pure intersection property if and only if (J^2 Γ$K \cap$ J^2 ΓF)=J^2 Γ($K \cap F$) for each Γ-ideal J and for all T_Γ_pure K, and F in Z Q/P is T_Γ_pure sub-module in Z/P, if P in Q.

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1. Introduction

Put M is a module over a non-zero ring associating with an identity that is left. Recall that a T_pure sub-module N, when for each J in R, such that $J^2M \cap N = J^2N$ [1]. Following [2 - 4], put R, Γ is abelian groups. Recall that R is a Γ –

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ring; if there is a function Ω: $R \times \Gamma \times R \to R$, $\Omega(x, \gamma, y) = x \gamma y$ which satisfies $(x + y) \gamma z = x \gamma z + y \gamma z$; $x(\gamma + \beta) y = x \gamma y + x \beta y$; $x \gamma (y + z) = x \gamma y + x \gamma z$; $(x \gamma y) \beta z = x \gamma (y \beta z)$ for all $x, y, z \in R$ and $\gamma, \beta \in \Gamma$. In [5,6] Γ–modules are introduced as a generalization of modules. Put R is a Γ −ring. An abelian group M with a function $\Omega : R \times \Gamma \times M \to M$, $\Omega(r, \gamma, m) = r \gamma m$, such that $r \gamma (m_1 + m_2) = r \gamma m_1 + r\gamma m_2$; $(r_1 + r_2) \gamma m = r_1 \gamma m + r_2 \gamma m$; $r(\gamma_1 + \gamma_2) m = r \gamma_1 m + r \gamma_2 m$ and $r_1 \gamma_1 (r_2 \gamma_2 m) = (r_1 \gamma_1 r_2) \gamma_2 m$ for all $m, m_1, m_2 \in M$and $\gamma, \gamma_1, \gamma_2 \in \Gamma$, $r, r_1, r_2 \in R$ is said to be (left) R_Γ−module. Prove that N is T_Γ_pure sub-module if and only if for each finite set { $\beta_i m_i$ } $\in M_\Gamma$, { $\beta_i n_i$ } $\in N_\Gamma$ with $\{r_{ij}\} \in R_\Gamma$ and $\beta_i n_j = \sum_{i=1}^{k} r_{ij}\beta_i m_i$, j = 1, 2, . . . , l, there is a{ $x_i \beta_i$} $\in$ N, which is a finite set, when $n_j - \sum_{i=1}^{k} r_{ij}\beta_i x_i \in N \cap L$.

2. Gamma T_Pure Sub-Modules (TΓ_PURE).

This segment introduces the connotation of the gamma T_pure (resp.TΓ_pure) sub-module.

Definition 2.1: Put Z is a Γ-module, and also K is a Γ-sub-module of Z, we have to say K is T_Γ_pure if $K \cap J^2\Gamma Z = J^2\Gamma K$ for each Γ-ideal J in R_Γ.

Remark 2.2: Put Z is Γ- module also Q is T_Γ_pure :

1. P is T_Γ_pure in Z , if P is T_Γ_pure in Q,.
2. Q is a T_Γ_pure in F if F in Z contains Q.
3. $\frac{Q}{P}$ is T_Γ_pure sub-module in $\frac{Z}{P}$, if P in Q.
4. Put Q and P are Γ-sub-modules in Z; if P is T_Γ_pure in Z and $\frac{Q}{P}$ is T_Γ_pure in $\frac{Z}{P}$, then Q is T_Γ_pure in Z.

Proof:

1. Put J is a Γ-ideal in Γ-ring R, we have Q is T_Γ_pure sub-module in Z and P is T_Γ_pure in Q, hence $Q\cap J^2\Gamma Z = J^2Q$ and $P\cap J^2Q = J^2\Gamma P$, but P is sub-module of Q, therefore $P \cap J^2\Gamma Z \subseteq Q \cap J^2\Gamma Z = J^2\Gamma Q$ and hence $P\cap J^2\Gamma Z \subseteq J^2\Gamma Q \cap P = J^2\Gamma P$. Since $J^2 \Gamma P \subseteq P \cap J^2 Z$, therefore $P \cap J^2 \Gamma Z = J^2\Gamma P$.
2. Assume that J is an Γ-ideal in R_Γ, since we have Q is T_Γ_pure in Z, therefore $Q \cap J^2\Gamma Z = J^2\Gamma Q$. Since F is a Γ-sub-module in Z, therefore $Q \cap J^2 \Gamma F \subseteq Q \cap J^2 \Gamma Z = J^2\Gamma Q$. Since $J^2 \Gamma Q \subseteq Q \cap J^2 \Gamma F$, thus $Q \cap J^2 \Gamma F = J^2 \Gamma Q$.
3. Put J is a Γ-ideal in R, we have Q is T_Γ_pure sub-module in Z, then $Q \cap J^2 \Gamma Z = J^2 \Gamma Q$. So $\frac{Q}{P} \cap J^2\Gamma\left(\frac{Z}{P}\right) = \frac{Q}{P} \cap \frac{J^2\Gamma Z+P}{P} = J^2\Gamma\left(\frac{Q}{P}\right) = J^2\Gamma\left(\frac{Q}{P}\right) = J^2\Gamma\left(\frac{Q}{P}\right) = J^2\Gamma\left(\frac{Q}{P}\right)$.

Proposition 2.3: Assume that M is Γ-module. Then N is T_Γ_pure sub-module if and only if for each finite sets { $\beta_i m_i$ } $\in M_\Gamma$, { $\beta_i n_i$ } $\in N_\Gamma$ with $\{r_{ij}\} \in R_\Gamma$ and

$\beta_i n_j = \sum_{i=1}^{k} r_{ij}\beta_i m_i$, j = 1, 2, . . . , l, there is a{ $x_i \beta_i$} ∈ N, which is a finite set, when $n_j - \sum_{i=1}^{k} r_{ij}\beta_i x_i$ ∈N ∩ L.

Proof: Put J is a Γ-ideal which is left, such that r_{ij} ($1 \le j \le l$, $1 \le i \le k$) is a generator of J . Therefore β_i n_j ∈ J^2 ΓM ∩ N. There is $\beta_i x_i$ ∈ N_Γ when $n_j = \sum_{i=1}^{k} r_{ij}\beta_i x_i + w_J$ al so β_i n_j - $\sum_{i=1}^{k} r_{ij}\beta_i x_i$ ∈ N ∩ L. The converse, take J is a Γ-ideal and also b' ∈ J^2ΓM ∩ N. Thus b' = $\sum_{i=1}^{k} a_i\beta_i m_i$ when a_i ∈ J and β_i m_i ∈ ΓM. Exists a finite set { β_i x_i}⊆ N_Γ, so that b' - $\sum_{i=1}^{k} a_i\beta_i m_i$ ∈ N ∩ L, since b' - $\sum_{i=1}^{k} a_i\beta_i m_i$ ∈ J^2 ΓM, thus b' ∈ J^2 ΓN. By looking carefully at the proof of the above proposition, we have, L is an Γ-sub-module of module M, then N is T_Γ_pure, if and only if N ∩J^2 ΓM = J^2ΓN for each Γ- ideal J.

Definition 2.4: Put M is a Γ-module, M_Γ is called have the T_Γ_pure sub-module intersection property, if we have the intersection of two T_Γ_pure sub-modules is again T_Γ_pure sub-module.

Proposition 2.5: Put M is a Γ-module.**1-**If M owns the T_Γ_pure sub-module intersection property, then each T_Γ_pure sub-module in M_Γ has the T_Γ_pure sub-module sub-module intersection property. **2-**Put N is T_Γ_pure sub-module in M. M has T_Γ_pure sub-module intersection property, if and only if, $\frac{M}{N}$ has T_Γ_pure sub-module intersection property.

Proof: **1-** It is obvious**. 2-** Assume that $\frac{K}{N}$,$\frac{F}{N}$ are two T_Γ_pure sub-modules in $\frac{M}{N}$, put J is a Γ-ideal, we want to show that $\left(\frac{K}{N} \cap \frac{F}{N}\right) \cap J^2\Gamma\left(\frac{M}{N}\right) = J^2\left(\frac{K}{N} \cap \frac{F}{N}\right)$. We claim that each one is a T_Γ_pure sub-module. To prove this matter, suppose that J is Γ- ideal also, let y ∈ $K \cap J^2\Gamma M$. Since $\frac{K}{N}$ is T_Γ_pure sub-module in $\frac{M}{N}$, hence $\frac{K}{N} \cap J^2\Gamma\frac{M}{N} = J^2\Gamma\frac{K}{N}$, therefore $\frac{K}{N} \cap \frac{J^2\Gamma M+N}{N} = \frac{J^2\Gamma K+N}{N}$, then $\frac{K \cap (J^2\Gamma M+N)}{N} = \frac{J^2\Gamma K+N}{N} = \frac{(K \cap J^2\Gamma M)+N}{N} = \frac{J^2\Gamma K+N}{N}$, thus $(K \cap J^2\Gamma M) + N = J^2\Gamma K + N$, and therefore $(K \cap J^2\Gamma M) = J^2\Gamma K$. Since $y \in K \cap J^2\Gamma M \subseteq (K \cap J^2\Gamma M) + N$, thus $y \in J^2\Gamma K + N$. Let z + w = y, $z \in J^2\Gamma K$ and $w \in N$. Now consider w = y – z ∈ $N \cap J^2\Gamma M = J^2\Gamma N \subseteq J^2\Gamma K$ and therefore K is T_Γ_pure sub-module. But M $_\Gamma$ has the T_Γ_pure intersection property; therefore, F∩K is the T_Γ_pure sub-module in M. Thus $(K \cap F) \cap J^2\Gamma M = J^2\Gamma(K \cap F)$. Now, let $y \in (\frac{K}{N} \cap \frac{F}{N}) \cap J^2\Gamma\frac{M}{N}$, therfore x + N = y , x ∈$J^2\Gamma M$ and y = k + N = f + N, when $k \in K, f \in F$. Therefore $y - k \in N \subseteq K, y - f \in N \subseteq F$ and therefore $x \in K \cap F$. Thus $x \in (K \cap F) \cap J^2\Gamma M = J^2\Gamma(K \cap F)$. Thus $y = x + N \in J^2\Gamma\left(\frac{K\cap F}{N}\right) = J^2\Gamma\left(\frac{K}{N} \cap \frac{F}{N}\right) \subseteq J^2\Gamma\left(\frac{K}{N} \cap \frac{F}{N}\right)$. On another side, put E, F is T_Γ_pure, suppose that N is a Γsub-module of E and

N be a Γ-sub-module in F, thus $\frac{E}{N}$, $\frac{F}{N}$ are T_Γ_pure sub-modules of $\frac{M}{N}$. But $\frac{M}{N}$ owns the T_Γ_pure intersection property, then $\frac{E}{N} \cap \frac{F}{N} = \frac{E \cap F}{N}$ is T_Γ_pure . Thus, F ∩E is T_Γ_pure Γsub-module in Γ-module M.

Theorem 2.6: *A Γ-module Z owns the T_Γ_pure intersection property if and only if $(J^2 \Gamma K \cap J^2 \Gamma F) = J^2 \Gamma(K \cap F)$ for each Γ-ideal J and for all T_Γ_pure K , F in Z.*

Proof: Put Z owns the T_Γ_pure sub-module intersection property then $K \cap F$ is T_Γ_pure sub-modules. Put J is a Γ-ideal, therefore $(K \cap F) \cap J^2 \Gamma Z = J^2 \Gamma(K \cap F)$. Clearly that $J^2 \Gamma(K \cap F) \subseteq (J^2 \Gamma K \cap J^2 \Gamma F)$. But $(J^2 \Gamma K \cap J^2 \Gamma F) \subseteq K \cap (F \cap J^2 \Gamma Z) = (K \cap F) \cap J^2 \Gamma Z = J^2 \Gamma(K \cap F)$. Then $J^2 \Gamma K \cap J^2 \Gamma F = J^2 \Gamma(K \cap F)$. On the other side, put J is an Γ-ideal of R, and put A, B are T_Γ_pure in Z, thus $A \cap B \cap J^2\Gamma Z = A \cap (B \cap J^2\Gamma Z) = A \cap J^2 \Gamma B$. Similarly, $A \cap B \cap J^2\Gamma Z = B \cap J^2 \Gamma A$. But A, B are T_Γ_pure sub-modules in an Γ-module Z. Then $A \cap B \cap J^2 \Gamma Z = J^2 \Gamma A \cap J^2\Gamma B = J^2 \Gamma(A \cap B)$

Conclusion

In this work have been introduced the concept of T_Γ_pure and it is proving the following relations

1. N is T_Γ_pure sub-module if and only if for each finite sets $\{ \beta_i m_i \} \in M_\Gamma$, $\{\beta_i n_i \} \in N_\Gamma$ with $\{r_{ij}\} \in R_\Gamma$ and $\beta_i n_j = \sum_{i=1}^{k} r_{ij}\beta_i m_i$, j = 1, 2, . . . , l, there is a$\{ x_i \beta_i\} \in N$, which is a finite set, when $n_j - \sum_{i=1}^{k} r_{ij}\beta_i x_i \in N \cap L$.
2. A Γ-module Z owns the T_Γ_pure intersection property if and only if $(J^2 \Gamma K \cap J^2 \Gamma F) = J^2 \Gamma(K \cap F)$ for each Γ-ideal J and for all T_Γ_pure K, F in Z.

Reference

[1] Shaymaa Abdul Kareem Muheyaddin, New Types Of Regular Fuzzy Modules And Pure Fuzzy Submodules, ph.D thesis University of Baghdad (2017).

[2] W.E. Barnes, On the gamma rings of Nobusawa, *Pacific J. Math.*, 18, 411-422 (1966).

[3] Mehdi Sadiq Abbas, Saad Abdul Kadhim Al-Saadi, Emad Allawi Shallal, Some Generalizations of Semisimple Gamma Rings, *Journal of Science*, Vol. 58, No.3C, pp: 1720-1728 (2017).

[4] Ali Abd Alhussein Zyarah, Nuhad Salim Al-Mothafar. On primary radical RΓ-submodules of RΓ-modules, *Journal of Discrete Mathematical Sciences and Cryptography*, Vol. 23, Issue 5 (2020)

[5] Mehdi Sadiq Abbas and Balsam M. Hamad, Duo Gamma Modules and Full Stability, *Iraqi Journal of Science* (2020)

[6] Hasan Abdulraheem Jubair Al-Yasiri, Ali Abd Alhussein Zyarah &, Nearly prime GR-submodules of GR-modules, *Journal of Discrete Mathematical Sciences and Cryptography*, Vol. 25, Issue 6 (2022).

[7] Taresh, M. R., & Hussain, A. K. : Some Weak Hereditary Properties. *Wasit Journal of Computer and Mathematics Science*, 1(2) (2022).

[8] G. R. Mohtashami Borzadaran & M. R. Miri. Some inequalities related to incomplete gamma function, *Journal of Interdisciplinary Mathematics*, 10:5, 619-623 (2007), DOI: 10.1080/09720502.2007.10700522.

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