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# On $\delta$- character in symmetric $t^{\omega}$-Δ- Banach-algebra

M. A. Neamah *
*Department of Mathematics*
*College of Education for Girls*
*University of Kufa*
*Najaf*
*Iraq*

Boushra Y. Hussein †
*Department of Mathematics*
*College of Science*
*University of Al-Qadisiyah*
*Al-Qadisiyah*
*Iraq*

**Abstract**

The aim of this paper is to provide information on new results from studies of symmetric $t^{\omega}$-Δ- Banach Algebra. A new function $\delta$- character has been defined and apparent to be symmetric $t^{\omega}$-Δ- Banaich Algebra. $t^{\omega}$- approach dual cone space is introduced, with its symmetry properties indicated $t^{\omega}$- Δ- Banaich algebra. Some new definitions, results and properties are given and discussed in this field.

***Subject Classification:*** *05E05, 19D23.*

***Keywords:*** *$t^{\omega}$-approach space, $t^{\omega}$-contraction, $t^{\omega}$-approach group, $t^{\omega}$-Δ- normed approach space $\delta$- character function, Symmetric $t^{\omega}$-Δ- Banach algebra.*

## 1. Introduction

W. Ambrose [5] gave the first definition of the term "Banach algebra" in 1945. S. Kurepa [2] published ON ROOTS OF AN ELEMENT OF A BANACH ALGEBRA in 1960, where he established that every Banach algebra might be

* *E-mail:* **maysoin.aziz.75@gimail.com (Corresponding Author)**
† *E-mail:* **boushri.alshebana@qu.edu.iq**

embedded isomorphically and isometrically in another Banach algebra, making any element in one Banach algebra an element in another Banach algebra.. In 1974, C. E. Reckart [1] introduced (General Theory of Banach Algebras) the first book with a detailed account of the theory of Banach algebra. In 1973, a monograph on Banach algebra was shown by F. F. Bonsall and J. Duncan [4]. In 1981, new results in the theory of real symmetric Banach--algebras were presented by J. Vukman[10]. In 1990, J. B. Conway [7], studied Banach algebra and spectrum theory, so he used them to study the operator on Banach space. Two unital Banach algebras $A$ and $B$ were studied by M. Bresar and P. Semrl [3] in 2008, such that $B$ is semisimple. In 2010, some properties of the spectral radius in complex-ordered Banach algebra were established by R. de John [8] in his thesis.

Moreover, He explained the conditions under which the spectrum contains the spectral radius of the positive element , he introduced the uniform continuity of the spectrum in commutative-ordered Banach algebra. Complex unital Banach algebra has its own Pseudospectrum and condition spectrum, the features of which G. K. Kumar and S. H. Kulkarni [6] described in 2012. They also established a large number of findings on linear maps that maintain the Pseudospectrum and the condition spectrum. In 2021, B. Hussain and R. Abbas[10] demonstrated several new findings in the normed approach space. B. Y. Hussein and H. A. Wshayeh [11] studied a symmetric Banach algebra in 2021.

There are four sections to this work: In the first section, we provide an overview of the study. Section 2 presents some introductory materials, including some fundamental definitions. Section 3 presents recent findings in symmetric $t^{\omega}$- Δ- approach Banach algebra is studied, and certain fundamental assertions are proved. In Section 4, we draw up to make connections between the study's primary results.

## 2. Preliminaries

We start this work by defining of important notion, namely $t^{\omega}$-approach distance.

**Definition 2.1 [9]:** Assume Y to be a set that is not empty. A collection $(t^{\omega})_{\omega<\infty}$ of function $t^{\omega} : T \times T \rightarrow [0,\infty]$ has an identifier of $t^{\omega}$-approach distance on $\chi$ if this function satisfies the following properties when g=R AND T=$2^{\chi}$:

$(t_1)$ $\forall\omega \in g^{+}, \forall K,W \in T: t^{\omega}(K,W) = 0 \Rightarrow A = B$,
$(t_2)$ $\forall\omega \in g^{+}, \forall K \in 2^{\chi}$, then $t^{\omega}(K,\emptyset) = \infty$,
$(t_3)$ $\forall\omega \in g^{+}, \forall K,W,F \in T: t^{\omega}(K, W\cap F) = max\{t^{\omega}(K,W), t^{\omega}(K,C)\}$,
$(t_4)$ $\forall\omega \in g^{+}, \forall K,W \in 2^{\chi}, \forall\varepsilon < \omega: t^{\omega}\left(A^{\omega}_{(\varepsilon)}, W\right) \leq t^{\omega}(K,W) + \varepsilon$ where

$K^{\omega}_{(\varepsilon)} = \{x \in Y | t^{\omega}(\{x\},W) \leq \omega - \varepsilon\}$. A pair $(Y, t^{\omega})$ where the function $t^{\omega}$ is a distance and The title for this pair is $t^{\omega}$- approach space and denoted by $t^{\omega}$- approach -spaces.

**Definition 2.2 [9]:** Let $( T, t^{\omega} )$ and $(W, t^{\omega\prime})$ are approach - spaces. A function $\varrho: T \rightarrow W$ identifies as $t^{\omega}-$ contraction if $\forall \ \omega < \infty$, and $\forall E, Y \in T, t^{\omega\prime}(\varrho((E), \varrho(Y)) \leq t^{\omega}(E, Y)$.

**Definition 2.3:** A set $O \in W$ supposed to be a cluster point in a $t^{\omega}$-approach space $( O, t^{\omega} )$ if $\exists$ a disjoint sequence $\{A_n\}_{n=1}^{\infty}$ in $E$ such that $inf_{x_i \in A_i, x \in B} t^{\omega}(A_n, Y) = 0$,it was composed by $\{A_n\}_{n=1}^{\infty} \rightarrow Y$. We denoted the set of all cluster points in $t^{\omega}$- approach space $\Gamma(\chi)$ .

**Definition 2.4:** A sequence $\{A_n\}_{n=1}^{\infty}$ in $w$ is said to be a Caucihy sequence in $t^{\omega}$-approach space ($t^{\omega}$–Cauchy sequence) if for every cluster point $B$, $lim_{n\rightarrow\infty} \inf_{x_i \in A_i, x \in B} t^{\omega}(A_n, B) = 0$, a sequence $\{A_n\}_{n=1}^{\infty}$ in $w$ is said to be $t^{\omega}$-convergent sequence in $t^{\omega}$-approach space if there exist $x \in w$ for all $B \in \Gamma(\chi)$ , $t^{\omega}(A_n, B) = 0$.

**Definition 2.5**: Let $w$ is a $t^{\omega}$-approach linear space over the field $F$. Δ- norm on $\chi$ is a function$\|.\|_{\Delta}: w \rightarrow R^{+}$ for all $x, y \in w$ ,$t^{\omega}$ is an approach distance on $\chi$ , then $(w, \|.\|_{\Delta}, t^{\omega}{}_{\|.\|_{\Delta}})$ is called $t^{\omega}$-Δ- approich normied space if the following conditions satisify:

1) $\| x \|_{\Delta} > 0$ for $x \neq 0, x \in w$;
2) $\| \alpha x \|_{\Delta} \leq \| x \|_{\Delta}$, $|\alpha| \leq 1, x \in w$;
3) $\lim_{\alpha\rightarrow 0} \| \alpha x \|_{\Delta} = 0$ for $x \in \chi$;
4) $\| x + y \|_{\Delta} \leq c \max(\| x \|_{\Delta} , \| y \|_{\Delta})$ for all $x, y \in w$ where $c$ is some constant such that $c > 1$; 5)$t^{\omega}{}_{\|.\|_{\Delta}}(A, B) = sup_{A,B \in 2^{\chi}} sup_{x\in B} \inf_{a \in A} \|x - a\|_{\Delta}$.

**Definition2.6:** Let $(w, t^{\omega})$ be $t^{\omega}$ app-space over the field, then $(w, t^{\omega}, +,\bullet)$ is called $t^{\omega}$-approach linear algebra if it satisfies the following conditions:

1) $(w, t^{\omega}, +,\bullet)$ is $t^{\omega}$-approach linear space;
2) The operation of multiplicative defined on $w$ satisifies the conditions for all $x, y, z \in w$ and $\alpha \in F$: i) $\alpha.(x.y) = (\alpha.x).y = x.(\alpha.y)$ , ii) $(x.y).z = x.(y.z)$;
3) $(x + y).z = (x.z + y.z)$ and $x.(y + z) = x.y + x.z$.

If any $x, y \in w$ satisifies $t^{\omega}(\{x.y\}, A) = t^{\omega}(\{y.x\}, A)$, then $(w, t^{\omega}, +,\bullet)$ is called commutative $t^{\omega}$-approach linear algebra

**Definition2.7:**$t^{\omega}$-approach linear space $w$ believed to be $t^{\omega}$-Δ- app- normed linear algebra if :

1) $w$ is $t^{\omega}$-Δ- approach normed space,
2) $w$ is $t^{\omega}$-app- linear algebra;
3) For all, $b \in w$, $\|ab\| \leq \|a\| \|b\|$, 4) If $w$ with identity, then $\|e\| = 1$.

**Definition 2.8:** Let $w$ be a $t^{\omega}$ approach -linear space, then $w$ is called a symmetric $t^{\omega}$-approach linear algebra if satisfying the following conditions:

1) $w$ is $t^{\omega}$-approach linear algebra;
2) An operation is defined on $w$, which assigns to each element $x, y \in w$ the element $x^*, y^* \in w$ satisfies the following conditions: 1) $(\alpha x + \beta y)^* = \bar{\alpha} x^* + \bar{\beta} y^*$, 2) $x^{**} = x$, 3) $(xy)^* = y^* x^*$,
   4) The function $*: x \to x^*$ is $t^{\omega}$-contraction.

**Definition2.9 (symmetric $t^{\omega}$- Δ- approach normed algebra) :** Let $w$ be a $t^{\omega}$– approach - linear space, then $w$ has been defined as symmetric $t^{\omega}$- Δ- approach normed algebra if it satisfies the conditions:

1) $\chi$ is $t^{\omega}$-Δ- approach normied algeibra,
2) $\chi$ is a symmetric $t^{\omega}$-approiach linear algebra,
3) $\| x^* \|_{\Delta} = \| x \|_{\Delta}, \forall\, x \in w.$

**Definition 2.10:** Let $w$ be a $t^{\omega}$-Δ- approach normed algebra, then $w$ is a complete $t^{\omega}$-Δ-approach algebra if every $t^{\omega}$-Cauichy sequence is convergent.

**Remark 2.1:** A complete $t^{\omega}$-Δ- approach normed algebra is called a $t^{\omega}$- Δ- Banach algebra.

**Remark 2.2:** A completely symmetric $t^{\omega}$- Δ- approach normed algebra is called symmetric $t^{\omega}$- Δ- Banach algebra

**3. New Result of Symmetric $t^{\omega}$-Δ- Banach Algebra.**

**Definition3.1:**Let $w$ be a symmetric $t^{\omega}$- Δ- approach Banaich algebra, $\forall x, y \in w, \sigma \in F$ and $\delta > 0$, a function $\psi$ is $\delta$- a character from $w$ into the complex field $F$ if satisfies the conditions:

1) $\psi(x + y) = \psi(x) + \psi(y)$;
2) $\psi(\lambda x) = \lambda \psi(x)$;
3) $\| \psi(xy) - \psi(x)\psi(y) \|_{\Delta} \leq \delta(\| x \|_{\Delta} \| y \|_{\Delta})$
4) $\psi(e) = 1$;
5) $\psi(x^*) = (\psi(x))^*$
6) The function $\psi$ defined over $w$ is satisfied$t^{\omega}$–contraction.

If $= \mathbb{C}$, then it is said that $\psi$ is a complex $\delta$- character and $\delta ch(w)$ denotes the set of all complex $\delta$- characters.

**Theorem 3.1:** *Let $w$ be a symmetric $t^{\omega}$- Δ- Banaich algebra with identity such that every non-zero element has an inverse. Then $\delta ch(\chi)$ is a symmetric $t^{\omega}$-Δ- Banaich algebra if $\| f_m \|_{\Delta} = sup\, |f_m(g)|$, and $\psi$ is a $\delta$- character.*

***Proof:*** Let $\psi: w \to \delta ch(w)$ defined as $\psi(m) = f_m$, such that $f_m(g) = g(m)$, with $t^{\omega}$-approach the usual distance $t^{\omega}: 2^{\delta ch(\chi)} \times 2^{\delta ch(\chi)} \to [0, \infty]$ defined on $\delta ch(w)$, where $\delta ch(w) = \{f_m | f_m: w \to \mathbb{C}\}$, such that, for all $\omega < \infty,\ f_m, f_n \in \delta ch(w), m, n \in w, A, B \in 2^{\delta ch(\chi)}$

$$t^{\omega}(T, Y) = \begin{cases} inf_{f_m \in A, f_n \in B} |f_m(g) - f_n(g)| & T \neq \emptyset, Y \neq \emptyset \\ \infty & T \neq \emptyset, Y = \emptyset \end{cases},$$

$+,.$ are addition and multiplication binary operations such that

$$(f_m + f_n)(g) = f_m(g) + f_n(g);\ (f_m . f_n)(g) = f_m(g) . f_n(g).$$

**First:**

(a) $(\delta ch(w), t^{\omega})$ is $t^{\omega}$-app-apace by[13].

(b) That is very clear $(\delta ch(w), +)$ belongs to a group of items.

(c) Now we have proof $+: \delta ch(w) \oplus \delta ch(w) \to \delta ch(w): (f_m, f_n) \to f_m + f_n$ is $t^{\omega}$ – contraction for all $\omega < \infty, f_m, f_n, f_r, f_s \in \delta ch(w), A, B, C, D \in 2^{\delta ch(\chi)}: f_m \in A, f_n \in B, f_r \in C, f_s \in D$.

$$t^{\omega\prime}(\varrho(t, y), \varrho(C, D)) = inf_{f_m \in A, f_n \in B, f_r \in C, f_s \in D} |(f_m + f_n)(g) - (f_r + f_s)(g)|$$
$$\leq inf_{f_m \in A, f_r \in C} (|f_m(g) - f_r(g)|) + inf_{f_n \in B, f_s \in D}(|f_n(g) - f_s(g)|)$$
$$= t^{\omega}(A, C) + t^{\omega}(B, D).$$

(d) $t^{\omega\prime}(\varrho(A), \varrho(B)) = t^{\omega\prime}(-t, -y) = inf_{f_m \in A, f_n \in B} |-f_m(g) - (-f_n(g))| = inf_{f_m \in A, f_n \in B} |f_m(g) - f_n(g)| = t^{\omega}(t, y)$. Then, $(\delta ch(w), t^{\omega}, +)$ is $t^{\omega}$- app- group.

Second: $(\delta ch(\chi), t^{\omega}, +, .)$ is $t^{\omega}$-app- linear algebra.

Third: Now, to prove $\delta ch(w)$ is $t^{\omega}$-Δ- approach normed algebra must be proved is $\|.\|_{\Delta}$ is the norm on $\delta ch(w)$.

1) Let $f_m \in \delta ch(w)$ such that $f_m \neq 0$, then $f_m > 0$ and $|f_m(g)| = |g(m)| > 0$ for all $m \in \chi$. Therefore, $\sup|f_m(g)| = \sup|g(m)| > 0$. Hence, $\| f_m \|_{\Delta} > 0$.

2) Let $f_m \in \delta ch(w)$ and $|\alpha| \leq 1, |\alpha||f_m(g)| = |\alpha f_m(g)| = |\alpha g(m)| \leq |g(m)| = |f_m(g)|$, then $\sup|\alpha f_m(g)| \leq \sup|f_m(g)|$. Thus, we get that $\| \alpha f_m \|_{\Delta} \leq \| f_m \|_{\Delta}$.

3) Let $f_m \in \delta ch(\chi)$, $lim_{\alpha \to 0} \| f_m \|_{\Delta} = lim_{\alpha \to 0} \sup|f_m(g)| = lim_{\alpha \to 0} \sup|\alpha g(m)| = \sup lim_{\alpha \to 0} |\alpha g(m)| = \sup lim_{\alpha \to 0} |\alpha f_m(g)| = 0$.

4) Let $f_m, f_n \in \delta ch(\chi w)$. Then, $\| f_m + f_n \|_{\Delta} \leq 2\, max\{\| f_m \|_{\Delta}, \| f_n \|_{\Delta}\}$, for all $f_m, f_n \in \delta ch(\chi)$,

5) $$t^{\omega}{}_{\|.\|_{\Delta}}(A, B) = inf_{f_m \in A, f_n \in B} \| f_m - f_n \|_{\Delta}$$
$$= sup_{A, B \in 2^{\delta ch(\chi)}} sup_{f_n \in B} \inf_{f_m \in A} \| f_m - f_n \|_{\Delta}.$$

6) For all $f_m, f_n \in \delta ch(w)$, $\| f_m . f_n \|_{\Delta} \leq \| f_m \|_{\Delta} . \| f_n \|_{\Delta}$.

7) If $\delta ch(\chi)$ with identity, then $\| f_m(I) \|_\Delta = \sup |f_m(I)| = \| I \|_\Delta = 1$.

8) $t^\omega\big(\{inf_{f_e, f_m \in A}(f_e . f_m)\}, B\big) = t^\omega\big(\{inf_{f_m \in A}(I f_m)\}, B\big)$
$= t^\omega(\{inf_{f_m \in A}(f_m)\}, B)$

Therefore, $\delta ch(w)$ is $t^\omega$-$\Delta$-approach normied space so it is $t^\omega$-$\Delta$-approach normied linear algebra.

Fourth: $*: \delta ch(\chi) \to \delta ch(\chi)$ defined by $*(f_m(g)) = \overline{(f_m(g))}$, is satisfying involution axioms such that $\|.\|_\Delta = \sup |f_m(g)|$.

Fifth: Now, to prove The function $*: \delta ch(w) \to \delta ch(w)$ such that $f_m \mapsto f_m^*$ is $t^\omega$- contraction for all

$\omega < \infty, f_m, f_n \in \delta ch(w), A, B \in 2^{\delta ch(\chi)}$ and for all $f_m \in A, f_n \in B$.

$t^{\omega\prime}\big(\varrho(A), \varrho(B)\big)$

$= inf_{f_m \in A, f_n \in B}|f_m^*(g) - f_n^*(g)| \leq inf_{f_m \in A, f_n \in B}|f_m(g) - f_n(g)| = t^\omega(A, B)$. Therefore, $\delta ch(w)$ is $t^\omega$-$\Delta$-approich- normied space.

Sixth: Let $A_n, B \in 2^{\delta ch(\chi)}$ , and let $B$ be a cluster point in $(\delta ch(w), t^\omega)$, we denoted the sum of all cluster points in $t^\omega$-approach space by $\Gamma\big(\delta ch(w)\big)$, $B \in \Gamma\big(\delta ch(\chi)\big)$.$t^\omega(A_n, B) =$

$$\begin{cases} inf_{f_{m_i} \in A_i, f_s \in B}\big|f_{m_i}(g) - f_s(g)\big| A_n \neq \emptyset, \text{forall } n \in Z^+, m, s \in \chi, B \neq \emptyset \\ \infty \quad A_n \neq \emptyset, \text{for some } n \in Z^+, B = \emptyset \end{cases}$$

Let $\{A_n\}_{n=1}^\infty$ be a $t^\omega$-Cauchy sequence in $(\delta ch(\chi), t^\omega)$. Then,

By def. 2.4$\{A_n\}_{n=1}^\infty$ is convergent in $(\delta ch(\chi), t^\omega)$ and $(\delta ch(\chi), \|.\|_\Delta, t^\omega{}_{\|.\|_\Delta})$.

This leads to $(\delta ch(\chi), \|.\|_\Delta, t^\omega{}_{\|.\|_\Delta})$ is a symmetric $t^\omega$-$\Delta$- Banach algebra.

The second part is to prove $\psi$ is a $\delta$- character. Let $\omega < \infty$, $f_m, f_n \in \delta ch(\chi), m, n \in \chi$

1) $\psi(m+n) = f_{m+n}$ such that $f_{m+n}(g) = g(m+n) = g(m) + g(n) = f_m(g) + f_n(g) = f_m + f_n = \psi(m) + \psi(m)$;
2) $\psi(\alpha m) = f_{\alpha m}$ such that $f_{\alpha m} = g(\alpha m) = \alpha g(m) = \alpha f_m(g) = \alpha f_m = \alpha\psi(m)$;
3) $\| \psi(mn) - \psi(m)\psi(n) \|_\Delta = \| f_{mn} - f_m f_n \|_\Delta = \| f_{mn}(g) - f_m(g) f_n(g) \|_\Delta = \| g(mn) - g(m)g(n) \|_\Delta \leq \delta(\| m \|_\Delta \| n \|_\Delta)$;
4) $\psi(m^*) = f_{m^*}$ such that $f_{m^*}(g) = g(m^*) = (g(m))^* = (f_m(g))^* = (f_m)^* = (\psi(m))^*$;
5) $\psi(e) = f_e$ such that $f_e(g) = g(e) = 1$; 6) Now, to prove that $\psi$ is $t^\omega$- contraction for all $\omega < \infty, f_m, f_n \in \delta ch(\chi), m, n \in \chi$, and $A, B \in 2^\chi$.
$t^{\omega\prime}\big(\psi(A), \psi(B)\big) = t^\omega(\{f_m(g)\}, \{f_n(g)\}) \leq t^\omega(A, B)$.

**Proposition 3.1:** Let $\chi$ be a symmetric $t^{\omega}$-Δ- Banach algebra, and let the set $g = \{f \in \chi^*: f(m^*m) \geq 0 \text{ for all } m \in \chi\}$.Then, $(g, t^{\omega})$ is $t^{\omega}$- app- cone space and $(g, \|.\|_{\Delta}, t^{\omega}{}_{\|.\|_{\Delta}})$ a symmetric $t^{\omega}$- Δ- Banach algebra if $\| f \|_{\Delta} = \sup_{m^* \neq 0, m \neq 0} \frac{|f(m^*m)|}{\|m^*\|\|m\|}$.

***Proof:*** We define $t^{\omega}$-app- distance $t^{\omega}: 2^{g} \times 2^{g} \to [0, \infty]$ on $g$, such that for all $\omega < \infty$, $f \in \chi^*, m \in \chi, f_1 \in A, f_2 \in B$, and $A, B \in 2^{g}$

$$t^{\omega}(A, B) = \begin{cases} inf_{f_1 \in A, f_2 \in B} \| f_1 - f_2 \|_c & A \neq \emptyset, B \neq \emptyset \\ \infty & A \neq \emptyset, B = \emptyset \end{cases},$$

with

$+,.$ are addition and multiplication binary operations such that for all $f_1, f_2 \in g, m \in \chi$.

$$(f_1 + f_2)(m^*m) = f_1(m^*m) + f_2(m^*m); (f_1.f_2)(m^*m) = f_1(m^*m).f_2(m^*m).$$

By the same way in Theorem 3.1we prove that $g$ is a $t^{\omega}$-app-symmetric cone and $(g, t^{\omega}, +, .)$ is $t^{\omega}$-approach linear algebra.

$$\begin{aligned} t^{\omega}{}_{\|.\|_{\Delta}}(A, B) &= inf_{f \in A, f_1 \in B} \| f - f_1 \|_{\Delta} \\ &= inf_{f \in A, f_1 \in B} \sup_{m^* \neq 0, m \neq 0} \frac{|f(m^*m) - f_1(m^*m)|}{\| m^* \| \| m \|} \\ &\leq sup_{A, B \in 2^{g}} sup_{f_1 \in B} \inf_{f \in A} \sup_{m^* \neq 0, m \neq 0} \frac{|f(m^*m) - f_1(m^*m)|}{\| m^* \| \| m \|} \\ &= sup_{A, B \in 2^{g}} sup_{f_1 \in B} \inf_{f \in A} \| f - f_1 \|_{\Delta}. \end{aligned}$$

Therefore, $g$ is $t^{\omega}$-Δ- approach normed space so it is $t^{\omega}$-Δ- approach normed linear algebra.

Also, $*: g \to g$ defined by $*(f(m^*m)) = \overline{(f(m^*m))}$ is satisfying involution axioms such that $\|.\|_{\Delta} = \sup |f(x)|$ and the function $* : g \to g: f \to f^*$ is $t^{\omega}$-contraction for all $\omega < \infty, f_1, f_2 \in g, A, B \in 2^{g}: f_1 \in A, f_2 \in B$.

Now, let $A_n, B \in 2^{g}$, and let $B$ is a cluster point in $(g, t^{\omega})$, we denoted the set of all cluster points in $t^{\omega}$-approach space by $\Gamma(g)$, $B \in \Gamma(2^{g})$.

$$t^{\omega}(A_n, B) = \begin{cases} inf_{f_i \in A_i, f \in B} \| f_i - f \|_c & A_n \neq \emptyset, \forall n \in Z^+, B \neq \emptyset \\ \infty & A_n \neq \emptyset, \text{for some } n \in Z^+, B = \emptyset \end{cases}$$

Let $\{A_n\}_{n=1}^{\infty}$ be a $t^{\omega}$-Cauchy sequience in $(g, t^{\omega})$. Thus, in the same way in Theorem 3.1, we prove $(g, \|.\|_{\Delta}, t^{\omega}{}_{\|.\|_{\Delta}})$a symmetric $t^{\omega}$-Δ- approach Banach algebra.

**The proof is clear.**

## Conclusion

In this paper, we defined a new function $\delta$- character and proved that it is a symmetric$t^{\omega}$-Δ- Banach Algebra. We introduced $t^{\omega}$- approach dual cone space and also proved that it is a symmetric $t^{\omega}$- Δ- Banach algebra. We put some important definitions, results and properties.

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