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Orthogonality of generalized Γ-m-derivations in semi-prime Γ-near-rings

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Abstract

We introduced the nomenclature of orthogonal Γ-m-derivations and orthogonal generalized Γ-m-derivations in semi-prime Γ-near-rings and provide a few essentials and enough provision for generalized Γ-n-derivations in semi-prime Γ-near-rings by orthogonal.

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1. Introduction

In this work. A Γ – near-ring was submitted in [3, 4, 5]. A Γ-near-ring N possesses the condition for s, r$\in$ N, s Γ N Γ r = 0 gives s = 0 or r = 0 is said prime Γ-near-ring and called semi-prime if N possesses the property $t\Gamma G\Gamma t$ = 0 gives t = 0 for every $t \in G$ [4, 5]. The additional commutators are as follows; $[s, r]_\rho$= sρr – rρs and (s, r) = s+r–s–r designate the commutator for additive groups [2, 4]. If (N, +) is abelian, N is said to be commutative and 2-torsion-free if 2s = 0 gives s = 0. [1].

An additive mapping h: N× N × . . .× N$\rightarrow$N called Γ-m-derivation if the relationships are:

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$h(k_1\gamma k_1', k_2, k_3 \ldots, k_m) = h(k_1, k_2, \ldots, k_{m'})\gamma k_1' + k_1\gamma h(k_1', k_2, k_3 \ldots, 'k_m)$
$h(k_1, k_2\gamma k_2', \ldots, k_m) = h(k_1, k_2, \ldots, k_m)\gamma k_2' + k_2\gamma h(k_1, k_2', \ldots, k_m)$
$h(k_1, k_2, \ldots, k_m\gamma k_m') = h(k_1, k_2, \ldots, k_m)\gamma k_m' + k_m\gamma h(k_1, k_2, k_3, \ldots, k_m')$, for every $k_{1i}, k_1', k_{2i}, k_3, k_2', \ldots, k_{m1}, ik_m' \in N, \gamma \in \Gamma$. [3, 4]

The mapping is additive $g: N\times N \times \ldots \times N \to N$ called right generalize Γ-m-derivation of N linked with Γ-m-derivation h if the eq.

$$g(k_1\gamma k_1', k_2, \ldots, k_m) = g(k_1, k_2, \ldots, k_m)\gamma k_1' + k_1\gamma h(k_1', k_2, \ldots, k_m)$$
$$g(k_1, k_2\gamma k_2', \ldots, k_m) = g(k_1, k_2, \ldots, k_m)\gamma k_2' + k_2\gamma h(ik_1, k_2', \ldots, tk_m)$$
$$\vdots$$
$$g(k_1, k_2, \ldots, k_m\gamma k_m') = g(ik_1, k_2, \ldots, tk_m)\gamma k_m' + k_n\gamma h(ik_1, k_2, \ldots, k_m')$$

Hold for every $k_1, k_1', k_2, k_2', \ldots, k_m, k_m' \in N$ and $\gamma \in \Gamma$. [4]. Lastly an Γ-m-additive mapping $h: N\times N \times \ldots \times N \to N$ called generalized Γ-m-derivation of N linked with Γ-m-derivation h if a right and left generalized Γ-m-derivation of N linked with Γ-m-additive h.[4].

Ashraf [2] introduces the orthogonal derivations in Γ-ring. Park [7] introduced in semi-prime near-rings, the concept of orthogonal generalized derivations is found. Kalyan and Akhil [5] insert in semi-prime gamma near-rings the concept of orthogonal generalized derivations. In the work, we introduced the definition of orthogonal of two generalized Γ-m-derivations in Γ-near-ring and orthogonal of two Γ-m-derivations in semi-prime Γ-near-ring and the current number of essential and sufficient circumstances for generalized Γ-m-derivations on semi-prime Γ-near-rings to be orthogonal. Also, we discuss some other related results.

2. Preliminaries

The following lemmas are required for the development of our primary proofs. We recall Γ-near-ring by Γ-(n-r) and Γ-m-derivations by Γ-m-der.

Lemma 2.1[5]: *When N is a 2-torsion-free semi-prime Γ-(n-r) and s, k $\in$ N, then The criteria listed below equal:*

(a) $s\alpha t\beta k = 0$ every $t \in N$, $\alpha, \beta \in \Gamma$.

(b) $k\alpha t\beta s = 0$ every $t \in N$, $\alpha, \beta \in \Gamma$.

(c) $s\alpha t\beta k + k\alpha t\beta s = 0$ every $t \in N$, $\alpha, \beta \in \Gamma$.

If the above of the three conditions listed are met, thus $s\Gamma k = k\Gamma s = 0$.

Lemma 2.2 [3, 4]: *When N be a Γ-(n-r) and h be Γ-m-der., thereafter for every $k_1, k_1', \ldots, k_m, s \in N$ and $\gamma, \beta \in \Gamma$.*

$$(h(ik_1', k_2, \ldots, k_m)\gamma k_1' + k_1\gamma h(k_1', ik_2, \ldots, ik_m))\beta s = h(ik_1, k_2 \ldots, k_m)\gamma k_1'\beta s + k_1\gamma h(k_1', k_2, \ldots, k_m)\beta s$$

Lemma 2.3 [4]: *When N is a Γ-(n-r), g be a right generalized Γ-m-der. linked with Γ-m-der. h of N and for every $k_1, k_2, \ldots, k_m, k, r \in N$ and $\gamma, \beta \in \Gamma$. Then*

$(g(ik_1, k_2, \ldots, ik_m)\gamma k + k_1\gamma h(ik_1, k_2, \ldots, 'k_m))\beta r =$

$g('k_1, ik_2, k_3 \ldots, k_m)\gamma k\beta r + k_1\gamma h(k, k_2, \ldots, k_m)\beta r$

Definition 2.4: Let h_1 and h_2 be two Γ-n-derivations of Γ-(n-r) . h_1 and h_2 are called orthogonal if $h_1(k_1, k_2, \ldots, k_m)\Gamma N\Gamma h_2(v_1, v_2, \ldots, v_m) = h_2(v_1, v_2, \ldots, v_m)\Gamma N\Gamma h_1(k_1, k_2, k_3 \ldots, k_m) = 0$ for every $k_1, k_2, k_3 \ldots, k_m, v_1, v_2, \ldots, v_m \square N$.

Definition 2.5: Let g_1 and g_2 be two generalized Γ-m-der. of Γ-(n-r). g_1 and g_2 are called orthogonal if $g_1(k_1, k_2, \ldots, k_m)\Gamma N\Gamma g_2(v_1, v_2, \ldots, v_m) = g_2(v_1, v_2, \ldots, v_m)\Gamma N\Gamma g_1(ik_1, k_2, k_3 \ldots, k_m) = 0$ for everyi$k_1, k_2, k_3 \ldots, ik_m, v_1, v_2, \ldots, v_m \in N$.

3. Main Results

Lemma 3.1: *When N is 2- torsion-free semi-prime Γ-(n-r), g_1, g_2 be two generalized Γ-m-der. of N linked with Γ-m-der. h_1 and h_2 of N progression. If g_1 and g_2 are orthogonal, thus go to the next conditions are:*

(a) *$g_1(k_1, k_2, \ldots, k_m)\Gamma g_2(v_1, v_2, \ldots, v_m) = g_2(k_1, k_2, \ldots, k_m)\Gamma g_1(v_1, v_2, \ldots, v_m) = 0$ for every $k_1, k_2, \ldots, k_m, v_1, v_2, \ldots, v_m \in N$.*

(b) *h_1 and g_2 are orthogonal and $h_1(k_1, k_2, \ldots, k_m)\Gamma g_2(v_1, v_2, \ldots, v_m) = g_2(v_1, v_2, \ldots, v_m)\Gamma h_1(ik_1, k_2, \ldots, tk_m) = 0$ for every $ik_1, k_2, k_3 \ldots, k_m, v_1, v_2, \ldots, v_m \in N$.*

(c) *h_2 and g_1 are orthogonal and $h_2(k_1, k_2, \ldots, k_m)\Gamma g_1(v_1, v_2, \ldots, v_m) = g_1(v_1, v_2, \ldots, v_m)\Gamma h_2(ik_1, k_2, \ldots, tk_m) = 0$ for every $k_1, ik_2, k_3 \ldots, k_m, v_1, v_2, \ldots, v_m \in N$.*

Proof: (a) Since g_1 and g_2 orthogonality, obtain $g_1(k_1, k_2, \ldots, k_m)\alpha k\beta g_2(v_1, v_2, \ldots, v_m) = 0$ for every $k, k_1, k_2, \ldots, k_m, v_1, v_2, \ldots, v_m \in N$ and $\alpha, \beta \in \mu$. By Lemma 2.1 obtain $g_1(k_1, k_2, \ldots, k_m)\alpha g_2(v_1, v_2, \ldots, v_m) = g_2(k_1, k_2, \ldots, k_m)\alpha g_1(v_1, v_2, \ldots, v_m) = 0$ (b) By (a) we get $g_1(k_1, k_2, \ldots, k_m)\alpha g_2(v_1, v_2, \ldots, v_m) = 0$ for every $k_1, k_2, \ldots, k_m, v_1, v_2, \ldots, v_m \in N$ and $\alpha \in \Gamma$. Replacing k_1 by $k_1\beta k_1'$, where $k_1' \in N$, in above eq. and taking Lemma 2.3 gives

$$0 = g_1(k_1'\beta k_1, k_2, \ldots, k_m)\alpha g_2(v_1, v_2, \ldots, v_m)$$
$$= g_1(k_1', k_2, \ldots, k_m)\beta t_1\alpha g_2(v_1, v_2, \ldots, v_m) + t_1'\beta h_1(t_1, t_2, \ldots, t_n)\alpha g_2(v_1, v_2, \ldots, v_m)$$
$$= k_1'\beta h_1(k_1, k_2, \ldots, k_m)\alpha g_2(v_1, v_2, \ldots, v_m).$$

It follows from the semiprimeness of N that $h_1(k_1, k_2, \ldots, k_m)\alpha g_2(v_1, v_2, \ldots, v_m) = 0$ for every $k_1, k_1', k_2, \ldots, k_m, v_1, v_2, \ldots, v_m \in N$ and $\alpha \in \Gamma$.

Thus $h_1(k_1, k_2, \ldots, k_m)\Gamma g_2(v_1, v_2, \ldots, v_m) = 0$. This is the first part of (b).

To show the second part we replace k_1 by $k_1\alpha k_1'$, where $k_1' \in N$, in above eq. and using Lemma 2.2, gives

$$0 = h_1(k_1\alpha k_1', k_2, \ldots, k_m)\beta g_2(v_1, v_2, \ldots, v_m)$$
$$= h_1(k_1, k_2, \ldots, k_m)\alpha k_1'\beta g_2(v_1, v_2, \ldots, v_m) + k_1\alpha h_1(k_1', k_2, \ldots, k_m)\beta g_2(v_1, v_2, \ldots, v_m)$$
$$= h_1(k_1, k_2, \ldots, k_m)\alpha t_1'\beta g_2(v_1, v_2, \ldots, v_m).$$

By Lemma 2.1 we obtain $g_2(v_1, v_2, \ldots, v_m)\gamma h_1(ik_1, tk_2, k_3\ldots, k_m) = 0$ for every $ik_1, k_2, k_3\ldots, ik_m, v_1, v_2, \ldots, v_m \in N$ and $\gamma \in \Gamma$. (c) The proof is similar to (b)

Theorem 3.2: *When N be a 2-torsion-free semi-prime Γ–(n-r). Suppose that g_1 and g_2 are two generalized Γ-n-der. of N linked with Γ-m-der. h_1 and h_2 respectively, the next term are equivalent:*

(a) *g_1 and g_2 are orthogonal.*

(b) *$g_1(ik_1, k_2, \ldots, k_m)\gamma g_2(v_1, v_2, \ldots, v_m) = h_1(k_1, k_2, \ldots, k_m)\gamma g_2(v_1, v_2, \ldots, v_m) = 0$ for every $k_1, k_2, \ldots, k_m, v_1, v_2, \ldots, v_m \in N, \gamma \in \Gamma$.*

(c) *$g_1(k_1, k_2, \ldots, k_m)\gamma g_2(v_1, v_2, \ldots, v_m) = h_1(k_1, k_2, \ldots, k_m)\gamma h_2(v_1, v_2, \ldots, v_m) = 0$ and $h_1(g_2(k_1, k_2, \ldots, k_m), v_2, \ldots, v_m) = h_1(h_2(k_1, k_2, \ldots, k_m), v_2, \ldots, v_m) = 0$*

for every $k_1, k_2, \ldots, k_m, v_1, v_2, \ldots, v_m \in N$.

Proof: The portion (a) $\Rightarrow$ (b) and (a) $\Rightarrow$ (c) are established by Lemma 3.1.

The portion, (a) $\Rightarrow$ (c) is immediate from Lemma 3.1.

(b) $\Rightarrow$ (a) We have, for every $k_1, k_2, \ldots, k_m, v_1, v_2, \ldots, v_m \in N$ and $\alpha \in \Gamma$.

$g_1(k_1, k_2, \ldots, k_m)\gamma g_2(v_1, v_2, \ldots, v_m)=h_1(k_1, k_2, \ldots, k_m)\gamma h_2(v_1, v_2, \ldots, v_m)=0$

Replacing k_1 by $k_1\alpha k_1'$ in the equation $g_1(k_1, k_2, \ldots, k_m)\gamma g_2(v_1, v_2, \ldots, v_m) = 0$ gives

$g_1(t_1\alpha t_1', t_2, \ldots, t_n)\gamma g_2(r_1, r_2, \ldots, r_n)=0$.Using hypothesis and Lemma 2.3 in last eq. obtain

$0 = g_1(k_1, k_2, \ldots, k_m)\alpha k_1'\gamma g_2(v_1, v_2, \ldots, v_m) + k_1\alpha h_1(k_1', k_2, \ldots, k_m)\gamma g_2(v_1, v_2, \ldots, v_m)$

$= g_1(k_1, k_2, \ldots, k_m)\alpha\, k_1'\gamma g_2(v_1, v_2, \ldots, v_m)$ for every $k_1, k_1', k_2, \ldots, k_m, v_1, v_2, \ldots, v_m \in N$, $\alpha, \gamma \in \Gamma$. By Lemma 2.1 we obtain g_1 and g_2 are orthogonal.

(c) $\Rightarrow$ (a) Since $h_1(g_2(k_1, k_2, \ldots, k_m), v_2, \ldots, v_m) = h_1(h_2(k_1, k_2, \ldots, k_m), v_2, \ldots, v_m) = 0$ for every $k_1, k_2, \ldots, k_m, v_2, \ldots, v_m \in N$. Replacing t_1 by $t_1\alpha t_1'$ in the equation $h_1(g_2(k_1, k_2, \ldots, k_m), v_2, \ldots, v_m) = 0$, we get

$0 = h_1(g_2(k_1\alpha k_1', k_2, \ldots, k_m), v_2, \ldots, v_m)$

$= h_1(g_2(k_1, k_2, \ldots, k_m)\alpha k_1', v_2, \ldots, v_m) + h_1(k_1\alpha h_2(k_1', k_2, \ldots, k_m), v_2, \ldots, v_m)$.

$= h_1(g_2(k_1, k_2, \ldots, k_m), v_2, \ldots, v_m)\alpha k_1' + g_2(k_1, k_2, \ldots, k_m)\alpha h_1(k_1', v_2, \ldots, v_m)+ h_1(k_1, v_2, \ldots, v_m)\alpha h_2(ik_1', ik_2, k_3\ldots, 'k_m)+k_1\alpha h_1(h_2(ik_1', 'k_2, k_3\ldots, 0k_m), v_2, \ldots, v_m)$. for every $'k_1, k_1', k_2, k_3\ldots, 'k_m, v_2, \ldots, v_m \in i N$. Hypothesis is being used once more obtain $g_2(k_1, k_2, \ldots, k_m)\alpha h_1(k_1', v_2, \ldots, v_m) = 0$. Replacing k_1' by $k_1'\gamma z$, where $z \in N$, in above eq. and using it once again, gives

$0 = g_2(k_1, k_2, \ldots, k_m)\alpha h_1(k_1'\gamma z, v_2, \ldots, v_m)$

$= g_2(k_1, k_2, \ldots, k_m)\alpha h_1(k_1', v_2, \ldots, v_m)\gamma z + g_2(k_1, k_2, \ldots, k_m)\alpha k_1'\gamma h_1(z, v_2, \ldots, v_m)$

$= g_2(k_1, k_2, \ldots, k_m)\alpha k_1'\gamma h_1(z, v_2, \ldots, v_m)$, for every $z, k_1, k_1', k_2, \ldots, k_n, v_2, \ldots, v_m \in N$. Hence by Lemma 2.1 we get

$h_1(z, v_2, \ldots, v_m)\alpha g_2(k_1, k_2, \ldots, k_m) = 0$, for every $k_1, k_2, \ldots, k_n, z, v_2, \ldots, v_m \in N$. Then (a) follows from (b)

Theorem 3.3: *When N a 2-torsionfree semi-prime Γ-(n-r), g be a generalized Γ-m-der. of N linked with Γ-m-der. h of N. If $g(k_1, k_2, \ldots, k_m)\ \Gamma\ g(v_1, v_2, \ldots, v_m) = 0$ for every $k_1, k_2, \ldots, k_m, v_1, v_2, \ldots, v_m \in N$, then $g = h = 0$.*

Proof: By supposition have

$g(k_1, k_2, \ldots, k_m)\gamma g(v_1, v_2, \ldots, v_m) = 0$, for every $k_1, k_2, \ldots, k_m, v_1, v_2, \ldots, v_m \in N, \alpha \in \Gamma$. (1)

Taking v_1 by $v_1\alpha v_1'$ in above eq. (1) and us it again, implies that

$0 = g(k_1, k_2, \ldots, k_m)\gamma g(v_1\alpha v_1', v_2, \ldots, v_m)$

$= g(k_1, k_2, \ldots, k_m)\gamma g(v_1, v_2, \ldots, v_m)\alpha v_1' + g(k_1, k_2, \ldots, k_m)\gamma v_1\alpha h(v_1', v_2, \ldots, v_m)$

$= g(k_1, k_2, \ldots, k_m)\gamma v_1\alpha h(v_1', v_2, \ldots, v_m)$

By Lemma 2.1 obtain $h(v_1', v_2, \ldots, v_m)\gamma g(k_1, k_2, \ldots, k_m) = 0$ (2)

Replacing k_1 by $k_1\beta v_1'$ in eq. (2) and when we use it again, give

$0 = h(v_1', v_2, \ldots, v_m)\gamma g(k_1\beta v_1', k_2, \ldots, k_m)$

$= h(v_1', v_2, \ldots, v_m)\gamma g(k_1, k_2, \ldots, k_m)\beta v_1' + h(v_1', \ldots, v_m)\gamma k_1\beta h(v_1', v_2, v_3\ldots, v_m)$

$= h(v_1', iv_2, \ldots, v_m)\gamma k_1\beta h(v_1', k_2, \ldots, k_m)$ for every $k_1, k_2, \ldots, k_m, v_1', v_2, \ldots, v_m \in N$.

In particular $h(v_1', iv_2, v_3\ldots, v_m)\gamma v_1\beta h(v_1', v_2, v_3\ldots, v_m)$. Semi-primeness of N give h = 0. By hypothesis and Lemma 2.3, obtain

$0 = g(k_1\beta v, k_2, \ldots, k_m)\gamma g(v_1, v_2, \ldots, v_m) = g(k_1, k_2, \ldots, k_m)\beta v\gamma g(v_1, v_2, \ldots, v_m) + k_1\beta h(v, k_2, \ldots, k_m)\gamma h(v_1, v_2, \ldots, v_m)$. By eq. (1) we conclude that $g(k_1, k_2, \ldots, k_m)\beta v\gamma g(v_1, v_2, \ldots, v_m) = 0$ for every $k_1, k_2, \ldots, k_m, v, v_1, v_1', v_2, \ldots, v_m \in N$. Thus we get g = 0 by the semi-primeness of N.

4. Conclusion

In the present work, we defined two generalized orthogonality Γ-m-der. In Γ-(n-r) also orthogonal of two Γ-m-der. in semi-prime Γ-(n-r). And give characterizations of Γ-m-der as a product of two Γ-m-der. in 2-torsion-free semi-prime Γ-(n-r).

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