Journal of Discrete Mathematical Sciences & Cryptography
ISSN 0972-0529 (Print), ISSN 2169-0065 (Online)
Vol. 26 (2023), No. 4, pp. 1157–1162
DOI : 10.47974/JDMSC-1553

On characterizations some of subgroups of the dihedral group D_{2n}

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Abstract

The finite group $\mathcal{M}$ is an $\mathcal{H}$-group if every subgroup in $\mathcal{M}$ is an $\mathcal{M}$-subgroup, and we say $\mathcal{M}$ is a φ-group if every subgroup in $\mathcal{M}$ is a φ-subgroup. In this study, we look at some of the characteristics of some dihedral group subgroups. As a result, we have student relationships between several subgroups, the quantity demand and investment in research

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and development, while the other model focuses on a more realistic relationship between the quantity demand and the price.

Subject Classification: *20F12, 20F14, 20F15.*
Keywords: *$\mathcal{H}$-group, φ-subgroup.*

1. Introduction

The dihedral group D_{2n} is define by $\langle a,b \mid a^n = b^2 = e,\ bab = a^{-1}\rangle$, $n \geq 3$, when $n=2$ it is isomorphic to $C_2 \times C_2$. Dived Jensen and Michael Keane in year 1990 studded and computed the subgroups structure of the dihedral group. Suppose that n is an integer number, we will write n by $n = 2^r \Pi_{i=1}^{s} p_i^{\alpha_i}$, where $p_1 < p_2 < \cdots < p_s$ are odd prime numbers and $r, \alpha_i \in N$ for all *i*. Let $\mathcal{H}$ a subgroup of a group $\mathcal{M}$ we can say that *S*-quasinormality in $\mathcal{M}$ if for each $\mathcal{H}$ with p-subgroup of $\mathcal{M}$ is permutes its mean $\mathcal{H}\mathcal{H}_s = \mathcal{H}_s\mathcal{H}$ where $\mathcal{H}_s$ is a Sylow subgroup of $\mathcal{M}$.

This student was concept in [1] and it was studied extensively by Deskins [2].

In [3] we can see some of the notion of *S*-quasinormal in $\mathcal{M}$ is embedded if $|\mathcal{H}|/p$ a sylow *p*–subgroup of $\mathcal{H}$ is also a sylow p–subgroup of some s–quaisnormality subgroup of G. For each finite group there exist sequences of subgroups of type $\mathcal{H}_i$ for each *i*such that $e \trianglelefteq \mathcal{H}_1 \trianglelefteq \mathcal{H}_2 \trianglelefteq \cdots \trianglelefteq \mathcal{H}_s \trianglelefteq \mathcal{M}$ we can say that $\mathcal{H}_i$ be a subnormal subgroup. A group G is called solvable group if it has subnormal subgroup such that $\frac{\mathcal{H}_i}{\mathcal{H}_{i-1}}$ is abelain group. Let *G* is a finite group, we can say $\mathcal{M}$ is T-group if for any subgroups $\mathcal{H}$ in *G* such that $\mathcal{H} \trianglelefteq K \trianglelefteq \mathcal{M}$ and $\mathcal{H} \trianglelefteq \mathcal{M}$. This mean normality is a transitive relation. There exists equivalent definition, it is for any subnormal subgroups are normal. $\mathcal{M}$ is $\hat{T}$-group then every subgroup of $\mathcal{M}$ is $\hat{T}$-group. We say $\mathcal{H}$-subgroup for any subgroup $\mathcal{H}$ is satisfy the condition $N_{\mathcal{M}}(\mathcal{H}) \cap \mathcal{H}^g \leq \mathcal{H}$ for all $g \in G$ and is called an NE-subgroup if it satisfies $N_{\mathcal{M}}(\mathcal{H}) \cap \mathcal{H}^G = \mathcal{H}$. $\tau(n)$ is a number of all divisors of *n* and by $\sigma(n)$ is a sum of all number divisors of *n* and we can denote foe all prime divisors of *n* by $\pi(n)$. In [11,13] could presented the structure of subgroups of the dihedral group D_{2n}, it was $\langle a^i \rangle$ for all $i \mid n$ and $\langle a^i, a^j b\rangle$ for all $i \mid n$ and $1 \leq j \leq n$ and in [11] shelash and ashrafi introduced the Sylow subgroups structure of D_{2n}. It was $\langle a^m, a^j b\rangle$ where $m = \Pi_{i=1}^{s} p_i^{\alpha_i}$ and $\langle a^{p_i^{\alpha_i}} \rangle$.

Shelash and Ashrafi in (12, 2018) showed that ever subgroup of type $\langle a^i \rangle$ for all $i \mid n$ and a subgroup of type $\langle a^t, a^j b\rangle$ when $t \mid 2^r$, $r \leq 1$ be subnormal subgroup in D_{2n}. The factor group $\frac{\langle a^{2^{r-i}}, a^j b\rangle}{\langle a^{2^{r-i+1}}, a^j b\rangle} \cong C_2$ is abelain group, thus the dihedral group is solvable group proved the following theorems:

Theorem 1.1: *The dihedral Group D_{2n} be T-group when $n = 2^r m$, and $2 \nmid m$.*

In [11], showed that if d is odd number, then $\langle a^d, a^j b\rangle$ be self-normalizing, this mean $N_{D_{2n}}(\langle a^d, a^j b\rangle) = \langle a^d, a^j b\rangle$.

Theorem 1.2: *The normalizer of a subgroup H in the dihedral group D_{2n} can bewritten as follows:*

$$N_{D_{2n}}(H) = \begin{cases} D_{2n} & \text{if } H \trianglelefteq D_{2n} \\ \langle a^t, a^j b\rangle & \text{if } H = \langle a^t, a^j b\rangle \,\&\, 2 \nmid t \\ \langle a^{\frac{t}{2}}, a^j b\rangle & \text{if } H = \langle a^t, a^j b\rangle \,\&\, 2 \mid t \end{cases}$$

2. The Main Result

In this section we shall present some of important results about compute the $\overline{T}$-group, *H*-group and NE-group and introduce some of relationship between those subgroups.

Theorem 2.1: *Let $n = 2^r m$ and $2 \nmid m$ be an integer number. Then D_{2n} be $\overline{T}$-group.*

Proof: (Case 1) $r = 0$, Suppose that a set $\mathcal{F}$ is contain subgroups, it is define by $\mathcal{F} = \{\langle a^i\rangle, \langle a^d, a^j b\rangle, i \mid n, d \mid n \,\&\, 2 \nmid d\}$, it is clear the all subgroups of type $\langle a^i\rangle, i \mid n$ are subnormal subgroups and T-groups. As well as the subgroups of type $\langle a^d, a^j b\rangle$ be isomorphic to $D_{\frac{2n}{d}}$ and $\frac{n}{d}$ is odd number, by theorem (1.2) then all subgroups of type $\langle a^d, a^j b\rangle$ are T-group, thus D_{2n} be $\overline{T}$-group when r=0.

(Case 2) $r = 1$, it is easy see that any subgroup of type $\langle a^i\rangle, i \mid n$ and $\langle a^d, a^j b\rangle, d = 2,1$ are subnormal subgroups and T-group.

Definition 2.2: A group $\mathcal{M}$ we can say that $\mathcal{H}$-group when all subgroup $\mathcal{H}$ in $\mathcal{M}$ is a $\mathcal{H}$-subgroup.

Definition 2.3: A group $\mathcal{M}$ we can say that φ-group when all subgroup $\mathcal{H}$ in $\mathcal{M}$ is a φ-subgroup.

Theorem 2.4: Let $\mathcal{M}$ be an abelain group. Thus:

1. *$\mathcal{M}$ is $\mathcal{H}$-group*
2. *$\mathcal{M}$ is φ-group*

Proof: Suppose that $\mathcal{M}$ is a finite group and $\mathcal{H}$ is a subgroup in $\mathcal{M}$, then:

$$\begin{aligned} N_{\mathcal{M}}(\mathcal{H}) \cap \mathcal{H}^g &= \{g \in \mathcal{M} \mid g\mathcal{H} = \mathcal{H}g\} \cap \{g\mathcal{H}g^{-1}\} \\ &= \{g \in \mathcal{M} \mid g\mathcal{H}g^{-1} = \mathcal{H}\} \cap \{\mathcal{H}\} \\ &= \mathcal{M} \cap \mathcal{H} = \mathcal{H} \end{aligned}$$

Theorem 2.5: *Suppose that $\mathcal{M}$ is a $\hat{T}$-group, all subgroup of $\mathcal{H}$-group is $\mathcal{H}$-group.*

Example 2.6: In this example, we will student certain of subgroups in dihedral group D_{2n}. Shelash and Ashrfai, proved that the dihedral group is $\hat{T}$-group when $n = 2^r m$ and $r \le 1$. Clear that the set of all subnormal subgroup are normal subgroups be $\mathcal{F} = \{\langle a^i \rangle, \langle a^d, a^j b \rangle\}$ where $i \mid n$, $d \mid 2$ and $1 \le j \le d$.

Theorem 2.7: *Let $n = 2^r m$ and $r \le 1$. The following are holds:*

(a) *Every subgroup H in $\mathcal{F}$ is a H-subgroup;*

(b) *Every subgroup H in $\mathcal{F}$ is an φ-subgroup.*

Proof: It is clear that by definition $\mathcal{H}$-subgroup the cyclic subgroups $\langle a^I \rangle$ for all $i \mid n$ are $\mathcal{H}$-subgroup, since they are normal subgroups and $N_{\mathcal{M}}(\mathcal{H}) = \mathcal{H}$ so $\mathcal{H}^g = g\mathcal{H}g^{-1} = \mathcal{H}$, thus for all $g \in \mathcal{M}$, we can see $N_{\mathcal{M}}(\mathcal{H}) \cap \mathcal{H}^g = \aleph$ and $\aleph \approx \mathcal{H} \le \mathcal{H}$. The second part of the set $\mathcal{F}$ are non-cyclic subgroups, they are isomorphic to $D_{\frac{2n}{d}}$, it is easy see that be $\mathcal{H}$-subgroup since are normal subgroups. Since $\mathcal{H}^{\mathcal{M}} = \mathcal{H}$ for any $\mathcal{H}$ normal subgroup. Clear that the any subgroups in set $\mathcal{F}$ be NE-subgroup. The proof is done.

Theorem 2.8: *If $\mathcal{M} \cong D_{2n}$ $n = 2^r m$ where $r \le 1$. Then D_{2n} is a $\hat{T}$-group and $\mathcal{H}$-group.*

Proof: Let H be a subgroup, then we can see for all subgroup is $\hat{T}$-group and H of $\mathcal{M}$ is a, then $\mathcal{M}$ be $\bar{T}$-group, and for every subgroup of $\mathcal{M}$ is a $\mathcal{H}$-subgroup, then $\mathcal{M}$ be $\mathcal{H}$-group. Since the all subgroups in set $\mathcal{F}$ be $\hat{T}$-group and H-subgroup thus the dihedral group D_{2n} is $\bar{T}$-group and $\mathcal{H}$- group when $n = 2^r m$ and $r \leq 1$.

5. Discussion and Conclusion

All of our prior findings demonstrate that employing subgroups embedding properties to investigate group classes is still fruitful. To characterize soluble T-groups and PT-groups. These subgroups have also been used in the literature to derive characterizations and criteria for group's membership. They will undoubtedly. Finally, we discuss some unsolved issues in this area.

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Received March, 2022
Revised May, 2022