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Fuzzy RG-ideals of RG-algebra

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Abstract

The aim of this paper is to introduce the terms fuzzy RG-subalgebra of RG -algebra, provide a few important examples, and look at a few aspects in this paper. The image and pre-image of fuzzy RG-subalgebra of RG-algebra are presented, as well as an introduction to the concept of fuzzy RG-ideal of RG-algebra, and several properties and theorems are proved.

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1. Introduction

A BCK-algebra is an important type of logical algebra submitted by C Is´eki K. and Tanaka S. and was extensively explored by researchers. The class of all KU-algebras is qualitative. The notion of a fuzzy subset and many operations on it was first introduced by L.A. Zadeh; then ,fuzzy subsets have been applied to diverse fields. Fuzzy algebra is an important branch of fuzzy mathematics. They introduced the concept of fuzzy subgroups in 1971 by A. Rosenfeld. Introduced the meaning of fuzzy ideals. Subalgebras of the BCK-algebras concerning a minimum, A.T. Hameed et. al. have introduced the concept of some types of algebras as (AT-algebras, AB-algebras, QS-algebras and SA-algebras) and gave the concept of ideals, subalgebra of them and investigated the relations among them. In this paper, the study of fuzzy algebraic structures was started with the concept of fuzzy RG-ideals of RG-algebras, and then we reconnoiter several properties which are related to fuzzy RG-ideal. We

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explain how to deal with the homomorphism of image and inverse image of fuzzy RG-ideals.

2. Preliminaries

Definition 2.1 [1, 2]: suppose(Ψ;$*$,0) be an algebra $\Rightarrow\Psi$ is **RG-algebra (RG-A)** if it meets: $\forall\, \nu, \varepsilon, \epsilon \in \Psi$, (RG_1): $\nu * 0 = \nu$, (RG_2): $\nu * \varepsilon = (\nu * \epsilon) * (\varepsilon * \epsilon)$, (RG_3): $\nu * \varepsilon = 0$ *and* $\varepsilon * \nu = 0 \Leftrightarrow \nu = \varepsilon$.
On RG-A (Ψ;$*$,0), the binary relation $\leq$ is $\nu \leq \varepsilon \Leftrightarrow \varepsilon * \nu = 0$.

Definition 2.2 ([1,2]): A set $S \neq \emptyset$ of RG-A $(\Psi;*\,,0)$ *is* named **RG-subalgebra (RG-S) of** Ψ (I), if any $\nu, \varepsilon \in$ S, $(\nu * \varepsilon) \in S$.

Definition 2.3 ([1,2]): A set $T \neq \emptyset$ of RG-A $(\Psi;*\,,0)$ *is* named **an RG-ideal (RG-I)of** Ψ , if any $\nu, \varepsilon, \epsilon \in \Psi$, (1) $0 \in T$, (2) $(\nu * \varepsilon) \in T$ and $(0 * \nu) \in T$ *imply* $(0 * \varepsilon) \in T$.

3. Fuzzy RG-Ideals of RG-Algebra

Definition 3.1: Let $(\Psi;*\,,0)$ be an RG-A, an FS ξ of Ψ is named **a fuzzy RG-subalgebra of Ψ(FRG-S)** if $\forall\, \nu, \varepsilon \in \Psi$, $\xi(\nu * \varepsilon) \geq min\,\{\xi(\nu), \xi(\varepsilon)\}$.

Definition 3.2: Let $(\Psi;*\,,0)$ be an RG-A, an FS ξ of Ψ is named **a fuzzy RG-ideal of Ψ(FRG-I)** if $\forall\, \nu, \varepsilon \in \Psi$,
(1) $\xi(0) \geq \xi(\nu)$, (2) $\xi(0 * \varepsilon) \geq min\,\{\xi(\nu * \varepsilon), \xi(0 * \nu)\}$.

Theorem 3.3: *Let A is an RG-S of an RG-A* $(\Psi;*\,,0)$*,ξ be an FRG-S of Ψ such that ξ is into {0, 1}. ξ is an FRG-I of Ψ $\Leftrightarrow$ A is an RG-I of Ψ, where ξ is the Characterization function of A.*

Proof: If ξ is an FRG-I of Ψ, since $\Psi(0) \geq \Psi(\nu)$, $\forall\, \nu \in \Psi$, clearly $\Psi(0) = 1$, so $0 \in A$. Let $\nu * \varepsilon, 0 * \nu \in A$, since ξ, is an FRG-I of $\Psi \Rightarrow$
$\xi(0 * \epsilon) \geq \min\{\xi(\nu * \varepsilon), \xi(0 * \nu)\}$, $\forall$ n, m$\in\psi$, and ξ(n*m) =1 $\Rightarrow 0 * \epsilon \in A$.
$\Rightarrow A$ is an RG-I ofΨ.
($\Leftarrow$), A is an RG-I of ψ, since $0 \in A$, $\xi(0)=1 \geq \xi(n)$, $\forall$ n$\in\Psi$. Let $\nu, \varepsilon \in \Psi$,
If $\varepsilon \notin A$, $\Rightarrow \xi(\varepsilon) = 0$, and so $\xi(\nu * \epsilon) \geq 0 = \min\{\xi(\nu * \varepsilon), \xi(0 * \varepsilon)\}$.
If $(\nu * \varepsilon) \notin A$, and $(0 * \varepsilon) \in A$, $\Rightarrow (0 * \epsilon) \notin A$(since A is an RG-I). Thus $\xi(0 * \epsilon) = 0 = \min\{\xi(\nu * \varepsilon), \xi(0 * \nu)\}$. Subsequently, ξ is an FRG-I of Ψ.

Proposition 3.4: In RG-A $(\Psi;*\,,0)$, ξ is a FS of Ψ. If ξ is an FRG-I of Ψ, *then* for any$\tau \in [0,1]$, $U(\xi, \tau)$is an RG-I of Ψ.

Proof: ξ is an FRG-I of Ψ, by Def (3.2(1)), we have $\xi(0) \geq \xi(\nu)$ $\forall\, \nu \in \Psi$, therefore $\xi(0) \geq \xi(\nu) \geq \tau$, $\forall\, \nu \in U(\xi, \tau)$ and so $0 \in U(\xi, \tau)$.

Let $\nu, \varepsilon \in \Psi$ be $\nu * \varepsilon, 0 * \nu \in U(\xi, \tau)$, $\Rightarrow \xi(\nu * \varepsilon) \geq \tau$ *and* $\xi(0 * \nu) \geq \tau$, since ξ *is* an FRG-I, it follows that $\xi(0 * \epsilon) \geq min\{\xi(\nu * \varepsilon), \xi(0 * \nu)\} \geq \tau$ and we have that $0 * \epsilon \in U(\xi, \tau)$. Subsequently $U(\xi, \tau)$ is an RG-I of Ψ.

Proposition 3.5: In RG-A $(\Psi; *, 0)$, ξ is a FS of Ψ. If for every $\tau \in [0,1]$, $U(\xi, \tau)$ is an RG-I of Ψ, then ξ is an FRG-I of Ψ

Proof: If (1) is not true, $\Rightarrow$ there exists $\nu` \in \Psi \ni \xi(0) < \xi(\nu`)$. If we take $t` = (\xi(\nu`) + \xi(0))/2, \Rightarrow \xi(0) < \tau`$ and $0 \leq \tau` < \xi(\nu`) \leq 1$,
$\Rightarrow \nu` \in U(\xi, \tau)$ and $\xi \neq \emptyset$. As $U(\xi, \tau)$ is an RG-I of $\Psi$, we have $0 \in U(\xi, \tau)$ and so $\xi(0) \geq \tau`$. This is a C.
Now, there exist $\nu`, \varepsilon` \Psi \ni \xi(0` * \epsilon`) < min\{\xi(\nu` * \varepsilon`), \xi(0` * \nu`)\}$.
Putting $\tau` = (\xi(0` * \epsilon`) + min\{\xi((\nu` * \varepsilon`)), \xi(0` * \nu`)\})/2, \Rightarrow$
$\xi(0` * \epsilon`) < \tau`$ and $0 \leq \tau` < min\{\xi(\nu` * \varepsilon`), \xi(0` * \nu`)\} \leq 1$,
Subsequently, $\xi(\nu` * \varepsilon`) > \tau`$ *and* $\xi(0` * \nu`) > \tau`, \Rightarrow$
$\nu' * \varepsilon`, 0' * \nu' \in U(\xi, \tau')$. Since $U(\xi, \tau')$ is an RG-I, it follows that
$0` * \epsilon` \in U(\xi, \tau')$ and $\xi(0` * \epsilon`) \geq \tau`$, this is also a C.
Subsequently, ξ is an FRG-I of Ψ.

Proposition 3.6: Let ξ be a FS of RG-A $(\Psi; *, 0)$. If ξ is an FRG-S of Ψ, then for any $\tau \in [0,1], U(\xi, \tau)$ is an FRG-S of Ψ.

Proof: ξ is an FRG-S of , assume $\nu, \varepsilon \in \Psi$ & $\nu \in U(\xi, \tau)$ &
$\varepsilon \in U(\xi, \tau), \Rightarrow \xi(\nu) \geq \tau \& \xi(\varepsilon) \geq \tau$.
Since ξ is an FRG-S, it follows that $\xi(\nu * \varepsilon) \geq min\{\xi(\nu), \xi(\varepsilon)\} \geq \tau$ and that $\nu * \varepsilon \in U(\xi, \tau)$. Hence $U(\xi, \tau)$ is an FRG-S of .

Proposition 3.7: Let ξ be a FS of RG-A $(\Psi; *, 0)$. If for all $\tau \in [0,1], U(\xi, \tau)$ is a FRG-S of X, then ξ is a FRG-S of X.

Proof: Assume $\xi(\nu * \varepsilon) \geq min\{\xi(\nu), \xi(\varepsilon)\}$ is not true, then there exist $\nu`$ and $\varepsilon` \in \Psi$ such that, $\xi(\nu` * \varepsilon`) < min\{\xi(\nu`), \xi(\varepsilon`)\}$. Putting $\tau` = (\xi(\nu` * \varepsilon`) + min\{\xi(\nu`), \xi(\varepsilon`)\})/2$, then $\xi(\nu`) < t`$ and $0 \leq t` < min\{\xi(\nu`), \xi(\varepsilon`)\} \leq 1$, hence $\xi(\nu`) > t`$ and $\xi(\varepsilon`) > t`$, which imply that $\nu` \in U(\xi, \tau')$ and $\varepsilon` \in U(\xi, \tau')$, since $U(\xi, \tau')$ is an RG-S, it follows that $\nu` * \varepsilon` \in U(\xi, \tau')$ and that $\xi(\nu` * \varepsilon`) \geq \tau`$, this is also a contradiction.
Hence ξ is an FRG-S of Ψ.

Proposition 3.8: Every FRG-I of RG-A $(\Psi; *, 0)$ is an FRG-S.

Theorem 3.9: *A homo. preimage of FRG-S of RG-A* $(\Psi; *, 0)$ *is also an FRG-S.*

Proof: Let $\Upsilon : (\Psi; *, 0) \rightarrow (\varsigma; *', 0')$ be a homo. of RG-As, β an FRG-S of ς and ξ the preimage of β under Υ, let $\nu, \varepsilon \in \Psi, \Rightarrow$
$\xi(\nu * \varepsilon) = \beta(\Upsilon(\nu * \varepsilon)) \geq min\{\beta(\Upsilon(\nu)), \beta(\Upsilon(\varepsilon))\}$
$= \min\{\xi(x), \xi(y)\}$ for all $\nu, \varepsilon \in \Psi$. $\Rightarrow$ Hence ξ is an FRG-S of Ψ.

Proposition 3.10: A homo. Pre-image of FRG-I of RG-A $(\Psi;*,0)$ is also FRG-I.

Theorem 3.11: *Let* $:(\Psi;*,0) \to (\varsigma;\, *', 0')$ *be an epimorphism between RG-As. For every FRG-S* ξ *of* Ψ*,* $\Upsilon(\xi)$ *is an FRG-S of* ς*.*

Proof: By def. $\beta\,(\varepsilon') = \Upsilon\,(\xi)(\varepsilon') = sup\{\xi(v)\,|v \in f^{-1}(\varepsilon)\}$, for all $\varepsilon' \in \varsigma$ $(sup\phi = 0)$. $\beta(v' * \varepsilon') \geq min\,\{\beta(v'), \beta(\varepsilon')\}$, for all $v', \varepsilon' \in \varsigma$. Let $\Upsilon:(\Psi;*,0) \to (\varsigma;\, *', 0')$ be an epimorphism of RG-As, ξ is an FRG-S of Ψ & β the image of ξ under Υ.

For any $v', \varepsilon' \in Y$, let $v_0 \in \Upsilon^{-1}(v')$, $\varepsilon_0 \in \Upsilon^{-1}(\varepsilon')$, be $\xi(v_0 * \varepsilon_0)$ $=\beta(\Upsilon(v_0 * \varepsilon_0)) = \beta(\Upsilon(v' * \varepsilon'))$ $= sup_{v_0 * \varepsilon_0 \in \Psi^{-1}(v' * \varepsilon')}\xi(v_0 * \varepsilon_0)$

$\Psi(v_0)$$=\beta(\Upsilon(v_0))$$=\beta(\Upsilon(v'))$$=sup_{v_0 \in \Upsilon^{-1}(v)}\xi(v_0)$, then

$\beta(v' * \varepsilon') = sup_{t \in \Upsilon^{-1}(v' * \varepsilon')}\xi(t) = \xi_{t \in \Upsilon^{-1}(v' * \varepsilon')}(v_0 * \varepsilon_0) \geq$

$\min\left\{\xi_{t \in \Upsilon^{-1}(v')}(\mathrm{t}), \xi_{t \in \Upsilon^{-1}(\varepsilon')}(\mathrm{t})\right\} = \min\{\beta(v'), \beta(\varepsilon')\}$.

Hence β is an FRG-S of ς.

Proposition 3.12: Let $\Upsilon :(\Psi;*,0) \to (\varsigma;\, *', 0')$ be an epimorphism between RG-As. For every FRG-I ξ of Ψ, $\Upsilon(\xi)$ is an FRG-I of ς.

Conclusion

In this paper, fuzzy RG-subalgebra and fuzzy RG-ideal of RG -algebra is given. We see that every fuzzy RG-ideal of RG –algebra will be fuzzy RG-subalgebra, and the inverse is not true we introduce some properties And gave Several properties, including this concept. We also discuss isomorphisms related properties on fuzzy RG-subalgebra and fuzzy RG-ideal of RG -algebra.

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