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Hyper and multiple f-index of nano star dendrimers

Sirous Ghobadi *
Department of Mathematics
Qaemshahr Branch
Islamic Azad University
Qaemshahr
Iran

Masoud Ghods †
Department of Mathematics
Semnan University
Faculty of Mathematics
Statistics and Computer Science
Semnan
Iran

Mobina Ghorbaninejad §
Department of Mathematics
Allame Tabarsi Institute
Qaemshahr
Iran

Abstract

Dendrimers are branching molecules that consist of a central part attached to tree-like branches which are repeatedly connected to each other. Topological indices of a molecular graph are real numbers associated with graphs, which represents some of the properties of the molecule. In this article, we obtaine precise formulas for the *hyper F-index* and *multiple F-index* of a number of nano star dendrimers.

Subject Classification: *05C09, 05C90, 05C92.*

Keywords: *F-index, Hyper F-index, Multiple F-index, Nano star dendrimer.*

E-mail:* **ghobadimath46@gmail.com or s.ghobadi@qaemiau.ac.ir (Corresponding Author)
†E-mail: **mghods@semnan.ac.ir**
§E-mail: **ghorbani325@gmail.com**

1. Introduction

A representation of a molecule that gives us information about the number of atoms composing it, and their connectivity, is called topological representation of a molecule. A topological representation of a molecule is defined molecular graph. The vertices of the molecular graph G is denoted by V correspond to the atoms of the molecule and the edges set of G denoted by E correspond to the chemical bonds between the atoms. If there exists a path between every pair of vertices of a graph G, then G is connected . The degree of a vertex y in G is denoted by $d_G(y)$.

Dendrimers are a novel class of globular nanoscaled macromolecules characterized by highly branched tree like structures that provide a high degree of surface functionality and versatility [22]. Due to their multivalent and monodisperse character, dendrimers have stimulated wide interest in the field of chemistry and biology. Dendrimers have applications in biomedical field [16], anticancer drugs [17], drug delivery [13], transdermal drug delivery [4], gene delivery [20], dendritic sensors [3], enhancing solubility [8] and photodynamic therapy [21].

Parts of macromolecules are called nano star dendrimer, which is used in the formation of nano latex, nano tubes, colored glasses, micro and macro capsules, and so on [2,6,14].

Applying graph theory in modeling chemical phenomena is called chemical graph theory. A topological index of a chemical graph, is a number dependent on the graph, which is structurally unchanged. Topological index was first introduced by Wiener in [23]. An exact formula for the Wiener polarity index of nanostar dendrimers was obtained by Alaeiyan et al. in [1]. For a graph $G=(V,E)$, the *first Zagreb index* was introduced by Gutman and coworker [12] and defined as,

$$M_1(G)=\sum_{x\in V(G)} d_G^2(x)=\sum_{xy\in E(G)}[d_G(x)+d_G(y)].$$

Shirdel et al. in [18] defined *hyper Zagreb index* as

$$HM(G)=\sum_{xy\in E(G)}[d_G(x)+d_G(y)]^2.$$

To see more results about the hyper-zagreb index for an infinite family of nanostar dendrimer please refer to [10].

Ghorbani and Azimi in [11] defined *first multiple Zagreb index* (PM_1) as

$$PM_1(G) = \prod_{xy \in E(G)} [d_G(x) + d_G(y)].$$

To see more results about PM_1 please refer to [7,11,15].

The forgotten topological index $(F-index)$ was another topological index introduced by Gutman and Trinajastic in [12].

For a graph $G=(V,E),$ in [9] Furtula and Gutman, defined this index as,

$$F(G) = \sum_{x \in V(G)} d_G^3(x) = \sum_{xy \in E(G)} [d_G^2(x) + d_G^2(y)].$$

Similar to the multiple first Zagreb index and hyper Zagreb index we define *multiple Forgotten index*, $(PF(G)),$ and *hyper forgotten index*, $(HF(G)),$ as follows,

$$PF(G) = \prod_{xy \in E(G)} [d_G^2(x) + d_G^2(y)],$$

and

$$HF(G) = \sum_{xy \in E(G)} [d_G^2(x) + d_G^2(y)]^2.$$

In [19], the multiple first Zagreb index and hyper Zagreb index of nano star dendrimers $NS_1[k]$ and $NS_2[k]$ was computed by Siddiqui et al. as follows,

$$PM_1(NS_1[k]) = 3^{2^{k+1}} \times 4^{4(2^k-1)} \times 4^{12\cdot 2^k - 11} \times 5^{14\cdot 2^k - 14},$$

$$HM(NS_1[k]) = 9\cdot 2^{k+1} + 64(2^k-1) + 16(12\cdot 2^k - 11) + 25(14\cdot 2^k - 14),$$

$$PM_1(NS_2[k]) = 3^{2^{k+1}} \times 4^{8\cdot 2^k - 5} \times 5^{6\cdot 2^k - 6},$$

and

$$HM(NS_2[k]) = 9\cdot 2^{k+1} + 16(8\cdot 2^k - 5) + 25(6\cdot 2^k - 6).$$

Further, N. De and his coworker in [5] introduced forgotten index, and forgotten polynomials of numbers of nano star dendrimers. In this article, we obtain *hyper forgotten index*, and *multiple forgotten index* of some nano star dendrimers namely $NS_1[k], NS_2[k],$ $NS_3[k], NS_4[k], NS_5[k], D_k$ and $Linear[k]-Tetracene$.

2. Main results

In this section, we suppose that graph G is one of the nano star dendrimers with $k \geq 2$ vertices. Let e_{ij} specifies the number of edges of graph connecting the vertices of degrees i and j such that $e_{ij} = e_{ji}$, as well, in each branch, e'_{ij} be the number of edges connecting vertices of degrees i and j, $(i, j \leq 4)$.

Let $NS_1[k]$ be the first type of nano star dendrimers with four alike branches and three additional edges, where k is the number of growth steps (see Figure 1). Therefore in this case, we have $e_{12} = 4e'_{12}$, $e_{22} = 4e'_{22} + 1$, $e_{13} = 4e'_{13}$, $e_{23} = 4e'_{23} + 2$.

It is easy to see that $e'_{12} = 2^{k-1}$, then $e_{12} = 4e'_{12} = 2 \cdot 2^k$. Using a similar argument, we have $e'_{22} = 3k - 3$, then $e_{22} = 12 \cdot 2^k - 11$; $e'_{13} = 2^k - 1$ then $e_{13} = 4e'_{13} = 4 \cdot 2^k - 4$; and finally $e'_{23} = 3(2^k - 1) + (2^{k-1} - 1)$, then $e_{23} = 4e'_{23} + 2 = 14 \cdot 2^k - 14$.

Theorem 2.1

$$HF(NS_1[k]) = 25 \times 2^{k+1} + 400(2^k - 1) + 64(12 \cdot 2^k - 11) + 169(14 \cdot 2^k - 14),$$

and

$$PF(NS_1[k]) = 5^{2^{k+1}} \times 10^{4(2^k - 1)} \times 8^{(12 \cdot 2^k - 11)} \times 13^{(14 \cdot 2^k - 14)}.$$

Proof: We divide the edge set of $NS_1[k]$ into four edge partitions based on the degrees of end vertices as follows,

$$E(NS_1[k]) = \bigcup_{i=1}^{4} E_i(NS_1[k]),$$

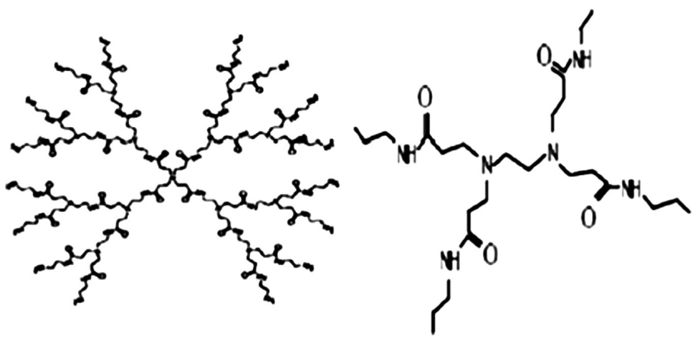

Figure 1

Nano star Dendrimer $NS_1[k]$.

where

$$E_1(NS_1[k]) = \{xy \in E(NS_1[k]) : d_{NS_1[k]}(x) = 1, \quad \text{and} \quad d_{NS_1[k]}(y) = 2\},$$

$$E_2(NS_1[k]) = \{xy \in E(NS_1[k]) : d_{NS_1[k]}(x) = 1, \ \text{and} \ d_{NS_1[k]}(y) = 3\},$$

$$E_3(NS_1[k]) = \{xy \in E(NS_1[k]) : d_{NS_1[k]}(x) = 2, \quad \text{and} \quad d_{NS_1[k]}(y) = 2\},$$

and

$$E_4(NS_1[k]) = \{xy \in E(NS_1[k]) : d_{NS_1[k]}(x) = 2, \quad \text{and} \quad d_{NS_1[k]}(y) = 3\}.$$

So that, $|E_1(NS_1[k])| = 2^{k+1}$, $|E_2(NS_1[k])| = 4(2^k - 1)$, $|E_3(NS_1[k])| = 12 \cdot 2^k - 11$, and $|E_4(NS_1[k])| = 14 \cdot 2^k - 14$.

By the definition of hyper F-index and multiple F-index we have,

$$\begin{aligned}
HF(NS_1[k]) &= \sum_{xy \in E_1(NS_1[k])} [d^2_{NS_1[k]}(x) + d^2_{NS_1[k]}(y)]^2 \\
&\quad + \sum_{xy \in E_2(NS_1[k])} [d^2_{NS_1[k]}(x) + d^2_{NS_1[k]}(y)]^2 \\
&\quad + \sum_{xy \in E_3(NS_1[k])} [d^2_{NS_1[k]}(x) + d^2_{NS_1[k]}(y)]^2 \\
&\quad + \sum_{xy \in E_4(NS_1[k])} [d^2_{NS_1[k]}(x) + d^2_{NS_1[k]}(y)]^2 \\
&= 25|E_1(NS_1[k])| + 100|E_2(NS_1[k])| + 64|E_3(NS_1[k])| \\
&\quad + 169|E_4(NS_1[k])| \\
&= 25 \times 2^{k+1} + 400(2^k - 1) + 64(12 \cdot 2^k - 11) + 169(14 \cdot 2^k - 14).
\end{aligned}$$

Now

$$\begin{aligned}
PF(NS_1[k]) &= \prod_{xy \in E_1(NS_1[k])} [d^2_{NS_1[k]}(x) + d^2_{NS_1[k]}(y)] \\
&\quad \times \prod_{xy \in E_2(NS_1[k])} [d^2_{NS_1[k]}(x) + d^2_{NS_1[k]}(y)] \\
&\quad \times \prod_{xy \in E_3(NS_1[k])} [d^2_{NS_1[k]}(x) + d^2_{NS_1[k]}(y)] \\
&\quad \times \prod_{xy \in E_4(NS_1[k])} [d^2_{NS_1[k]}(x) + d^2_{NS_1[k]}(y)] \\
&= 5^{|E_1(NS_1[k])|} \times 10^{|E_2(NS_1[k])|} \times 8^{|E_3(NS_1[k])|} \times 13^{|E_4(NS_1[k])|} \\
&= 5^{2^{k+1}} \times 10^{4(2^k - 1)} \times 8^{12 \cdot 2^k - 11} \times 13^{14 \cdot 2^k - 14}.
\end{aligned}$$

$\square$

In the second type of nano star dendrimer $(NS_2[k])$, where k is the number of growth steps (see Figure 2), we have four alike branches, and five additional edges, in which $e_{12} = 4e'_{12}$, $e_{22} = 4e'_{22} + 3$, and $e_{23} = 4e'_{23} + 2$.

By simple computation we have, $e'_{12} = 2^{k-1}$, $e'_{22} = 2(2^k - 1)$, and $e'_{23} = 3 \cdot 2^k - 2$. Thus $e_{12} = 2 \cdot 2^{k+1}$, $e_{22} = 8 \cdot 2^k - 5$, and $e_{23} = 6 \cdot 2^k - 6$.

Theorem 2.2

$$HF(NS_2[k]) = 25 \cdot 2^{k+1} + 64(8 \cdot 2^k - 5) + 169(6 \cdot 2^k - 6),$$

and

$$PF(NS_2[k]) = 5^{2^{(k+1)}} \times 8^{(8 \cdot 2^k - 5)} \times 13^{(6 \cdot 2^k - 6)}.$$

***Proof*:** Suppose, we divide the edge set of $NS_2[k]$ into three edge partitions based on the degrees of end vertices. We have

$$E_1(NS_2[k]) = \{xy \in E(NS_2[k]) : d_{NS_2[k]}(x) = 1, \quad \text{and} \quad d_{NS_2[k]}(y) = 2\},$$

$$E_2(NS_2[k]) = \{xy \in E(NS_2[k]) : d_{NS_2[k]}(x) = 2, \quad \text{and} \quad d_{NS_2[k]}(y) = 2\},$$

and

$$E_3(NS_2[k]) = \{xy \in E(NS_2[k]) : d_{NS_2[k]}(x) = 2, \quad \text{and} \quad d_{NS_2[k]}(y) = 3\}.$$

Thus, $|E_1(NS_2[k])| = 2^{k+1}$, $|E_2(NS_2[k])| = 8 \cdot 2^k - 5$, and $|E_3(NS_2[k])| = 6 \cdot 2^k - 6$. From definitions of hyper and multiple F-index we have,

$$\begin{aligned} HF(NS_2[k]) = & \sum_{xy \in E_1(NS_2[k])} [d^2_{NS_2[k]}(x) + d^2_{NS_2[k]}(y)]^2 \\ & + \sum_{xy \in E_2(NS_2[k])} [d^2_{NS_2[k]}(x) + d^2_{NS_2[k]}(y)]^2 \\ & + \sum_{xy \in E_3(NS_2[k])} [d^2_{NS_2[k]}(x) + d^2_{NS_2[k]}(y)]^2 \\ & = 25|E_1(NS_2[k])| + 64|E_2(NS_2[k])| + 169|E_3(NS_2[k])| \\ & = 25 \times 2^{k+1} + 64(8 \cdot 2^k - 5) + 169(6 \cdot 2^k - 6), \end{aligned}$$

and

$$\begin{aligned} PF(NS_2[k]) = & \prod_{xy \in E_1(NS_2[k])} [d^2_{NS_2[k]}(x) + d^2_{NS_2[k]}(y)] \\ & \times \prod_{xy \in E_2(NS_2[k])} [d^2_{NS_2[k]}(x) + d^2_{NS_2[k]}(y)] \end{aligned}$$

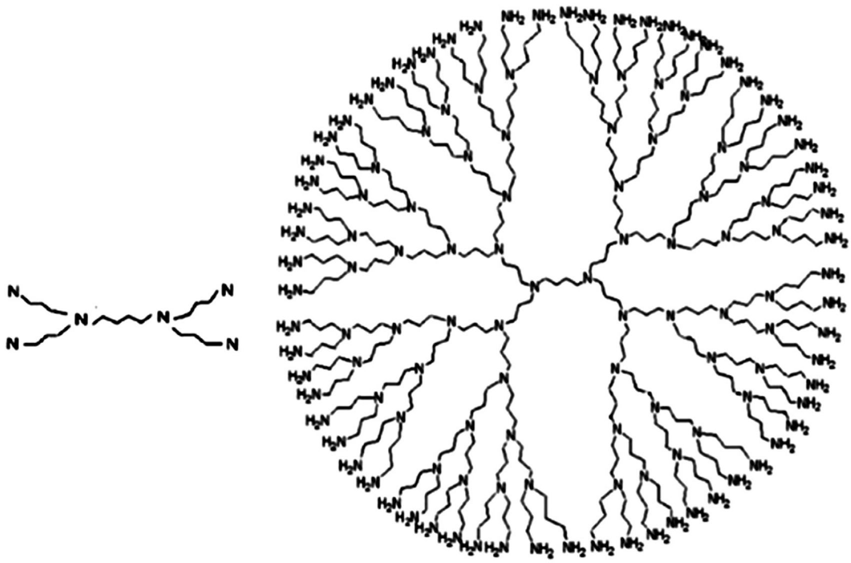

Figure 2

Nano star Dendrimer $NS_2[k]$.

$$\times \prod_{xy \in E_3(NS_2[k])} [d^2_{NS_2[k]}(x) + d^2_{NS_2[k]}(y)]$$
$$= 5^{|E_1(NS_2[k])|} \times 8^{|E_2(NS_2[k])|} \times 13^{|E_3(NS_2[k])|}$$
$$= 5^{2^{k+1}} \times 8^{8 \cdot 2^k - 5} \times 13^{6 \cdot 2^k - 6}.$$

□

In the third class of nano star dendrimer $NS_3[k]$ (see Figure 3), with simple calculation, we can see that $e_{12} = 3 \cdot 2^k$, $e_{22} = 54 \cdot 2^{k-1} - 24$, $e_{23} = 66(2^{k-1} - 1) + 48$, and $e_{33} = 3 \cdot 2^{k+1}$.

Theorem 2.3 :

$$HF(NS_3[k]) = 169(66(2^{k+1} + 1)48) + 64(54 \cdot 2^{k-1} - 24) + 324(3 \cdot 2^{k+1}) + 25(3 \cdot 2^k),$$

and

$$PF(NS_3[k]) = 13^{66(2^{k-1}+1)48} \times 8^{(54 \cdot 2^{k-1} - 24)} \times 18^{3 \cdot 2^{k+1}} \times 5^{3 \cdot 2^k}.$$

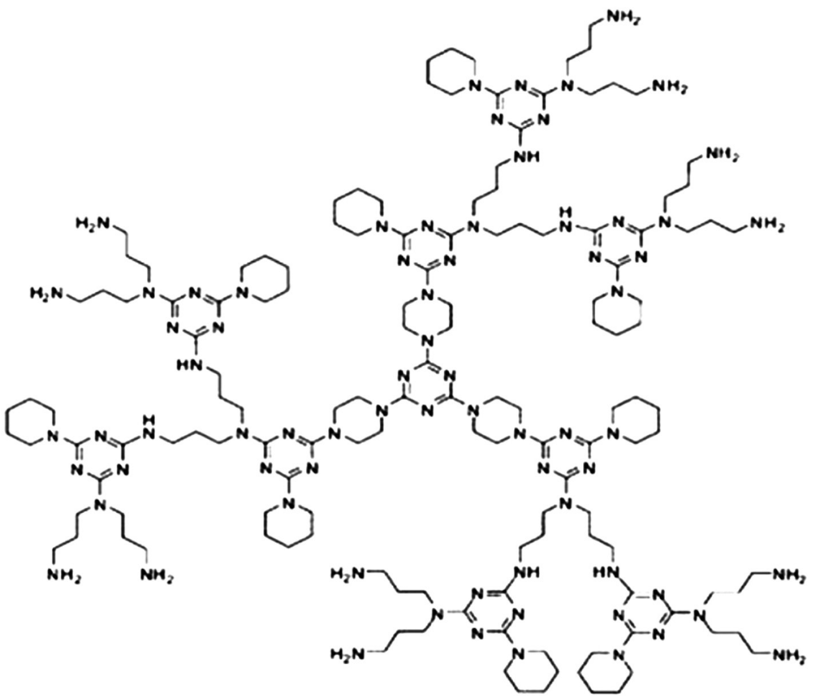

Figure 3
Nano star Dendrimer $NS_3[k]$.

Proof : Due to the above theorems we divide the edge set of $NS_3[k]$ into four edge partitions, based on degrees of end vertices. Thus, we write

$$E_1(NS_3[k]) = \{xy \in E(NS_3[k]) : d_{NS_3[k]}(x) = 2, \text{ and } d_{NS_3[k]}(y) = 3\},$$

$$E_2(NS_3[k]) = \{xy \in E(NS_3[k]) : d_{NS_3[k]}(x) = 2, \text{ and } d_{NS_3[k]}(y) = 2\},$$

$$E_3(NS_3[k]) = \{xy \in E(NS_3[k]) : d_{NS_3[k]}(x) = 3, \text{ and } d_{NS_3[k]}(y) = 3\},$$

and

$$E_4(NS_3[k]) = \{xy \in E(NS_3[k]) : d_{NS_3[k]}(x) = 1, \text{ and } d_{NS_3[k]}(y) = 2\}.$$

Hence, $|E_1(NS_3[k])| = 66(2^{k-1}+1)48$, $|E_2(NS_3[k])| = 54\cdot 2^{k-1} - 24$, $|E_3(NS_3[k])| = 3\cdot 2^{k+1}$, and $|E_4(NS_3[k])| = 3\cdot 2^{k}$. We have,

$$
\begin{aligned}
HF(NS_3[k]) &= \sum_{xy \in E_1(NS_3[k])} [d^2_{NS_3[k]}(x) + d^2_{NS_3[k]}(y)]^2 \\
&\quad + \sum_{xy \in E_2(NS_3[k])} [d^2_{NS_3[k]}(x) + d^2_{NS_3[k]}(y)]^2 \\
&\quad + \sum_{xy \in E_3(NS_3[k])} [d^2_{NS_3[k]}(x) + d^2_{NS_3[k]}(y)]^2 \\
&\quad + \sum_{xy \in E_4(NS_3[k])} [d^2_{NS_3[k]}(x) + d^2_{NS_3[k]}(y)]^2 \\
&= 169\,|E_1(NS_3[k])| + 64\,|E_2(NS_3[k])| + 324\,|E_3(NS_3[k])| \\
&\quad + 25\,|E_4(NS_3[k])| \\
&= 169(66(2^{k+1}+1)48) + 64(54 \cdot 2^{k-1} - 24) + 324(3 \cdot 2^{k+1}) + 25(3 \cdot 2^{k}),
\end{aligned}
$$

and

$$
\begin{aligned}
PF(NS_3[k]) &= \prod_{xy \in E_1(NS_3[k])} [d^2_{NS_3[k]}(x) + d^2_{NS_3[k]}(y)] \\
&\quad \times \prod_{xy \in E_2(NS_3[k])} [d^2_{NS_3[k]}(x) + d^2_{NS_3[k]}(y)] \\
&\quad \times \prod_{xy \in E_3(NS_3[k])} [d^2_{NS_3[k]}(x) + d^2_{NS_3[k]}(y)] \\
&\quad \times \prod_{xy \in E_4(NS_3[k])} [d^2_{NS_3[k]}(x) + d^2_{NS_3[k]}(y)] \\
&= 13^{|E_1(NS_3[k])|} \times 8^{|E_2(NS_3[k])|} \times 18^{|E_3(NS_3[k])|} \times 5^{|E_4(NS_3[k])|} \\
&= 13^{66(2^{k-1}+1)48} \times 8^{(54 \cdot 2^{k-1} - 24)} \times 18^{3 \cdot 2^{k+1}} \times 5^{3 \cdot 2^{k}}.
\end{aligned}
$$

$\square$

Since, $NS_4[k]$ (see Figure 4) has two similar branches, then by simple computation we can see that $e_{22} = 2^{k+2} + 2$, $e_{23} = 32 \cdot 2^{k-1} - 8$, $e_{33} = 86$, $e_{34} = 6$, and $e_{44} = 3$.

Theorem 2.4 :

$$HF(NS_4[k]) = 169(32 \cdot 2^{k-1} - 8) + 64(2^{k+1} + 2) + 100(2^{k+1}) + 34686,$$

and

$$PF(NS_4[k]) = 13^{(32 \cdot 2^{k-1} - 8)} \times 8^{(2^{k+1}+2)} \times 10^{2^{k+1}} \times 18^{86} \times 25^{6} \times 32^{3}.$$

Proof : As we saw in the above theorems, we divide the edge set of $NS_4[k]$ as follows,

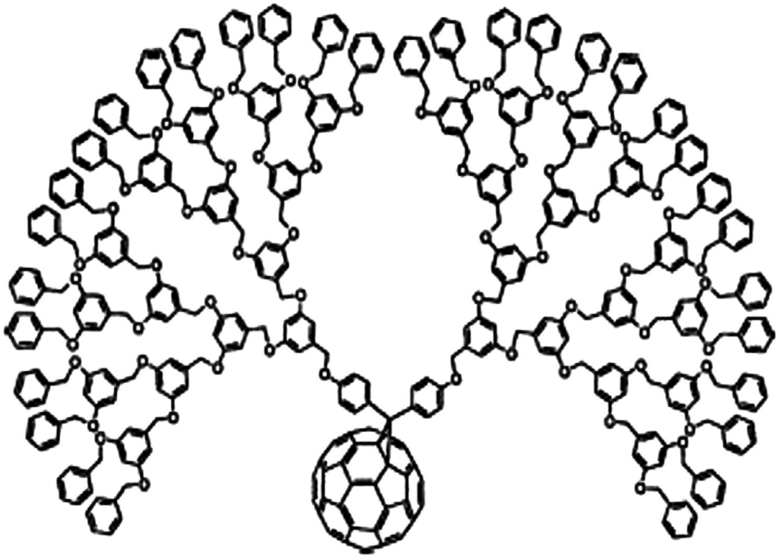

Figure 4

Nano star Dendrimer $NS_4[k]$.

$$E_1(NS_4[k]) = \{xy \in E(NS_4[k]) : d_{NS_4[k]}(x) = 2, \text{ and } d_{NS_4[k]}(y) = 3\},$$

$$E_2(NS_4[k]) = \{xy \in E(NS_4[k]) : d_{NS_4[k]}(x) = 2, \text{ and } d_{NS_4[k]}(y) = 2\},$$

$$E_3(NS_4[k]) = \{xy \in E(NS_3[k]) : d_{NS_4[k]}(x) = 1, \text{ and } d_{NS_4[k]}(y) = 3\},$$

$$E_4(NS_4[k]) = \{xy \in E(NS_4[k]) : d_{NS_4[k]}(x) = 3, \text{ and } d_{NS_4[k]}(y) = 3\},$$

$$E_5(NS_4[k]) = \{xy \in E(NS_4[k]) : d_{NS_4[k]}(x) = 3, \text{ and } d_{NS_4[k]}(y) = 4\},$$

and

$$E_6(NS_4[k]) = \{xy \in E(NS_4[k]) : d_{NS_4[k]}(x) = 4, \text{and } d_{NS_4[k]}(y) = 4\}.$$

So, $|E_1(NS_4[k])| = 32 \cdot 2^{k-1} - 8$, $|E_2(NS_4[k])| = 2^{k+1} + 2$, $|E_3(NS_4[k])| = 2^{k+1}$, $|E_4(NS_4[k])| = 86$, $|E_5(NS_4[k])| = 6$, and $|E_4(NS_4[k])| = 3$.

From the definitions we have,

$$
\begin{aligned}
HF(NS_4[k]) &= \sum_{xy\in E_1(NS_4[k])}[d^2_{NS_4[k]}(x)+d^2_{NS_4[k]}(y)]^2 \\
&+ \sum_{xy\in E_2(NS_4[k])}[d^2_{NS_4[k]}(x)+d^2_{NS_4[k]}(y)]^2 \\
&+ \sum_{xy\in E_3(NS_4[k])}[d^2_{NS_4[k]}(x)+d^2_{NS_4[k]}(y)]^2 \\
&+ \sum_{xy\in E_4(NS_4[k])}[d^2_{NS_4[k]}(x)+d^2_{NS_4[k]}(y)]^2 \\
&+ \sum_{xy\in E_5(NS_4[k])}[d^2_{NS_4[k]}(x)+d^2_{NS_4[k]}(y)]^2 \\
&+ \sum_{xy\in E_6(NS_4[k])}[d^2_{NS_4[k]}(x)+d^2_{NS_4[k]}(y)]^2 \\
&= 169\,|E_1(NS_4[k])| + 64\,|E_2(NS_4[k])| + 100\,|E_3(NS_4[k])| \\
&+ 324\,|E_4(NS_4[k])| + 625\,|E_5(NS_4[k])| + 1024\,|E_6(NS_4[k])| \\
&= 169(32\cdot 2^{k-1}-8)+64(2^{k+1}+2)+100(2^{k+1})+34686,
\end{aligned}
$$

and

$$
\begin{aligned}
PF(NS_4[k]) &= \prod_{xy\in E_1(NS_4[k])}[d^2_{NS_4[k]}(x)+d^2_{NS_4[k]}(y)] \\
&\times \prod_{xy\in E_2(NS_4[k])}[d^2_{NS_4[k]}(x)+d^2_{NS_4[k]}(y)] \\
&\times \prod_{xy\in E_3(NS_4[k])}[d^2_{NS_4[k]}(x)+d^2_{NS_4[k]}(y)] \\
&\times \prod_{xy\in E_4(NS_4[k])}[d^2_{NS_4[k]}(x)+d^2_{NS_4[k]}(y)] \\
&\times \prod_{xy\in E_5(NS_4[k])}[d^2_{NS_4[k]}(x)+d^2_{NS_4[k]}(y)] \\
&\times \prod_{xy\in E_6(NS_4[k])}[d^2_{NS_4[k]}(x)+d^2_{NS_4[k]}(y)] \\
&= 13^{|E_1(NS_4[k])|}\times 8^{|E_2(NS_4[k])|}\times 10^{|E_3(NS_4[k])|}\times 18^{|E_4(NS_4[k])|} \\
&\times 25^{|E_5(NS_4[k])|}\times 32^{|E_6(NS_4[k])|} \\
&= 13^{(32\cdot 2^{k-1}-8)}\times 8^{(2^{k+1}+2)}\times 10^{2^{k+1}}\times 18^{86}\times 25^{6}\times 32^{3}.
\end{aligned}
$$

□

Finally, for the fifth class of nano star dendrimers, $NS_5[k]$ (see Figure 5) there are three similar branches, and obviously $e_{13}=3e'_{13}+3$, $e_{22}=3e'_{23}+12$, $e_{23}=3e'_{23}+48$, and $e_{33}=24$, in which $e'_{13}=2^{k+1}$,

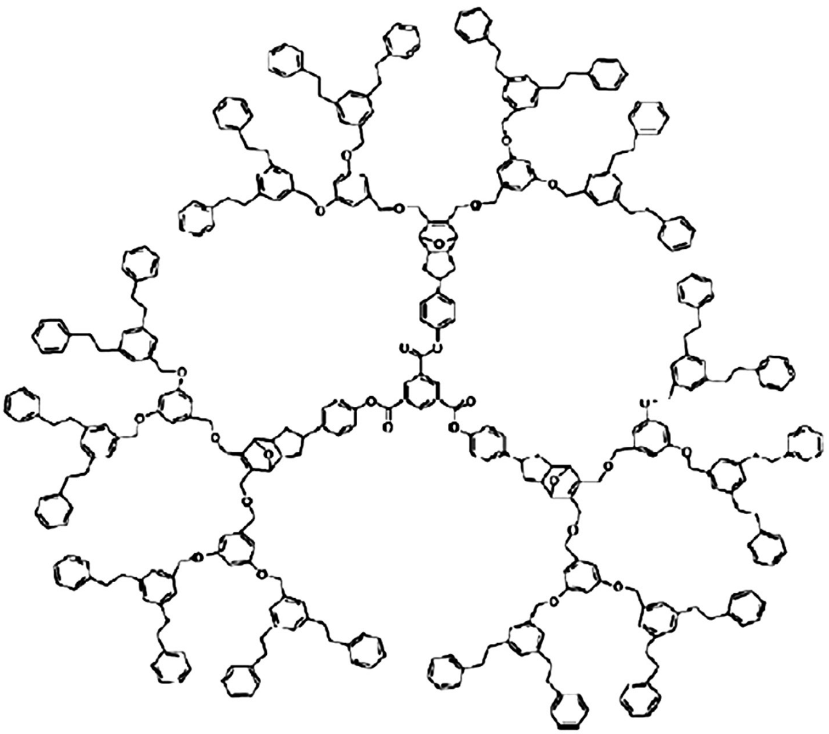

Figure 5

Nano star Dendrimer $NS_5[k]$.

$e'_{22} = 2^{k+1} - 2$, and $e'_{23} = 9(2^{k+1} - 2) - 2^{k+1}$. Hence, $e_{13} = 3(2^{k+1} + 1)$, $e_{22} = 6(2^k + 1)$, $e_{23} = 6(2^{k+3} - 1)$, and $e_{33} = 24$. Also for the another two nano star dendrimers D_k (see Figure 6) and T, (Linear [k]-Tetracene) (see Figure 7), we can divide the edge set into three edge partitions based on degrees of end vertices.

In the three next theorems, since the proof is similar to the above theorems then we omit the proof.

Theorem 2.5 :

$$HF(NS_5[k]) = 169(6(2^{k+3} - 1)) + 64(6(2^{k+1} + 1)) + 100(3(2^{k+1} + 1)) + 7776,$$

and

$$PF(NS_5[k]) = 13^{6(2^{k+3}-1)} \times 8^{6(2^k+1)} \times 18^{24} \times 10^{3(2^{k+1}+1)}.$$

H

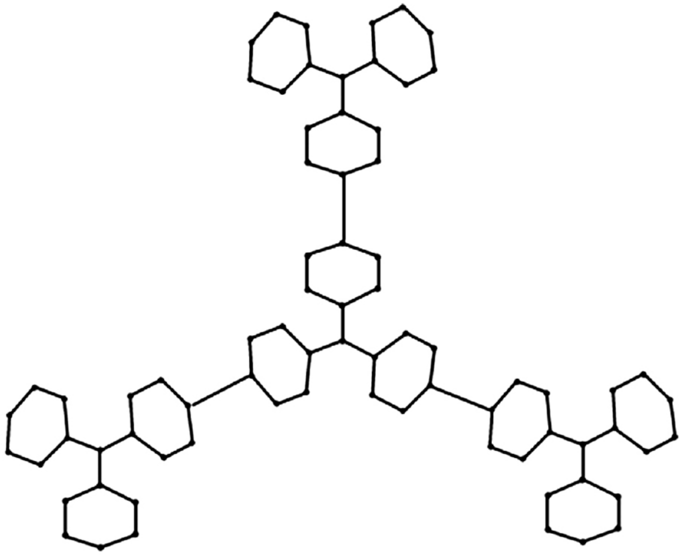

Figure 6

Nano star Dendrimer D_k ($k = 2$).

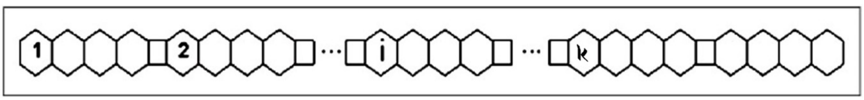

Figure 7

The molecular graph of a linear [k]-Tetracene.

Theorem 2.6 :

$$HF(D_k) = 64(24 \cdot 2^{k-1} - 12) + 169(30 \cdot 2^{k-1} - 24) + 324(12 \cdot 2^{k-1} - 9),$$

and

$$PF(D_k) = 8^{24 \cdot 2^{k-1} - 12} \times 13^{30 \cdot 2^{k-1} - 24} \times 18^{12 \cdot 2^{k-1} - 9}.$$

Theorem 2.7 :

$$HF(T) = 384 + 169(16k - 4) + 324(7k - 4),$$

and

$$PF(T) = 8^6 \times 13^{16k-4} \times 18^{7k-4}.$$

3. Conclusion

Dendrimers are characterized by individual characteristics that make them suitable for many applications. The great importance of dendrimers in chemistry became apparent immediately after their prepared. Dendrimers are quite definite artificial macromolecules, which are characterized by a combination of a large number of functional groups and a compact molecular structure. Work was established to determine hyper F-index and multiple F-index of nano star dendrimers namely $NS_1[k], NS_2[k], NS_3[k], NS_4[k], NS_5[k], D_k$ and $Linear[k]-Tetracene$.

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