Journal of Discrete Mathematical Sciences & Cryptography
ISSN 0972-0529 (Print), ISSN 2169-0065 (Online)
Vol. 26 (2023), No. 1, pp. 87–101
DOI : 10.1080/09720529.2021.1914426

Computing certain topological indices of thorny graphs

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Abstract

The topological index has transformed the chemical structure into a numeric number and they make a connection between chemistry and mathematics.

In this paper, we compute some certain topological indices like First Zagreb index, First Leap Zagreb index and F-Leap index of certain generalized graphs such as special families of thorny graphs.

Subject Classification: *(2010) 05C40, 05C12, 05C07, 05C35.*

Keywords: *Topological index, Thorny graph.*

Introduction

We recommend [5, 6, 7, 8] for general symbols and ideas in graph theory. In this paper, we have assumed the graphs that are simple, connected with no directed or weighted edges. For an instance $G = (V, E)$, we assume the number of vertices and edges are n, m and denote by $|V| = n$, $|E| = m$. The symbols $d_G(v)$ represents the degree of a vertex v which is the number of vertices are adjacent to v.

Consider two vertices u and v of G and the shortest path which connect these two vertices, the distance $d(u, v)$ denotes the number of edges in this minimum path. In graph G, for a $k \in Z^+$ and a $v \in V(G)$, we define $N_k(v)$, the open k-distance neighbourhood of v, as $N_k(v) = \{u \in V(G) : d(u, v) = k\}$.

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Also, we define $d_k(v)$, the k-distance degree of a vertex v, as $d_k(v) = |N_k(v)|$. For further information, see [9].

There are a lot of topological indices that are studied.

Scientist are interested to find these topological indices to know the chemical molecular behave, to see more details one can refer to [1, 2, 11, 13].

Among these topological indices, the most significant in which scientists were interested in studying, is Zagreb indices that correlates with molecules and their chemical properties. Gutman and Trinajsti [8] introduced the First and Second Zagreb index which were defined as

$$M_1(G) = \sum_{v \in V(G)} d_G(v)^2 = \sum_{uv \in E(G)} \left[d_G(u) + d_G(v) \right]$$

In [10], F- index was introduced and defined as

$$F(G) = \sum_{u \in V(G)} d^3(u) = \sum_{uv \in E(G)} (d^2(u) + d^2(v)).$$

In [9], by depending on second vertex degrees, Gutman introduced the First Leap Zagreb index and defined as

$$LM_1(G) = \sum_{u \in V(G)} d_2^2(u).$$

In [12], Kulli introduced the F-Leap index of a graph as follows:

$$FL(G) = \sum_{u \in V(G)} d_2^3(u).$$

Consider a graph $G = (V, E)$, with $|V| = n, |E| = m$. In this paper we define thorn graph. The thorn graph of G, G_p with n non-negative integers like $p_1, p_2, ..., p_n$, is achieved by connecting p_i vertices of degree one to the vertex v_i of G, for $i = 1, ..., n$. Thorns of v_i are the p_i vertices of degree one that have been joined to the vertex of v_i. Obviously, $V(G_p) = V(G) \cup V_1 \cup V_2 \cup ... \cup V_n$. The idea of introducing the thorn graph belongs to Gutman and finally the difference of applications in chemistry was found, see [3,4,14].

Now we classify thorn graphs by different types of graphs like path, cycle, complete graph, complete bipartite graph, etc.

First type

Consider a path P of order greater than 2 and let the vertex v_i be the beginning vertex of such paths in graph G. The thorn path graph, G_p is achieved by joining p_i, sample of such paths at each vertex v_i of G.

Second type

Consider a cycle C of length p and let the vertex v_i be a vertex in each cycle in graph G. The thorn cycle graph, G_C is achieved by joining p_i, sample of such cycle at each vertex v_i of G.

Third type

Consider a complete graph K_p of order greater than 3 and let the vertex v_i be a vertex in K_p in graph G. The thorn complete graph, G_K is achieved by joining p_i, sample of such complete graph K_p at each vertex v_i of G

Fourth type

Consider a complete bipartite graph $K_{r,s}$ and let the vertex v_i be a vertex in a partition of $K_{r,s}$ in graph G. The thorn complete bipartite graph, G_A is achieved by joining p_i sample of such complete bipartite graph $K_{r,s}$ at each vertex v_i of G.

Fifth type

Consider a graph K_p and let the vertex v_i be a vertex in graph G. The thorn graph, G'_K is achieved by attaching p_i sample of K_p at each edge of vertex v_i of G.

Sixth type

Consider a complete bipartite graph $K_{r,s}$ and let the vertex v_i be a vertex in graph G. The thorn graph, G''_K is achieved by attaching p_i sample of $K_{r,s}$ at each edge of vertex v_i of G.

In this paper, we like to compute the First Zagreb index, First Leap Zagreb index and F-Leap index for some classifications of thorn graphs.

Results for first zagreb index

We consider the graph $G = (V, E)$. It has an order n and size m, $|V| = n$, $|E| = m$. For thorn graphs, we compute the First and Second Zagreb index which were defined as

$$M_1(G) = \sum_{v \in V(G)} d_G(v)^2 = \sum_{uv \in E(G)} \left[d_G(u) + d_G(v) \right].$$

Theorem 1 : *Consider a graph G that is connected. Then we have:*

$$M_1(G_P) = M_1(G) + 2\sum_{i=1}^{n} d_G(v_i)\, p_i + \sum_{i=1}^{n} p_i^2 + (4p-7)\sum_{i=1}^{n} p_i.$$

Proof : Commonly, consider $V(G) = \{v_1, v_2, ..., v_n\}$. Denoting the K^{th} path of order greater than 2 by $P^{(k)}$ and let the vertex v_i be the beginning vertex of a path $P^{(k)}$. Now we join $P^{(k)}$ at each vertex v_i of G. Suppose in G_P by considering $v_i = v_{i1}^{(k)}$, we have $V(P^{(k)}) = \{v_{i1}^{(k)}, v_{i2}^{(k)}, ..., v_{ip}^{(k)}\}$, for $1 \le i \le n,\ 1 \le \mathrm{k} \le p_i$.

With a little care we notice that:

1) $d_{G_P}(v_i) = d_G(v_i) + p_i$,
2) $d_{G_P}(v_{ij}^{(k)}) = 2$ and $d_{G_P}(v_{ip}^{(k)}) = 1;\ 1 \le k \le p_i,\ 2 \le j \le p-1,\ 1 \le i \le n.$

So we have:

$$\begin{aligned} M_1(G_P) &= \sum_{v \in V(G_p)} d_{G_P}^2(v) \\ &= \sum_{i=1}^{n} \left[d_G(v_i) + p_i \right]^2 + (4(p-2)+1)\sum_{i=1}^{n} p_i \\ &= M_1(G) + 2\sum_{i=1}^{n} d_G(v_i)\, p_i + \sum_{i=1}^{n} p_i^2 + (4p\text{-}7)\sum_{i=1}^{n} p_i. \end{aligned}$$

Note 1 : Let consider $p_i = b$ for all $1 \le i \le n$, then:

$$M_1(G_P) = M_1(G) + 4mb + nb^2 + (4p-7)nb.$$

Theorem 2 : *Consider a graph G that is connected. Then we have:*

$$M_1(G_C) = M_1(G) + 4\sum_{i=1}^{n} d_G(v_i)\, p_i + 4\sum_{i=1}^{n} p_i^2 + 4(p-1)\sum_{i=1}^{n} p_i.$$

Proof : Commonly, consider $V(G) = \{v_1, v_2, ..., v_n\}$. Denoting the K^{th} sample of a cycle of length p by $C^{(k)}$ and let v_i is a vertex of G. Now we join $C^{(k)}$ at each vertex v_i of G. Suppose in G_C by considering $v_i = v_{i1}^{(k)}$, we have $V(C^{(k)}) = \{v_{i1}^{(k)}, v_{i2}^{(k)}, ..., v_{ip}^{(k)}\}$, for $1 \le i \le n,\ 1 \le \mathrm{k} \le p_i$.

With a little care we notice that:

1) $d_{G_C}(v_i) = d_G(v_i) + 2p_i$,

2) $d_{G_C}(v_{ij}^{(k)}) = 2$ for all $1 \le k \le p_i,\ 2 \le j \le p,\ 1 \le i \le n.$

So we have:

$$\begin{aligned} M_1(G_C) &= \sum_{v \in V(G_C)} d_{G_C}^2(v) \\ &= \sum_{i=1}^{n} (d_G(v_i) + 2p_i)^2 + 4(p-1)\sum_{i=1}^{n} p_i \\ &= M_1(G) + 4\sum_{i=1}^{n} d_G(v_i)\, p_i + 4\sum_{i=1}^{n} p_i^2 + 4(p-1)\sum_{i=1}^{n} p_i. \end{aligned}$$

Note 2 : Let consider $p_i = b$ for all $1 \le i \le n$, then:

$$M_1(G_C) = M_1(G) + 8mb + 4nb^2 + 4(p-1)nb.$$

Example 1 : Consider the path P_n as graph G and let $P_i = b$ for all $1 \le i \le n$, then we have:

1) $M_1(G_p) = 4n + 4mb + nb^2 + (4p-7)nb - 6,$
2) $M_1(G_C) = 4n + 8mb + 4nb^2 + 4(p-1)nb - 6.$

Example 2 : Consider the cycle C_n as a graph G and let $P_i = b$ for all $1 \le i \le n$, then we have:

1) $M_1(G_p) = 4n + 4mb + nb^2 + (4p-7)nb,$
2) $M_1(G_C) = 4n + 8mb + 4nb^2 + 4(p-1)nb.$

Theorem 3 : *Consider a graph G that is connected. Then we have:*

$$M_1(G_K) = M_1(G) + 2(p-1)\sum_{i=1}^{n} d_G(v_i)p_i + (p-1)^2\sum_{i=1}^{n} p_i^2 + (p-1)^3\sum_{i=1}^{n} p_i.$$

Proof : Commonly, consider $V(G) = \{v_1, v_2, ..., v_n\}$. Denoting the K^{th} sample of a complete graph K_p by $K_p^{(k)}$ and let v_i is a vertex of G. Then we join $K_p^{(k)}$ at each vertex v_i of G. Suppose in G_K by considering $v_i = v_{i1}^{(k)}$, we have $V(K_p{}^{(k)}) = \{v_{i1}^{(k)}, v_{i2}^{(k)}, ..., v_{ip}^{(k)}\}$, for $1 \le i \le n,\ 1 \le \mathrm{k} \le p_i$.

With a little care we notice that:

1) $d_{G_K}(v_i) = d_G(v_i) + (p-1)p_i$,
2) $d_{G_K}(v_{ij}^{(k)}) = p-1$ for all $1 \le k \le p_i$, $2 \le j \le m$ and $1 \le i \le n$.

So we have:

$$\begin{aligned} M_1(G_K) &= \sum_{v \in V(G_K)} d^2_{G_K}(v) \\ &= \sum_{i=1}^{n} (d_G(v_i) + (p-1)p_i)^2 + (p-1)^3 \sum_{i=1}^{n} p_i \\ &= M_1(G) + 2(p-1)\sum_{i=1}^{n} d_G(v_i)p_i + (p-1)^2 \sum_{i=1}^{n} p_i^2 + (p-1)^3 \sum_{i=1}^{n} p_i. \end{aligned}$$

Note 3 : Let consider $p_i = b$ for all $1 \le i \le n$, then:

$$M_1(G_K) = M_1(G) + 4mb(p-1) + (p-1)^2 nb^2 + (p-1)^3 nb.$$

Theorem 4 : *Consider a graph G that is connected. Then we have:*

$$M_1(G_A) = M_1(G) + 2s\sum_{i=1}^{n} d_G(v_i)p_i + s^2 \sum_{i=1}^{n} p_i^2 + (r-1)s^2 \sum_{i=1}^{n} p_i + sr^2 \sum_{i=1}^{n} p_i.$$

Proof : Commonly, consider $V(G) = \{v_1, v_2, ..., v_n\}$. Denoting the K^{th} sample of a complete bipartite graph $K_{r,s}$ by $K_{r,s}^{(k)}$ and let v_i is a vertex of *G*. Now we join $K_{r,s}^{(k)}$ at each vertex v_i of *G*. Consider the $K_{r,s}^{(k)}$ has the bipartition of $(X^{(k)}, Y^{(k)})$

Suppose in G_A by considering $v_i = v_{i1}^{(k)}$, we have $V(X^{(k)}) = \{v_{i1}^{(k)}, v_{i2}^{(k)}, ..., v_{ir}^{(k)}\}$ and $V(Y^{(k)}) = \{u_{i1}^{(k)}, u_{i2}^{(k)}, ..., u_{is}^{(k)}\}$ for $1 \le i \le n$, $1 \le \mathrm{k} \le p_i$.

With a little care we notice that:

1) $d_{G_A}(v_i) = d_G(v_i) + sp_i$,
2) $d_{G_A}(v_{ij}^{(k)}) = s \quad 2 \le j \le r$,
3) $d_{G_A}(u_{ij}^{(k)}) = r \quad 1 \le j \le s$.

So we have:

$$\begin{aligned} M_1(G_A) &= \sum_{v \in V(G_A)} d^2_{G_A}(v) \\ &= \sum_{i=1}^{n} \left[d_G(v_i) + sp_i \right]^2 + (r-1)s^2 \sum_{i=1}^{n} p_i + sr^2 \sum_{i=1}^{n} p_i \end{aligned}$$

$$= M_1(G) + 2s\sum_{i=1}^{n} d_G(v_i)p_i + s^2\sum_{i=1}^{n} p_i^2 + (r-1)s^2\sum_{i=1}^{n} p_i + sr^2\sum_{i=1}^{n} p_i.$$

Theorem 5 : *Consider a graph G that is connected. Then we have:*

$$M_1(G'_K) = M_1(G) + 2\sum_{i=1}^{n} d_G(v_i)p_i + \sum_{i=1}^{n} p_i^2 + (p-1)^3\sum_{i=1}^{n} p_i + p^2\sum_{i=1}^{n} p_i.$$

Proof : Commonly, consider $V(G) = \{v_1, v_2, ..., v_n\}$. Denoting the K^{th} sample of a complete graph K_p of order p by K_p^k and let v_i be a vertex of G. Then we join $v_{i1}^{(k)}$ to the vertex v_i of G and we obtain G'_K. We have $V(K_P{}^{(k)}) = \{v_{i1}^{(k)}, v_{i2}^{(k)}, ..., v_{ip}^{(k)}\}$, for $1 \le i \le n$, $1 \le \mathrm{k} \le p_i$.

With a little care we notice that in G'_K :

1) $d_{G'_K}(v_i) = d_G(v_i) + p_i$,
2) $d_{G'_K}(v_{ij}^{(k)}) = p - 1 \quad \text{for} \quad 2 \le j \le p.$
3) $d_{G'_K}(v_{i1}^{(k)}) = p,$

So we have:

$$\begin{aligned} M_1(G'_K) &= \sum_{v \in V(G'_K)} d^2_{G'_K}(v) \\ &= \sum_{i=1}^{n} (d_G(v_i) + p_i)^2 + (p-1)^3\sum_{i=1}^{n} p_i + p^2\sum_{i=1}^{n} p_i \\ &= M_1(G) + 2\sum_{i=1}^{n} d_G(v_i)p_i + \sum_{i=1}^{n} p_i^2 + (p-1)^3\sum_{i=1}^{n} p_i + p^2\sum_{i=1}^{n} p_i. \end{aligned}$$

Note 4 : Let consider $p_i = b$ for all $1 \le i \le n$, then:

$$M_1(G'_K) = M_1(G) + 4mb + nb^2 + (p-1)^3 nb + p^2 nb.$$

Theorem 6 : *Consider a graph G that is connected. Then we have:*

$$\begin{aligned} M_1(G''_K) = M_1(G) &+ 2\sum_{i=1}^{n} d_G(v_i)p_i + \sum_{i=1}^{n} p_i^2 + (s+1)^2\sum_{i=1}^{n} p_i \\ &+ (r-1)s^2\sum_{i=1}^{n} p_i + sr^2\sum_{i=1}^{n} p_i. \end{aligned}$$

Proof : Commonly, consider $V(G)=\{v_1,v_2,....,v_n\}$. Denoting the K^{th} sample of a complete bipartite graph $K_{r,s}$ by $K_{r,s}^{(k)}$ and let v_i is a vertex of G. Now we join $v_{i1}^{(k)}$ to the vertex v_i of G in G_K''. Consider the $K_{r,s}^{(k)}$ has the bipartition of $(X^{(k)},Y^{(k)})$.

We have $V(X^{(k)})=\{v_{i1}^{(k)},v_{i2}^{(k)},...,v_{ir}^{(k)}\}$ and $V(Y^{(k)})=\{u_{i1}^{(k)},u_{i2}^{(k)},...,u_{is}^{(k)}\}$ for $1\le i\le n,\ 1\le k\le p_i$.

With a little care we notice that:

1) $d_{G_K''}(v_i)=d_G(v_i)+p_i \qquad 1\le i\le n,$
2) $d_{G_K''}(v_{i1}^{(k)})=s+1,$
3) $d_{G_K''}(v_{ij}^{(k)})=s \qquad 2\le j\le r,$
4) $d_{G_K''}(u_{ij}^{(k)})=r \qquad 1\le j\le s.$

So we have:

$$\begin{aligned}M_1(G_K'')&=\sum_{v\in V(G_K'')}d_{G_K''}^2(v)\\&=\sum_{i=1}^{n}\left[d_G(v_i)+p_i\right]^2+(s+1)^2\sum_{i=1}^{n}p_i+(r-1)s^2\sum_{i=1}^{n}p_i+sr^2\sum_{i=1}^{n}p_i\\&=M_1(G)+2\sum_{i=1}^{n}d_G(v_i)p_i+\sum_{i=1}^{n}p_i^2+(s+1)^2\sum_{i=1}^{n}p_i+(r-1)s^2\sum_{i=1}^{n}p_i+sr^2\sum_{i=1}^{n}p_i.\end{aligned}$$

Note 5 : Let consider $p_i=b$ for all $1\le i\le n$, then:

$$M_1(G_K'')=M_1(G)+4mb+nb^2+(s+1)^2nb+(r-1)s^2nb+sr^2nb.$$

Results for first leap zagreb index and f-leap index

We consider the First Leap Zagreb index as defined $LM_1(G)=\sum_{u\in V(G)}d_2^2(u)$ and F-Leap index as defined $FL(G)=\sum_{u\in V(G)}d_2^3(u)$. Now we compute them for some thorn graphs.

It's good to review the definition of thorn graph. Commonly, let $G=(V,E)$ be a simple connected graph in which $V=\{v_1,v_2,....,v_n\}$. For nonnegative integers like $p_1,p_2,...,p_n$, a thorn graph is a graph achieved by joining p_i vertices with degree one to each vertex v_i of G. Specifically, if $p_i=b$, then we have a b-thorny graph and denote it by G^b.

Consider the path P_n with n vertices, by joining b thorns (vertex with degree one) to each vertex of P_n, we achieved a b-thorny path graph P_n^b.

Before we state the theorems, it's better to give a vertex partition of P_n^b according to 2-degree.

Table 1

Vertex partition of P_n^b according to 2-degree

n	2-degree	Number of vertices
2	b	2b+2
3	b	2b
	b+1	b+2
	2b	1
$n \geq 4$	b	2b
	b+1	(n-2)b+2
	2b+1	2
	2b+2	n-4

Theorem 7 : *Consider the b-thorny path graph P_n^b, the First Leap Zagreb index is computed as follows:*

$$LM_1(P_2^b) = (2b+2)b^2,$$
$$LM_1(P_3^b) = 2b^3 + (b(b+1)+2)^2 + 4b^2,$$
$$LM_1(P_n^b) = 2b^3 + [(n(b+1)-2)b(b+1)+2]^2 + 2(2b+1)^2 + (n(2b+2)-4)^2.$$

Proof : By considering the table 1, we have:

$$LM_1(P_2^b) = \sum_{v \in V(P_2^b)} d_2(v)^2 = (2b+2)b^2,$$
$$LM_1(P_3^b) = \sum_{v \in V(P_3^b)} d_2(v)^2 = 2b(b^2) + (b+2)(b+1)^2 + (2b)^2 = 2b^3 + (b(b+1)+2)^2 + 4b^2,$$
$$LM_1(P_n^b) = \sum_{v \in V(P_n^b)} d_2(v)^2 = 2b.b^2 + [(n-2)b+2](b+1)^2 + 2(2b+1)^2 + (n-4)(2b+2)^2$$
$$= 2b^3 + [(n(b+1)-2)b(b+1)+2]^2 + 2(2b+1)^2 + (n(2b+2)-4)^2.$$

Theorem 8 : *Consider the b-thorny path graph* P_n^b, *the F-Leap index is computed as follows:*

$$FL(P_2^b) = (2b+2)b^3,$$
$$FL(P_3^b) = 2b^4 + (b+2)(b+1)^3 + 8b^3,$$
$$FL(P_n^b) = 2b^4 + [(n-2)b+2](b+1)^3 + 2(2b+1)^3 + (n-4)(2b+2)^3.$$

Proof : By considering the definition of F-Leap index, $FL(G) = \sum_{u \in V(G)} d_2^3(u)$ and table 1 the proof is obvious.

Consider the cycle C_n with n vertices, by joining b thorns (a vertex with degree one) to each vertex of C_n, we achieved a b-thorny cycle graph C_n^b.

Before we state the theorems, it's better to give a vertex partition of C_n^b according to 2-degree.

Table 2

Vertex partition of C_n^b according to 2-degree

n	2-degree	Number of vertices
3	b+1	3b
	2b	3
4	b+1	4b
	2b+1	4
$n \geq 5$	b+1	nb
	2b+2	n

Theorem 9 : *Consider the b-thorny cycle graph* C_n^b, *the First Leap Zagreb index is computed as follows:*

$$LM_1(C_3^b) = 3b(b+1)^2 + 12b^2,$$
$$LM_1(C_4^b) = 4b(b+1)^2 + 4(2b+1)^2,$$
$$LM_1(C_n^b) = nb(b+1)^2 + n(2b+2)^2.$$

Proof **:** By considering the table 2, we have:

$$LM_1(C_3^b) = \sum_{v \in V(C_2^b)} d_2(v)^2 = 3b(b+1)^2 + 12b^2,$$

$$LM_1(C_4^b) = \sum_{v \in V(C_3^b)} d_2(v)^2 = 4b(b+1)^2 + 4(2b+1)^2,$$
$$LM_1(C_n^b) = \sum_{v \in V(C_n^b)} d_2(v)^2 = nb(b+1)^2 + n(2b+2)^2.$$

Theorem 10 : *Consider the b-thorny cycle graph* C_n^b, *the F-Leap index is computed as follows:*

$$FL(C_3^b) = 3b(b+1)^3 + 24b^3,$$
$$FL(C_4^b) = 4b(b+1)^3 + 4(2b+1)^3,$$
$$FL(C_n^b) = nb(b+1)^3 + n(2b+2)^3.$$

Proof : By considering the definition of F-Leap index, $FL(G) = \sum_{u \in V(G)} d_2^3(u)$ and table 2 the proof is obvious.

Consider the star S_{n+1} with $n+1$ vertices, by joining b thorns(vertex with degree one) to each vertex of S_{n+1}, we achieved a b-thorny star graph S_{n+1}^b.

Before we state the theorems, it's better to give a vertex partition of S_{n+1}^b according to 2-degree.

Table 3

Vertex partition of S_{n+1}^b according to 2-degree

2- degree	Number of vertices
b	nb
(n-1+b)	n+b
nb	1

Theorem 11 : *Consider the b-thorny star graph* S_{n+1}^b, *the First Leap Zagreb index is computed as follows:*

$$LM_1(S_{n+1}^b) = nb^3 + (n+b)(n-1+b)^2 + n^2b^2.$$

Proof : By considering the table 3 and the definition of First Leap Zagreb index**,** one can prove it easily.

Theorem 12 : *Consider the b-thorny star graph* S_{n+1}^b, *the F-Leap index is computed as follows:*

$$FL(S_{n+1}^b) = nb^4 + (n+b)(n-1+b)^3 + n^3b^3.$$

Proof : By considering the table 3 and the definition of F-Leap index**,** one can prove it easily.

Consider the complete graph K_n with *n* vertices, by joining b thorns (vertex with degree one) to each vertex of K_{n+1}, we achieved a b-thorny complete graph K_n^b.

Before we state the theorems, it's better to give a vertex partition of K_n^b according to 2-degree.

Table 4

Vertex partition of K_n^b according to 2-degree

2-degree	Number of vertices
b+n-1	nb
(n-1)b	n

Theorem 13 : *Consider the b-thorny complete graph K_n^b, the First Leap Zagreb index is computed as follows:*

$$LM_1(K_n^b) = nb(b+n-1)^2 + n((n-1)b)^2.$$

Proof : By considering the table 4 and the definition of First Leap Zagreb index**,** one can prove it easily.

Theorem 14 : *Consider the b-thorny complete graph K_n^b, the F-Leap index is computed as follows:*

$$FL(K_n^b) = nb(b+n-1)^3 + n((n-1)b)^3.$$

Proof : By considering the table 4 and the definition of F-Leap index**,** one can prove it easily.

Consider the complete bipartite graphs $K_{r,s}$, by joining b thorns (a vertex with degree one) to its each vertex of $K_{r,s}$, we achieved a b-thorny complete bipartite graphs $K_{r,s}^b$.

Before we state the theorems, it's better to give a vertex partition of $K_{r,s}^b$ according to 2-degree.

Table 5
Vertex partition of $K^b_{r,s}$ according to 2-degree

2-degree	Number of vertices
b+s-1	rb
b+r-1	rs
rb+s-1	s
sb+r-1	r

Theorem 15 : *Consider the b-thorny complete bipartite graph $K^b_{r,s}$, the First Leap Zagreb index is computed as follows:*

$$LM_1(K^b_{r,s}) = br(b+s-1)^2 + bs(b+r-1)^2 + s(br+s-1)^2 + r(bs+r-1)^2.$$

Proof : By considering the table 5 and the definition of First Leap Zagreb index**,** one can prove it easily.

Theorem 16 : *Consider the b-thorny complete bipartite graph $K^b_{r,s}$, the F-Leap index is computed as follows:*

$$FL(K^b_{r,s}) = br(b+s-1)^3 + bs(b+r-1)^3 + s(br+s-1)^3 + r(bs+r-1)^3.$$

Proof : By considering the table 5 and the definition of F-Leap index**,** one can prove it easily.

Conclusion

In this paper, we compute some topological indices like First Zagreb index, Firs Leap Zagreb index and F-Leap index and give some examples for some kind of particular structure in chemical graph recognized as thorn graph.

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Received September, 2020

Revised January, 2021