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Generalized (α, β)-reverse derivations in semi prime near rings

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Abstract

In this paper, our aim is to study the right generalized (α, β)-reverse derivations on semi-prime near-ring and generalized dependent element of right generalized (α, β)-reverse

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derivation and right generalized β-reverse derivation on semi-prime near-ring. We generalize some useful results by using these notions.

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1. Introduction

Near rings are generalization of rings. Dikson initiated the research on near-rings in 1905. In 1987, the derivations of near ring was studied by Bell and Mason and proved some commutativity theorem for near rings [1]. Numerous researchers have studied commutativity of prime and semi prime rings, accepting sufficiently constrained derivations [2, 3, 5, 6, 7, 8, 11, 13, 15, 16, 17].

Herstein introduced the concept of reverse derivation [9]. Later, the idea of reverse derivation and some properties of reverse derivation were studied by Bresar and Vukman [4]. The reverse derivation on semi prime rings have been studied in [12]. Recently Sirivoravit and Leerawat have generalized the idea of reverse derivation to a reverse *f*-derivation on generalized Γ-ring and obtained some conditions which concludes that the zero map is the only reverse *f*-derivation on M [14]. In this paper, we prove some useful results by using the idea of generalized (α,β)-reverse derivations on semi prime near-rings. We also prove some results by using the notion of generalized β-reverse derivations on semi prime near-rings.

2. Preliminaries

In this section, we present some basic definitions.

Definition 2.1 : Let $W \neq \{\}$ with binary operations "." (multiplication) and "+" (addition) if:

(a) $(W,+)$ a group.

(b) $(W,.)$ a semi-group.

(c) $l(m+n) = lm + ln \quad ((m+n)l = ml + nl) \quad \forall\, l,m,n \in \mathrm{W}$,

are satisfied then it is a left(right) near ring.

Definition 2.2 : The mapping $k : W \to W$ is known to be (α, β)-derivation if $k(lm) = k(l)\alpha(m) + \beta(l)k(m)$ hold for all $l, m \in W$, where α and β are mappings.

Definition 2.3 : A near ring W is known as semi prime if $lWl = 0$ for all $l \in W$ implies $l = 0$.

Definition 2.4 : The Jordan product or anti-commutator is defined as $lom = lm + ml$ for all $l, m \in W$ and the Lie product or commutator is defined as $[l, m] = lm - ml$.

Definition 2.5 : Some identities are defined as below for any $l, m, n \in W$:

(a) $[l, mn] = m[l, n] + [l, m]n$
(b) $[lm, n] = l[m, n] + [l, n]m$
(c) $lo(mn) = (lom)n - m[l, n] = m(lon) + [l, m]n$
(d) $(lm)on = l(mon) - [l, n]m = (lon)m + l[m, n]$.

Definition 2.6 : A near ring W is called a 2-torsion free if $2l = 0$, with $l \in W$, then $l = 0$.

Definition 2.7 : An additive subgroup L of W satisfying $[l, n] \in L$ for all $l \in L, n \in W$ is said to be a lie ideal of W.

Definition 2.8 : A lie ideal L of W is called square closed lie ideal of W. If $l^2 \in L$ for all $l \in L$. Moreover, if L is a square closed lie ideal of W, then $2lm \in L$ for all $l, m \in L$.

Definition 2.9 : The multiplicative center of W is defined as:

$$Z(W) = \{l \in W : lm = ml, \text{for all } m \in W\}.$$

Definition 2.10 : The derivation k will be commuting if $0 = [l, k(l)]$, for all $l \in W$.

3. Generalized (α, β)-Reverse Derivations on Semi Prime Near Rings

In this section, we prove some results on generalized (α, β)-reverse derivations on semi prime near rings.

Definition 3.1 : An additive mapping $k: W \to W$ is known as (α, β)-reverse derivation if $k(lm) = k(m)\alpha(l) + \beta(m)k(l)$, for all $l, m \in W$, where α and β are mappings on W.

Example 1 : Let S be any near ring and $W = \left\{ \begin{pmatrix} 0 & l & m \\ 0 & 0 & n \\ 0 & 0 & 0 \end{pmatrix} : l, m, n \in S \right\}$.

Then W is a near ring under addition and multiplication. We define the following mappings on W:

Let $\alpha : W \to W$ is a mapping defined by:

$$\alpha \begin{pmatrix} 0 & l & m \\ 0 & 0 & n \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} l & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

Let $\beta : W \to W$ is a mapping defined by:

$$\beta \begin{pmatrix} 0 & l & m \\ 0 & 0 & n \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & m \\ 0 & 0 & n \end{pmatrix},$$

and let $k : W \to W$ is a mapping defined by:

$$k \begin{pmatrix} 0 & l & m \\ 0 & 0 & n \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & l & n-l \\ 0 & 0 & -n \\ 0 & 0 & 0 \end{pmatrix}.$$

Then k is a nonzero (α, β)-reverse derivation on W.

Definition 3.2 : An additive mapping $H : W \to W$ is a right (resp., left) generalized (α, β)-reverse derivation if there exist (α, β)-reverse derivation k on W such that $H(lm) = H(m)\alpha(l) + \beta(m)k(l)(H(lm) = k(m)\alpha(l) + \beta(m)H(l))$ for all $l, m \in W$, where α and β are mappings on W.

Example 2 : Let S be any near ring and $W = \left\{ \begin{pmatrix} 0 & l & m \\ 0 & 0 & n \\ 0 & 0 & 0 \end{pmatrix} : l, m, n \in S \right\}$. Then W is a near ring under addition and multiplication. We define the following mappings on W:

Let $\alpha : W \to W$ is a mapping defined by:

$$\alpha \begin{pmatrix} 0 & l & m \\ 0 & 0 & n \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} l & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

Let $\beta : W \to W$ is a mapping defined by:

$$\beta\begin{pmatrix} 0 & l & m \\ 0 & 0 & n \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & m \\ 0 & 0 & n \end{pmatrix},$$

Let $k : W \to W$ is a mapping defined by:

$$k\begin{pmatrix} 0 & l & m \\ 0 & 0 & n \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & l & n-l \\ 0 & 0 & -n \\ 0 & 0 & 0 \end{pmatrix},$$

and let $H : W \to W$ is a mapping defined by:

$$H\begin{pmatrix} 0 & l & m \\ 0 & 0 & n \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & l \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Then H is a nonzero generalized (α,β)-reverse derivation on W associated with a nonzero (α,β)-reverse derivation k.

Theorem 3.3 : *Let L be a square closed commutative lie ideal of a semi prime near ring W such that* $L \not\subseteq Z(W)$. *Suppose that W admits a right generalized* (α,β)*-reverse derivation H associated with a nonzero map k, where* α,β *are automorphism. If*

(a) $H([l,m]) = 0$, *for all* $l \in L, m \in W$,

(b) $H([l,m]) = \pm[l,m]$, for all $l \in L, m \in W$,

then k is a β-commuting map on L.

Proof:

(a) Suppose that

$$H([l,m]) = 0, \text{ for all } l \in L, m \in W \tag{3.1}$$

Substituting lm for m in (3.1) alongwith the definition (2.5)(a), we get

$$H(l[l,lm]) = 0, \text{ for all } l \in L, m \in W$$

By definition (3.2) and alongwith equation (3.1), we obtain

$$\beta[l,m]k(l) = 0, \text{for all } l \in L, m \in W \tag{3.2}$$

This gives

$$[\beta(l),\beta(m)]k(l) = 0, \text{ for all } l \in L, m \in W$$

Replacing m by mr in last relation, we get

$$[\beta(l),\beta(m)]\beta(r)k(l)=0$$

Since β is automorphism replacing r by $\beta^{-1}(r)$ and m by $\beta^{-1}(m)$ in last equation, we have

$$[\beta(l),m]rk(l)=0$$

Replacing m by $k(l)$ in last relation, we get

$$[\beta(l),k(l)]rk(l)=0 \tag{3.3}$$

Right multiplying the equation (3.3) by $\beta(l)$, we have

$$[\beta(l),k(l)]rk(l)\beta(l)=0 \tag{3.4}$$

Replacing r by $r\beta(l)$ in equation (3.3), we obtain

$$[\beta(l),k(l)]r\beta(l)k(l)=0 \tag{3.5}$$

Subtracting equation (3.4) from equation (3.5), we get

$$[\beta(l),k(l)]r[\beta(l),k(l)]=0, \text{ for all } l,r\in W$$

This implies

$$[\beta(l),k(l)]W[\beta(l),k(l)]=0$$

Since W is semi prime near ring, we have

$$[\beta(l),k(l)]=0, \text{ for all } l\in L$$

Hence k is β-commuting map on L.

(b) Suppose that

$$H[l,m]=\pm[l,m], \text{ for all } l\in L, m\in W \tag{3.6}$$

Substituting lm for m in equation (3.6) and alongwith the definition (2.5)(a), we get

$$H(l[l,m])=\pm l[l,m]$$

By definition (3.2) and alongwith equation (3.6), we obtain

$$\pm[l,m]\alpha(l)+\beta[l,m]k(l)=\pm l[l,m]$$

Since α is automorphism replacing l by $\alpha^{-1}(l)$ in last equation, we have

$$\pm[l,m]l+\beta[l,m]k(l)=\pm l[l,m]$$

Since L is commutative, we obtain

$$[\beta(l), \beta(m)]k(l) = 0$$

Replacing m by mr in last equation and alongwith definition (2.5)(a), we get

$$[\beta(l), \beta(m)]\beta(r)k(l) = 0$$

Since β is automorphism replacing m by $\beta^{-1}(m)$ in last equation, we obtain

$$[\beta(l), m]\beta(r)k(l) = 0$$

Replacing m by $k(l)$ in last relation, we get

$$[\beta(l), k(l)]\beta(r)k(l) = 0$$

Since β is automorphism replacing r by $\beta^{-1}(r)$ in last equation, we have

$$[\beta(l), k(l)]rk(l) = 0 \quad (3.7)$$

Right multiplying the last equation by $\beta(l)$, we obtain

$$[\beta(l), k(l)]rk(l)\beta(l) = 0 \quad (3.8)$$

Replacing r by $r\beta(l)$ in equation (3.7), we get

$$[\beta(l), k(l)]r\beta(l)k(l) = 0 \quad (3.9)$$

Subtracting equation (3.8) from equation (3.9), we haves

$$[\beta(l), k(l)]r[\beta(l), k(l)] = 0, \text{ for all } r, l \in W$$

This implies

$$[\beta(l), k(l)]W[\beta(l), k(l)] = 0$$

Since W is semi prime near ring, we get

$$[\beta(l), k(l)] = 0, \text{ for all } l \in L$$

Hence k is β-commuting map on L. □

Theorem 3.4 : *Let L be a square closed commutative lie ideal of a semi prime near ring W such that $L \not\subseteq Z(W)$. Suppose that W admits a right generalized (α, β)-reverse derivation H associated with a nonzero map k, where β is automorphism. If $H(lom) = 0$, $\forall\, l \in L, m \in W$, then k is β-commuting map on L.*

Proof: Suppose that

$$H(lom) = 0, \quad \text{for all } l \in L, m \in W \quad (3.10)$$

Replacing lm for m in equation (3.10) and by definition (2.5)(c), we get

$$H(l(lom)) = 0$$

By definition (3.1) and alongwith equation (3.10), we have

$$\beta(lom)k(l) = 0$$

Replacing m by r in last relation, we have

$$\beta(lor)k(l) = 0$$

Replacing r by mr in last equation and alongwith definition (2.5)(c), we obtain

$$\beta(m)(\beta(l)o\beta(r))k(l) + [\beta(l), \beta(m)]\beta(r)k(l) = 0$$

From equation (3.11), we obtain

$$[\beta(l), \beta(m)]\beta(r)k(l) = 0$$

Since β is automorphism put r by $\beta^{-1}(r)$ and m by $\beta^{-1}(m)$ in last equation, we have

$$[\beta(l), m]rk(l) = 0$$

Replacing m by $k(l)$ in the last relation, we obtain

$$[\beta(l), k(l)]rk(l) = 0 \tag{3.12}$$

Right multiplying the last equation by $\beta(l)$, we have

$$[\beta(l), k(l)]rk(l)\beta(l) = 0 \tag{3.13}$$

Replacing r by $r\beta(l)$ in equation (3.12), we get

$$[\beta(l), k(l)]r\beta(l)k(l) = 0 \tag{3.14}$$

Subtracting equation (3.13) from equation (3.14), we obtain

$$[\beta(l), k(l)]r[\beta(l), k(l)] = 0, \text{ for all } l, r \in W$$

This gives

$$[\beta(l), k(l)]W[\beta(l), k(l)] = 0$$

Since W is semi prime near ring, we have

$$[\beta(l), k(l)] = 0$$

Hence k is β-commuting map on L. □

Theorem 3.5 : *Suppose that W is a semi prime near ring and $H : W \to W$ is a right generalized (α, β)-reverse derivation associated with (α, β)-reverse derivation k, where β is automorphism and $k(\beta(s)) = \beta(k(s))$, then* $k(t) \in Z(W)$ for every $t \in W$.

Proof: Suppose that

$$H(l^2m) = H(m)\alpha(l^2) + \beta(m)k(l^2), \text{ for all } l, m \in W \quad (3.15)$$

This implies

$$H(l^2m) = H(m)\alpha(l^2) + \beta(m)k(l)\alpha(l) + \beta(m)\beta(l)k(l) \quad (3.16)$$

Also

$$H(l^2m) = H(l(lm)) = H(lm)\alpha(l) + \beta(lm)k(l)$$

We arrive at

$$H(l^2m) = H(m)\alpha(l)\alpha(l) + \beta(m)k(l)\alpha(l) + \beta(l)\beta(m)k(l) \quad (3.17)$$

From equations (3.16) and equation (3.17), we get

$$H(m)\alpha(l^2) + \beta(m)k(l)\alpha(l) + \beta(m)\beta(l)k(l)$$
$$= H(m)\alpha(l)\alpha(l) + \beta(m)k(l)\alpha(l) + \beta(l)\beta(m)k(l)$$

By simplifying the last relation, we have

$$[\beta(l), \beta(m)]k(l) = 0$$

Replacing m by ml in last equation alongwith definition (2.5)(a), we obtain

$$[\beta(l), \beta(m)]\beta(l)k(l) = 0$$

Replacing $\beta(l)k(l)$ by t in last equation, we have

$$[\beta(l), \beta(m)]t = 0$$

Putting t by $k(t)$ in last equation, we obtain

$$[\beta(l), \beta(m)]k(t) = 0, \quad \text{for all } l, m, t \in W \quad (3.18)$$

Replacing l by lw in last relation and alongwith equation (3.18), we get

$$[\beta(l), \beta(m)]\beta(w)k(t) = 0 \quad (3.19)$$

Replacing m by $k(s)$ in relation (3.19), we have

$$[\beta(l), \beta(k(s))]\beta(w)k(t) = 0$$

By using hypothesis, we obtain

$$[\beta(l), k(\beta(s))]\beta(w)k(t) = 0$$

Since β is automorphism replacing w by $\beta^{-1}(w)$ and s by $\beta^{-1}(s)$ in last relation, we get

$$[\beta(l), k(s)]wk(t) = 0$$

Replacing s by t in last relation, we obtain

$$[\beta(l), k(t)]wk(t) = 0 \tag{3.20}$$

Replacing w by $r\beta(l)$ in equation (3.20), we get

$$[\beta(l), k(t)]r\beta(l)k(t) = 0 \tag{3.21}$$

Right multiplying the equation (3.20) by $\beta(t)$, we have

$$[\beta(l), k(t)]rk(t)\beta(l) = 0 \tag{3.22}$$

Subtracting equation (3.22) from equation (3.21), we obtain

$$[\beta(l), k(t)]r[\beta(l), k(t)] = 0, \text{ for all } l, t, r \in W$$

This implies

$$[\beta(l), k(t)]W[\beta(l), k(t)] = 0$$

Since W is semi prime near ring, we get

$$[\beta(l), k(t)] = 0, \quad \text{for all } l, t \in W$$

This gives

$$k(t) \in Z(W), \quad \text{for all } t \in W.$$

□

Theorem 3.6 : *Suppose that W is a semi prime near ring and $H : W \to W$ is a right generalized (α, β)-reverse derivation associated with (α, β)-reverse derivation k, where α is automorphism, then $H(m) \in Z(W)$, for every $m \in W$.*

Proof: Suppose that

$$H(lm^2) = H(m^2)\alpha(l) + \beta(m^2)k(l), \quad \text{for all } l, m \in W$$

This gives

$$H(lm^2) = H(m)\alpha(m)\alpha(l) + \beta(m)k(m)\alpha(l) + \beta(m^2)k(l)$$

By Theorem 3.5, we have

$$H(lm^2) = H(m)\alpha(m)\alpha(l) + \beta(m)\alpha(l)k(m) + \beta(m^2)k(l) \tag{3.23}$$

Also

$$H(lm^2) = H(lmm) = H(m)\alpha(lm) + \beta(m)k(lm)$$

This implies

$$H(lm^2) = H(lmm) = H(m)\alpha(l)\alpha(m) + \beta(m)k(m)\alpha(l) + \beta(m)\beta(m)k(l)$$

By Theorem 3.5, we obtain

$$H(lm^2) = H(m)\alpha(l)\alpha(m) + \beta(m)\alpha(l)k(m) + \beta(m^2)k(l) \tag{3.24}$$

Comparing equation (3.23) and equation (3.24), we get

$$H(m)\alpha(m)\alpha(l) + \beta(m)\alpha(l)k(m) + \beta(m^2)k(l)$$
$$= H(m)\alpha(l)\alpha(m) + \beta(m)\alpha(l)k(m) + \beta(m^2)k(l)$$

By simplifying the last relation, we have

$$H(m)[\alpha(l), \alpha(m)] = 0, \quad \text{for all } l, m \in W \tag{3.25}$$

Replacing l by mt in equation (3.25)and alongwith definition (2.5)(b), we obtain

$$H(m)\alpha(m)[\alpha(t), \alpha(m)] = 0$$

Replacing $H(m)\alpha(m)$ by n in last relation, we have

$$n[\alpha(t), \alpha(m)] = 0, \quad \text{for all } n, t, m \in W \tag{3.26}$$

Replacing t by rt in last relation, we get

$$n\alpha(r)[\alpha(t), \alpha(m)] = 0 \tag{3.27}$$

Since α is automorphism replacing t by $\alpha^{-1}(t)$ in equation (3.27), we have

$$n\alpha(r)[t, \alpha(m)] = 0$$

Replacing t by $H(m)$ in last equation, we obtain

$$n\alpha(r)[H(m), \alpha(m)] = 0 \tag{3.28}$$

Replacing r by mr in last relation, we get

$$n\alpha(m)\alpha(r)[H(n), \alpha(m)] = 0$$

Since α is automorphism replacing r by $\alpha^{-1}(r)$ in last relation, we have

$$n\alpha(m)r[H(m),\alpha(m)]=0 \tag{3.29}$$

Replacing $\alpha(m)r$ by w in last relation, we obtain

$$nw[H(m),\alpha(m)]=0$$

Replacing n by $H(m)$ in last relation, we obtain

$$H(m)w[H(m),\alpha(m)]=0$$

Left multiplying the last equation by $\alpha(m)$, we get

$$\alpha(m)H(m)w[H(m),\alpha(m)]=0 \tag{3.30}$$

Replacing r by w and n by $H(m)$ in equation (3.29), we have

$$H(m)\alpha(m)w[H(m),\alpha(m)]=0 \tag{3.31}$$

Subtracting equation (3.30) from equation (3.31), we obtain

$$[H(m),\alpha(m)]w[H(m),\alpha(m)]=0, \text{ for all } m,w\in W$$

This implies

$$[H(m),\alpha(m)]W[H(m),\alpha(m)]=0$$

Since W is semi prime near ring, we get

$$[H(m),\alpha(m)]=0, \quad \text{for all } m\in W$$

This implies

$$H(m)\in Z(W), \quad \text{for all } m\in W.$$

□

Lemma 3.7 : *Suppose W is a semi prime near ring with no nonzero divisor of zero. Let k be a commuting* (α,β) *-reverse derivation of W, where* β *is automorphism. Then* $[\alpha(l),\beta(m)]Wk(w)=0$, *for all* $l,m,w\in W$.

Proof:

Given that

$$[\alpha(l),k(l)]=0 \tag{3.32}$$

By linearizing the last relation and alongwith definition (2.4), we obtain

$$(\alpha(l)+\alpha(m))(k(l)+k(m))=(k(l)+k(m))(\alpha(l)+\alpha(m))$$

By simplifying the last relation, we have

$$[\alpha(l),k(m)]=[k(l),\alpha(m)] \tag{3.33}$$

Replacing m by lm in last relation, we obtain

$$[\alpha(l),k(lm)]=[k(l),\alpha(lm)]$$

By simplifying the last equation, we get

$$[\alpha(l),k(m)\alpha(l)+\beta(m)k(l)]=\alpha(l)[k(l),\alpha(m)]+[k(l),\alpha(l)]\alpha(m)$$

In view of equation (3.32) and alongwith equation (3.33), we have

$$[k(l),\alpha(m)]\alpha(l)+[\alpha(l),\beta(m)]k(l)=[k(l),\alpha(m)]\alpha(l)$$

This implies

$$[\alpha(l),\beta(m)]k(l)=0 \tag{3.34}$$

Replacing m by mn in last relation, we get

$$[\alpha(l),\beta(m)\beta(n)]k(l)=0$$

By using the definition (2.5)(a) and alongwith equation (3.34), we get

$$[\alpha(l),\beta(m)]\beta(n)k(l)=0$$

Since β is automorphism replacing n by $\beta^{-1}(n)$ in the last relation, we obtain

$$[\alpha(l),\beta(m)]nk(l)=0 \tag{3.35}$$

Replacing l by w in equation (3.35), we have

$$[\alpha(w),\beta(m)]nk(w)=0 \tag{3.36}$$

Replacing l by $l+w$ in equation (3.5), we get

$$[\alpha(l),\beta(m)]nk(l)+[\alpha(w),\beta(m)]nk(l)$$
$$+[\alpha(l),\beta(m)]nk(w)+[\alpha(w),\beta(m)]nk(w)=0$$

In view of equation (3.5), we obtain

$$[\alpha(l),\beta(m)]nk(w)+[\alpha(w),\beta(m)]nk(l)+[\alpha(w),\beta(m)]nk(w)=0$$

From equation (3.6), we have

$$[\alpha(l),\beta(m)]nk(w)+[\alpha(w),\beta(m)]nk(l)=0$$

This implies

$$[\alpha(l), \beta(m)]nk(w) = -[\alpha(w), \beta(m)]nk(l) \tag{3.7}$$

Left multiplying on both side of the last equation by $[\alpha(l), \beta(m)]nk(w)r$, we get

$$[\alpha(l), \beta(m)]nk(w)r[\alpha(l), \beta(m)]nk(w) = -[\alpha(l), \beta(m)]nk(w)r[\alpha(w), \beta(m)]nk(l)$$

From equation (3.37), we obtain

$$[\alpha(l), \beta(m)]nk(w)r[\alpha(l), \beta(m)]nk(w) = [\alpha(l), \beta(m)]nk(w)r[\alpha(l), \beta(m)]nk(w)$$

This gives

$$[\alpha(l), \beta(m)]nk(w)W[\alpha(l), \beta(m)]nk(w) = [\alpha(l), \beta(m)]nk(w)W[\alpha(l), \beta(m)]nk(w)$$

The last relation alongwith (3.35), we get

$$[\alpha(l), \beta(m)]nk(w)W[\alpha(l), \beta(m)]nk(w) \subseteq [\alpha(l), \beta(m)]Wk(l) = 0$$

This gives

$$[\alpha(l), \beta(m)]nk(w)W[\alpha(l), \beta(m)]nk(w) = 0$$

Since W is semi prime near ring, we have

$$[\alpha(l), \beta(m)]nk(w) = 0, \text{ for all } l, m, n, w \in W$$

This gives

$$[\alpha(l), \beta(m)]Wk(w) = 0, \text{ for all } l, m, w \in W.$$

□

4. Generalized Dependent Element of Right Generalized β-Reverse Derivations on Semi Prime Near Ring

In this section, we prove some results about generalized dependent element and right generalized β-reverse derivations on semi prime near ring. By taking $\alpha = \beta$ in Theorem 3.5, Theorem 3.6 and Lemma 3.7 we obtain the following results:

Theorem 4.1 : *Suppose that W is a semi prime near ring and $H: W \to W$ is a right generalized β-reverse derivation associated with β-reverse derivation k, where β is automorphism and $k(\beta(s)) = \beta(k(s))$, then $k(t) \in Z(W)$ for every $t \in W$.*

Theorem 4.2 : *Suppose that W is a semi prime near ring and $H: W \to W$ is a right generalized β-reverse derivation associated with β-reverse derivation k, where β is automorphism, then $H(m) \in Z(W)$, for every $m \in W$.*

Lemma 4.3 : *Suppose W is a semi prime near ring with no nonzero divisor of zero. Let k be a commuting* β *-reverse derivation of W, where* β *is automorphism. Then* $[\beta(l), \beta(m)]Wk(w) = 0,$ *for all* $l, m, w \in W$.

Theorem 4.4 : *Suppose W is a semi prime near ring with no nonzero divisor of zero and* ψ *be a* β *-inner derivation of W and if* ψ *is* β *-commuting on W, where* β *is a automorphism on W, then* $\psi = 0$.

Proof: Let $a \in W$, since ψ is a β -inner derivation of *W*, defined by

$$\psi(l) = [\beta(a), \beta(l)], \text{ for all } l \in W$$

By using Lemma 4.3 the last relation implies

$$[\beta(a), \beta(l)]W[\beta(a), \beta(l)] = 0, \text{ for all } l \in W.$$

Since *W* is semi prime near ring, we get

$$[\beta(a), \beta(l)] = 0$$

This gives

$$\psi(l) = 0, \text{ for all } l \in W.$$

Hence

$$\psi = 0.$$

□

Definition 4.5 : [10] Let $H : W \to W$ be a map, if there exist an element $g \in W$ such that $H(l)g = [l, g]g,$ hold for every $l \in W,$ then g is called generalized dependent element of *H* and $GD(H)$ is the collection of all generalized dependent elements of *H*.

Definition 4.6 : An additive mapping $k : W \to W$ is known as β -reverse derivation if $k(lm) = k(m)\beta(l) + \beta(m)k(l),$ for all $l, m \in W,$ where β is mapping.

Example 3 : Let *S* be any near ring and let $W = \left\{ \begin{pmatrix} 0 & 0 & 0 \\ l & 0 & 0 \\ m & n & 0 \end{pmatrix} : l, m, n \in S \right\}.$

We define the following mappings on *W*:
Let $\beta : W \to W$ is a mapping defined by:

$$\beta \begin{pmatrix} 0 & 0 & 0 \\ l & 0 & 0 \\ m & n & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & m \end{pmatrix},$$

Let $k: W \to W$ is a mapping defined by:

$$k\begin{pmatrix} 0 & 0 & 0 \\ l & 0 & 0 \\ m & n & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ l & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Then k is a nonzero β-reverse derivation on W.

Definition 4.7 : An additive mapping $H: W \to W$ is a right (resp., left) generalized β-reverse derivation if there exist β-reverse derivation k on W such that $H(lm) = H(m)\beta(l) + \beta(m)k(l)(H(lm) = k(m)\beta(l) + \beta(m)H(l))$ for all $l, m \in W$, where β is mapping on W.

Example 4 : Let S be any near ring and let $W = \left\{ \begin{pmatrix} 0 & 0 & 0 \\ l & 0 & 0 \\ m & n & 0 \end{pmatrix} : l, m, n \in S \right\}.$

We define the following mappings on W:
Let $\beta: W \to W$ is a mapping defined by:

$$\beta\begin{pmatrix} 0 & 0 & 0 \\ l & 0 & 0 \\ m & n & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & m \end{pmatrix},$$

Let $k: W \to W$ is a mapping defined by:

$$k\begin{pmatrix} 0 & 0 & 0 \\ l & 0 & 0 \\ m & n & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ l & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

Let $H: W \to W$ is a mapping defined by:

$$H\begin{pmatrix} 0 & 0 & 0 \\ l & 0 & 0 \\ m & n & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ n & 0 & 0 \end{pmatrix}.$$

Then H is a nonzero generalized β-reverse derivation on W associated with β-reverse derivation k.

Theorem 4.8 : *Suppose W is a semi prime near ring with no nonzero divisor of zero. If $H: W \to W$ is a right generalized β-reverse derivation associated with β-reverse derivation k, where β is automorphism, then $\beta(g) \in GD(H)$ if and only if $\beta(g) \in Z(W)$ and $H(l)\beta(g) = 0$, for all $l \in W$.*

Proof: Let $\beta(g) \in GD(H)$, for all $l \in W$

$$H(l)\beta(g) = [\beta(l), \beta(g)]\beta(g) \tag{4.1}$$

Replacing l by lm in equation (4.1) alongwith definition (4.7), we get

$$H(m)\beta(l)\beta(g)+\beta(m)k(l)\beta(g)=\beta(l)[\beta(m),\beta(g)]\beta(g)+[\beta(l),\beta(g)]\beta(m)\beta(g)$$

By Theorem 3.6, we have

$$\beta(l)H(m)\beta(g)+\beta(m)k(l)\beta(g)=\beta(l)[\beta(m),\beta(g)]\beta(g)+[\beta(l),\beta(g)]\beta(m)\beta(g)$$

From equation (4.1), we obtain

$$\beta(m)k(l)\beta(g)=[\beta(l),\beta(g)]\beta(m)\beta(g) \tag{4.2}$$

Right multiplying the last equation by $\beta(n)$, we obtain

$$\beta(m)k(l)\beta(g)\beta(n)=[\beta(l),\beta(g)]\beta(m)\beta(g)\beta(n) \tag{4.3}$$

Replacing m by mn in equation (4.2), we have

$$\beta(m)\beta(n)k(l)\beta(g)=[\beta(l),\beta(g)]\beta(m)\beta(n)\beta(g)$$

By Theorem 4.1, we get

$$\beta(m)k(l)\beta(n)\beta(g)=[\beta(l),\beta(g)]\beta(m)\beta(n)\beta(g) \tag{4.4}$$

Subtracting equation (4.3) from equation (4.4), we obtain

$$\beta(m)k(l)[\beta(n),\beta(g)]=[\beta(l),\beta(g)]\beta(m)[\beta(n),\beta(g)] \tag{4.5}$$

Left multiplying the last relation by $\beta(r)$, we have

$$\beta(r)\beta(m)k(l)[\beta(n),\beta(g)]=\beta(r)[\beta(l),\beta(g)]\beta(m)[\beta(n),\beta(g)] \tag{4.6}$$

Replacing m by rm in equation (4.5), we get

$$\beta(r)\beta(m)k(l)[\beta(n),\beta(g)]=[\beta(l),\beta(g)]\beta(r)\beta(m)[\beta(n),\beta(g)] \tag{4.7}$$

By comparing the relation (4.6) and (4.7), we obtain

$$\beta(r)[\beta(l),\beta(g)]\beta(m)[\beta(n),\beta(g)]=[\beta(l),\beta(g)]\beta(r)\beta(m)[\beta(n),\beta(g)]$$

By simplifying the last equation, we have

$$[\beta(r),[\beta(l),\beta(g)]]\beta(m)[\beta(n),\beta(g)]=0$$

Replacing r by l in last equation, we get

$$[\beta(l),[\beta(l),\beta(g)]]\beta(m)[\beta(n),\beta(g)]=0 \tag{4.8}$$

Replacing m by mr in last equation, we obtain

$$[\beta(l),[\beta(l),\beta(g)]]\beta(m)\beta(r)[\beta(n),\beta(g)]=0 \tag{4.9}$$

Right multiplying the equation (4.8) by $\beta(r)$, we have

$$[\beta(l),[\beta(l),\beta(g)]]\beta(m)[\beta(n),\beta(g)]\beta(r)=0 \tag{4.10}$$

Subtracting equation (4.10) from equation (4.9), we get

$$[\beta(l),[\beta(l),\beta(g)]]\beta(m)[\beta(r),[\beta(n),\beta(g)]]=0$$

Since β is automorphism replacing m by $\beta^{-1}(m)$ in last equation, we obtain

$$[\beta(l),[\beta(l),\beta(g)]]m[\beta(r),[\beta(n),\beta(g)]]=0$$

Replacing n and r by l in last relation, we have

$$[\beta(l),[\beta(l),\beta(g)]]m[\beta(l),[\beta(l),\beta(g)]]=0 \text{ for all } l,m\in W$$

This implies

$$[\beta(l),[\beta(l),\beta(g)]]W[\beta(l),[\beta(l),\beta(g)]]=0$$

Since W is semi prime near-ring, we get

$$[\beta(l),[\beta(l),\beta(g)]]=0, \quad \text{for all } l\in W$$

Let $\psi: W\to W$ is an inner derivation defined by:

$$\psi(g)=[\beta(l),\beta(g)]$$

If $\psi(g)$ is commuting then by Theorem 4.4, we get $\psi=0$
This implies

$$\psi(g)=0,\ g\in W$$

This gives

$$[\beta(l),\beta(g)]=0, \quad \text{for all } l\in W$$

Then $\beta(g)\in Z(W)$. From equation (4.1), we obtain

$$H(l)\beta(g)=0, \quad \text{for all } l\in W$$

Conversely assume that

$$H(l)\beta(g)=0, \text{ for all } l\in W \tag{4.11}$$

and $\beta(g)\in Z(W)$

$$[\beta(l),\beta(g)]=0, \text{ for all } l\in W \tag{4.12}$$

Right multiplying equation (4.12) by $\beta(g)$, we obtain

$$[\beta(l),\beta(g)]\beta(g)=0 \tag{4.13}$$

From equation (4.11) and equation (4.13), we get

$$H(l)\beta(g)=0=[\beta(l),\beta(g)]\beta(g)$$

This implies

$$H(l)\beta(g) = [\beta(l), \beta(g)]\beta(g), \quad \text{for all } l, g \in W.$$

□

Theorem 4.9 : *Suppose W is a semi prime near ring with no nonzero divisor of zero. If* $H: W \to W$ *is a right generalized* β*-reverse derivation associated with* β*-reverse derivation k. Then* $\beta(g) \in GD(H)$ *if and only if* $\beta(g) \in Z(W)$ *and* $\beta(g)H(l) = 0, \text{ for all } l \in W.$

Proof: Let

$$\beta(g) \in GD(H)$$

By Theorem 4.8, we have

$$\beta(g) \in Z(W) \text{ and } H(l)\beta(g) = 0, \text{ for all } l \in W$$

This gives

$$\beta(g) \in Z(W) \subset W$$

This implies

$$\beta(g) \in W$$

Since by Theorem 4.2, we obtain

$$\beta(g)H(l) = 0, \text{ for all } l \in W$$

Conversely assume that $\beta(g)H(l) = 0.$ Then by Theorem 4.2, we get

$$H(l)\beta(g) = 0, \quad \text{for all } l \in W \tag{4.14}$$

Assume that $\beta(g) \in Z(W).$ This gives

$$[\beta(l), \beta(g)] = 0, \text{ for all } l \in W \tag{4.15}$$

Right multiplying equation (4.15) by $\beta(g),$ we have

$$[\beta(l), \beta(g)]\beta(g) = 0, \text{ for all } l \in W \tag{4.16}$$

From equation (4.14) and (4.16), we obtain

$$H(l)\beta(g) = 0 = [\beta(l), \beta(g)]\beta(g)$$

This gives

$$H(l)\beta(g) = [\beta(l), \beta(g)]\beta(g), \text{ for all } l \in W$$

This implies

$$\beta(g) \in GD(H).$$

□

Theorem 4.10 : *Suppose W is a semi prime near ring with no nonzero divisor of zero. Let $H: W \to W$ be a right generalized β-reverse derivation associated with β-reverse derivation k, where β is automorphism and $\beta(k(r)) = k(\beta(r))$. If $\beta(g) \in GD(H)$, then $k(\beta(g)) = 0$.*

Proof : Given that $\beta(g) \in GD(H)$ then by Theorem 4.9, we have

$$\beta(g)H(l) = 0, \quad \text{for all } l \in H \tag{4.17}$$

Replacing l by rl in equation (4.17), we obtain

$$\beta(g)H(rl) = 0, \quad \text{for all } l \in H$$

By definition (4.7) and alongwith equation (4.17), we get

$$\beta(g)\beta(l)k(r) = 0 \tag{4.18}$$

Replacing l by $k(w)lg$ in relation (4.18) and using the hypothesis, we obtain

$$\beta(g)k(\beta(w))\beta(l)\beta(g)k(r) = 0$$

Since β is automorphism replacing w by $\beta^{-1}(w)$ and l by $\beta^{-1}(l)$ in last equation, we get

$$\beta(g)k(w)l\beta(g)k(r) = 0, \text{ for all } l, r, w \in W$$

Replacing w by r in last relation, we obtain

$$\beta(g)k(r)W\beta(g)k(r) = 0$$

Since W is semi prime, we get

$$\beta(g)k(r) = 0, \text{ for all } r \in W \tag{4.19}$$

Replacing r by $k(r)$ in equation (4.19), we have

$$\beta(g)k(k(r)) = 0 \tag{4.20}$$

Right multiplying the equation (4.18) by $\beta(g)$, we obtain

$$\beta(g)\beta(l)k(r)\beta(g) = 0$$

Left multiplying the last relation by $k(r)$, we get

$$k(r)\beta(g)\beta(l)k(r)\beta(g) = 0$$

Since β is automorphism replacing l by $\beta^{-1}(l)$ in last equation, we have

$$k(r)\beta(g)lk(r)\beta(g)=0, \text{ for all } l,r\in W$$

This implies

$$k(r)\beta(g)Wk(r)\beta(g)=0$$

Since W is semi prime, we obtain

$$k(r)\beta(g)=0$$

This gives

$$k(k(r)\beta(g))=0, \text{ for all } r\in W$$

By definition (4.6) and alongwith equation (4.20), we have

$$k(\beta(g))\beta(k(r))=0$$

By hypothesis, we get

$$k(\beta(g))k(\beta(r))=0$$

Since β is automorphism replacing r by $\beta^{-1}(r)$ in last equation, we get

$$k(\beta(g))k(r)=0 \tag{4.21}$$

Replacing r by $\beta(g)r$ in equation (4.21), we have

$$k(\beta(g))k(\beta(g)r)=0$$

By definition (4.6) and alongwith equation (4.21), we obtain

$$k(\beta(g))\beta(r)k(\beta(g))=0$$

Sine β is automorphism replacing r by $\beta^{-1}(r)$ in last equation, we have

$$k(\beta(g))rk(\beta(g))=0, \text{ for all } r\in W$$

This implies

$$k(\beta(g))Wk(\beta(g))=0$$

Since W is semi prime near ring, we obtain

$$k(\beta(g))=0.$$

□

Theorem 4.11 : *Suppose W is a semi prime near ring with no nonzero divisor of zero. If* $H: W \to W$ *be a right generalized* β *-reverse derivation associated with* β *-reverse derivation k, where* β *is automorphism and* $H(\beta(l)) = \beta(H(l))$. *If* $\beta(g) \in GD(H)$, *then* $H(g) = 0$.

Proof: Suppose that $\beta(g) \in GD(H)$ then by Theorem 4.8, we get

$$H(l)\beta(g) = 0, \quad \text{for all } l \in W \tag{4.22}$$

By definition (4.7) and alongwith the last relation, we have

$$H(\beta(g))\beta(H(l)) + \beta(\beta(g))k(H(l)) = 0$$

Since β is automorphism replacing g by $\beta^{-1}(g)$ in last relation, we get

$$H(e)\beta(H(l)) + \beta(g)k(H(l)) = 0 \tag{4.23}$$

By definition (4.6) and alongwith the equation (4.22), we have

$$k(\beta(g))\beta(H(l)) + \beta(\beta(g))k(H(l)) = 0$$

By Theorem 4.10, we get

$$\beta(\beta(g))k(H(l)) = 0$$

Since β is automorphism replacing g by $\beta^{-1}(g)$ in last equation, we have

$$\beta(g)k(H(l)) = 0 \tag{4.24}$$

Substituting equation (4.24) in equation (4.23) and by hypothesis, we obtain

$$H(g)H(\beta(l)) = 0, \quad \text{for all } l, r \in W$$

Since β is automorphism replacing l by $\beta^{-1}(l)$ in last equation, we get

$$H(g)H(l) = 0, \quad \text{for all } l, r \in W$$

Right multiplying the last relation by r, we have

$$H(g)H(l)r = 0$$

By Theorem 4.2, we obtain

$$H(g)rH(l) = 0, \quad \text{for all } l, r \in W$$

Replacing l by g in the last equation, we get

$$H(g)rH(g) = 0, \quad \text{for all } l, r \in W$$

This implies

$$H(g)WH(g) = 0$$

Since W is semi prime near ring, we have

$$H(g) = 0.$$

□

Theorem 4.12 : *Suppose W is a semi prime near ring with no nonzero divisor of zero. If $H : W \to W$ is a right generalized β-reverse derivation associated with β-reverse derivation k, where β is automorphism, then $GD(H)$ is commutative semi prime sub near ring of W.*

Proof: Assume that $\beta(g) \in GD(H)$. By Theorem 4.8, we get $\beta(g) \in Z(W)$

$$\beta(g)\beta(l) = \beta(l)\beta(g), \quad \text{for all } l \in W \tag{4.25}$$

Let $\beta(b) \in GD(H)$ then by Theorem 4.8, we obtain

$$\beta(b)\beta(l) = \beta(l)\beta(b), \quad \text{for all } b \in W \tag{4.26}$$

Subtracting equation (4.25) and equation (4.26), we get

$$(\beta(g) - \beta(b))\beta(l) = \beta(l)(\beta(g) - \beta(b))$$

This implies

$$(\beta(g) - \beta(b)) \in Z(W) \tag{4.27}$$

Since $\beta(g), \beta(b) \in GD(H)$, then by Theorem 4.8, we have

$$H(l)\beta(g) = 0 \quad \text{and} \quad H(l)\beta(b) = 0$$

We arrive at

$$H(l)(\beta(g) - \beta(b)) = 0 \tag{4.28}$$

From equation (4.27) and equation (4.28), we obtain

$$(\beta(g) - \beta(b)) \in GD(H) \tag{4.29}$$

Since $\beta(b) \in GD(H)$ by Theorem 4.9, we get

$$\beta(b)H(l) = 0, \quad \text{for all } l \in W$$

Left multiplying the last equation by $\beta(g)$, we have

$$\beta(g)\beta(b)H(l) = 0, \quad \text{for all } l \in W \tag{4.30}$$

Right multiplying the equation (4.25) by $\beta(b)$, we obtain

$$\beta(g)\beta(l)\beta(b) = \beta(l)\beta(g)\beta(b) \tag{4.31}$$

Left multiplying the equation (4.26) by $\beta(g)$, we get

$$\beta(g)\beta(b)\beta(l) = \beta(g)\beta(l)\beta(b), \quad \text{for all } b \in W \tag{4.32}$$

By comparing the equation (4.31) and equation (4.32), we obtain

$$\beta(g)\beta(b) \in Z(W) \tag{4.33}$$

From equation (4.30) and equation (4.33) with Theorem 4.9, we have

$$\beta(g)\beta(b) \in GD(H) \tag{4.34}$$

Now from equation (4.29) and (4.34), we obtain $GD(H)$ is sub near ring of W. Since $\beta(g)\beta(b) \in Z(W) \subset W$, we get $\beta(g)\beta(b) = \beta(b)\beta(g)$. Therefore $GD(H)$ is a commutative sub near ring of W.

Let us take $\beta(g)GD(H)\beta(g) = 0$, for all $\beta(g) \in GD(H)$

This implies

$$\beta(g)\beta(l)\beta(g) = 0, \quad \text{for all } \beta(l) \in GD(H), l \in W$$

Since β is automorphism replacing l by $\beta^{-1}(l)$ in last equation, we have

$$\beta(g)l\beta(g) = 0, \quad \text{for all } \beta(l) \in GD(H), l \in W$$

This gives

$$\beta(g)W\beta(g) = 0$$

This implies

$$\beta(g) = 0.$$

Since W is semi prime, hence $GD(H)$ is a commutative semi prime sub near ring of W. □

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