

## A connection between fuzzy $\Gamma$ -hypergroups and $\Gamma$ -groups

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### Abstract

By applying the notion of fuzzy  $\Gamma$ -hypergroups, we demonstrate that the defined equivalence relation  $\varepsilon$  is the smallest fuzzy strongly regular relation and the quotient  $H / \varepsilon^*$  is a  $\Gamma / \beta^*$ -group. Especially, the homomorphism of fuzzy  $\Gamma$ -hypergroups is introduced to obtain some related conclusions. Also, the functor from the category of fuzzy  $\Gamma$ -hypergroups is assigned to the category of fundamental  $\Gamma / \beta^*$ -groups in order to show that the functor is covariant.

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### 1. Introduction

Fuzzy algebraic hyperstructures are considered as the main and noticeable subject and it is a well-developed branch of fuzzy mathematics theory. One of the classes that deal with this subject is fuzzy hypergroups. This idea was first introduced by Sen et al. in [1], where fuzzy semihypergroups were defined and investigated. After that, by using the fuzzy semihypergroup notion, Leoreanu-Fotea and Davvaz [2] introduced the concept of fuzzy hyperring and studied its connections with hyperrings and also Leoreanu-Fotea presented fuzzy hypermodules [3].

Sen et al. [4] presented the theory of  $\Gamma$ -semigroup as an extension of semigroup and then Nobosawa [5] studied  $\Gamma$ -rings for classical structures. Ameri et al. [6], introduced the notion of  $\Gamma$ -module over a  $\Gamma$ -ring. They also introduced the fuzzy  $\Gamma$ -hypermodules for the generalization purpose

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of  $\Gamma$ -hypermultiplication, and fuzzy modules and obtained basic results. Further study by Leoreanu-Fotea et al. [7] on Fuzzy  $\Gamma$ -hyperring and fuzzy  $\Gamma$ -hypermultiplication was done in finding important features.

Fundamental relation plays an important role in fuzzy hyperstructure theory, it can be defined as the smallest equivalence relation such that the quotient is universal algebra (such as group, ring, module, etc). Lately, Davvaz and Firouzkouhi analyzed the fundamental relation on fuzzy hyperring and fuzzy hypermultiplication. They assigned the fundamental functor between the category of fuzzy hyperrings (hypermodules) and the category of rings (modules), (for more information refer to [8]-[11]).

In this paper, we apply the notion of fuzzy  $\Gamma$ -hypergroups and show that the defined equivalence relation  $\varepsilon$  is the smallest fuzzy strongly regular relation and the quotient  $H / \varepsilon^*$  can be  $\Gamma / \beta^*$ -group that is not abelian. Next, we convey the homomorphism of fuzzy  $\Gamma$ -hypergroups to gain related results. Also, we state a functor from the category of fuzzy  $\Gamma$ -hypergroups to the category of fundamental  $\Gamma / \beta^*$ -groups to demonstrate that the functor is covariant.

## 2. Preliminaries

**Definition 2.1:** [6] Assume  $G$  and  $\Gamma$  be two nonempty sets. Denote  $F^*(G)$  be the set of all nonzero fuzzy subset of  $G$ . A Fuzzy  $\Gamma$ -hyperoperation on  $G$  is a map  $\uplus: G \times \Gamma \times G \rightarrow F^*(G)$ , which associates a nonzero subset  $a \uplus \gamma \uplus b$  for all  $a, b \in G$  and  $\gamma \in \Gamma$ .  $(G, \uplus)$  is called a Fuzzy  $\Gamma$ -hypergroupoid. A fuzzy  $\Gamma$ -hypergroupoid is described as a fuzzy  $\Gamma$ -hypersemigroup if for all  $a, b, c \in G$  and  $\alpha, \beta \in \Gamma$ , we have  $a \uplus \alpha \uplus (b \uplus \beta \uplus c) = (a \uplus \alpha \uplus b) \uplus \beta \uplus c$ , where for any  $\mu \in F^*(G)$ , we have  $(a \uplus \gamma \uplus \mu)(r) = \bigvee_{t \in G} ((a \uplus \gamma \uplus t)(r) \wedge \mu(t))$  and  $(\mu \uplus \gamma \uplus a)(r) = \bigvee_{t \in G} (\mu(t) \wedge (t \uplus \gamma \uplus a)(r))$  for all  $r \in G, \gamma \in \Gamma$ .

If  $A$  is a nonempty subset of  $G$  and  $x \in G$ , then for all  $r \in G$ , and  $\gamma \in \Gamma$ , we have  $(x \uplus \gamma \uplus A)(r) = \bigvee_{a \in A} (x \uplus \gamma \uplus a)(r)$ , and  $(A \uplus \gamma \uplus x)(r) = \bigvee_{a \in A} (a \uplus \gamma \uplus x)(r)$ .

A fuzzy  $\Gamma$ -hypersemigroup  $(G, \uplus)$  is called a fuzzy  $\Gamma$ -hypergroup if for all  $a \in G$  and  $\gamma \in \Gamma$ , we have  $a \uplus \gamma \uplus G = G \uplus \gamma \uplus a = \chi_G$ .

Let  $\mu, \nu \in F^*(G)$ , we state

$$(\mu \uplus \gamma \uplus \nu)(t) = \bigvee_{p, q \in G} (\mu(p) \wedge (p \uplus \gamma \uplus q)(t) \wedge \nu(q)), \forall t \in G, \gamma \in \Gamma.$$

**Definition 2.2 :** [4] Assume  $H = \{a_1, a_2, a_3, \dots\}$  and  $\Gamma = \{\gamma_1, \gamma_2, \gamma_3, \dots\}$  be two non-empty sets. Then  $H$  is called a  $\Gamma$ -semigroup if there exists a map  $H \times \Gamma \times H \rightarrow H$  written as  $(a_1, \gamma_1, a_2) \rightarrow a_1 \gamma_1 a_2$ , satisfying  $(a_1 \gamma_1 a_2) \gamma_2 a_3 = a_1 \gamma_1 (a_2 \gamma_2 a_3)$ .

Let  $M$  be a non-empty subset of  $H$ . Then  $M$  is named a sub  $\Gamma$ -semigroup of  $H$  if  $a_1 \gamma_1 a_2 \in M$ .

**Definition 2.3 :** [1] Suppose  $\eta$  be an equivalence relation on a fuzzy semihypergroup  $(G, \oplus)$ . We denote  $\mu \bar{\eta} \nu$ , for  $\mu, \nu \in F^*(G)$ , if  $\mu(a) > 0$  implies there exists  $b \in G$  such that  $\nu(b) > 0$  and  $a \eta b$ , and if  $\nu(x) > 0$  implies there exists  $y \in G$  such that  $\mu(y) > 0$  and  $x \eta y$ . Also, we denote  $\mu \bar{\bar{\eta}} \nu$ , if for all  $x \in G$  such that  $\mu(x) > 0$ , and for all  $y \in G$  such that  $\nu(y) > 0$ , we have  $x \eta y$ . The relation  $\eta$  on a fuzzy semihypergroup  $(G, \oplus)$  is called a fuzzy (strongly) regular relation if  $a \eta a', b \eta b'$  implies  $a \oplus b \bar{\eta} a' \oplus b' \bar{\bar{\eta}} (a \oplus b \bar{\eta} a' \oplus b')$ .

**Definition 2.4 :** [6] The relation  $\eta$  on a fuzzy  $\Gamma$ -hypersemigroup  $(G, \oplus)$  is considered as a fuzzy regular relation on  $(G, \oplus)$  if, for all  $a, b, c, a_0, b_0 \in G, \gamma \in \Gamma$ , the next condition holds:

$$a \eta a_0, b \eta b_0 \Rightarrow (a \oplus \gamma \oplus b) \eta (a_0 \oplus \gamma \oplus b_0).$$

### 3. Fundamental Relation on Fuzzy $\Gamma$ -hypergroup

**Definition 3.1 :** Let  $(H, \odot)$  be a fuzzy  $\Gamma$ -hypergroup and  $\beta^*$  the fundamental relation on  $(\Gamma, \otimes)$ . We define the relation  $\varepsilon^*$  as the smallest equivalence relation such that the quotient  $H / \varepsilon^*$  is a  $\Gamma / \beta^*$ -group. Then the relation  $\varepsilon^*$  is called the fundamental equivalence relation on fuzzy  $\Gamma$ -hypergroup  $H$ , and  $H / \varepsilon^*$  is called the fundamental  $\Gamma / \beta^*$ -group.

Denote the set  $U_{(H, \Gamma)} = U$  in the following manner:

$$U = \left\{ \prod_{i=1}^n (a_i \odot \gamma_i \odot b_i) \mid a_i, b_i \in H, \gamma_i \in \Gamma, n \in \mathbb{N} \right\}$$

Now, we define the relation  $\varepsilon$  on  $H$  as follows:

$$x \varepsilon y \Leftrightarrow \exists u \in U : u(x) > 0, u(y) > 0.$$

Evidently,  $\varepsilon$  is symmetric. Introduce  $\forall m \in H, m(m) = \chi_m(m) = 1 > 0$ , thus the relation  $\varepsilon$  is reflexive. Consider  $\varepsilon^*$  be the transitive closure of  $\varepsilon$ . For all  $a, b \in H$  and  $\gamma \in \Gamma$ , we determine:

$$\varepsilon^*(a) \oplus \beta^*(\gamma) \oplus \varepsilon^*(b) = \{\varepsilon^*(d) \mid (a' \odot \gamma' \odot b')(d) > 0; a\varepsilon^*a', \gamma\beta^*\gamma', b\varepsilon^*b'\}.$$

**Definition 3.2:** Let  $H$  be a fuzzy  $\Gamma$ -hypergroup, for every map  $\varphi : H \rightarrow H'$ , where  $H'$  is a group, then we define  $\varphi(u) = \{\varphi(t) : u(t) > 0\}$  for  $u \in U$  such that  $u = \prod_{i=1}^n (a_i \odot \gamma_i \odot b_i)$ .

**Theorem 3.3 :** If  $(H, \odot)$  is fuzzy  $\Gamma$ -hypergroup, then  $\varepsilon^*$  is fuzzy strongly regular relation.

**Proof:** The relation  $\varepsilon$  is reflexive and symmetric, so  $\varepsilon^*$  is an equivalence relation. To illustrate that it is fuzzy strongly regular, we should verify

$$a\varepsilon a', \gamma\beta^*\gamma', b\varepsilon b' \Rightarrow a \odot \gamma \odot b \overline{\varepsilon} a' \odot \gamma' \odot b'. \quad (1)$$

To verify (1), for every  $x \in H, (a \odot \gamma \odot b)(x) > 0$  and for every  $y \in H, (a' \odot \gamma' \odot b')(y) > 0$ , we show that  $x\varepsilon y$ . Since  $a\varepsilon a'$ , then

$$\exists u \in U, u = \prod_{i=1}^n (a_i \odot \gamma_i \odot b_i) : u(a) > 0, u(a') > 0,$$

and since  $b\varepsilon b'$ , then

$$\exists u' \in U, u' = \prod_{i=1}^n (a'_i \odot \gamma'_i \odot b'_i) : u'(b) > 0, u'(b') > 0,$$

Thus,

$$(u \odot \gamma \odot b)(x) = \bigvee_{a \in H} u(a) \wedge (a \odot \gamma \odot b)(x) > 0,$$

and

$$(u \odot \gamma \odot u')(x) = \bigvee_{b \in H} (u \odot \gamma \odot b)(x) \wedge u'(b) > 0.$$

Then, we have  $(u \odot \gamma \odot u')(x) > 0$ . Similarly, if  $(a' \odot \gamma' \odot b')(y) > 0$ , so  $(u \odot \gamma \odot u')(y) > 0$ , while  $(u \odot \gamma \odot u') \in U$ , we have  $x\varepsilon y$ .

In addition, if  $x\varepsilon^*x', \gamma\beta^*\gamma', y\varepsilon^*y'$ , we verify that  $x \odot \gamma \odot y \overline{\varepsilon}^* x' \odot \gamma' \odot y'$ . For every  $l, l' \in H, (x \odot \gamma \odot y)(l) > 0, (x' \odot \gamma' \odot y')(l') > 0$ , there exist  $m \in \mathbb{N}$  and  $(x = v_0, v_1, \dots, v_{m-1}, v_m = x') \in H^{m+1}, (\gamma = w'_0, w'_1, \dots, w'_{m-1}, w'_m = \gamma') \in \Gamma^{m+1}$  and  $(y = v'_0, v'_1, \dots, v'_{m-1}, v'_m = y') \in H^{m+1}$  such that  $x = v_0 \varepsilon v_1 \varepsilon \dots \varepsilon v_{m-1} \varepsilon v_m = x', \gamma = w'_0 \beta w'_1 \beta \dots \beta w'_{m-1} \beta w'_m = \gamma'$ , and  $y = v'_0 \varepsilon v'_1 \varepsilon \dots \varepsilon v'_{m-1} \varepsilon v'_m = y'$ , then by (1), we have  $x \odot \gamma \odot y \overline{\varepsilon} v_1 \odot w'_1 \odot v'_1$

$\bar{\varepsilon} \dots \bar{\varepsilon} x' \odot \gamma' \odot y'$ . If  $(x \odot \gamma \odot y)(l) > 0$ ,  $(x' \odot \gamma' \odot y')(l') > 0$ , with considering  $t_i \in H$  for  $i \in \{1, 2, \dots, m-1\}$ , we have  $(v_1 \odot w'_1 \odot v'_1)(t_1) > 0$ ,  $(v_2 \odot w'_2 \odot v'_2)(t_2) > 0, \dots, (v_{m-1} \odot w'_{m-1} \odot v'_{m-1})(t_{m-1}) > 0$ , so  $l \varepsilon t_1 \varepsilon t_2 \varepsilon \dots \varepsilon t_{m-1} \varepsilon l'$ , therefore  $l \varepsilon^* l'$ , hence  $\varepsilon^*$  is strongly regular relation.

**Lemma 3.4 :** *The set  $\varepsilon^*(a) \oplus \beta^*(\gamma) \oplus \varepsilon^*(b)$  is singleton.*

**Proof :** By Theorem 3.3,  $\varepsilon^*$  is a fuzzy strongly regular equivalence, if  $a \varepsilon^* a'$  and,  $b \varepsilon^* b'$ , then  $a \odot \gamma \odot b \varepsilon^* a' \odot \gamma' \odot b'$ . With the notion of fuzzy strongly regular relation,  $\forall c' \in H, (a \odot \gamma \odot b)(c') > 0$  and  $\forall c \in H, (a' \odot \gamma' \odot b')(c) > 0$ , we have  $c \varepsilon^* c'$ , i.e.,  $\varepsilon^*(c) = \varepsilon^*(c')$ , and we have  $\varepsilon^*(a) \oplus \beta^*(\gamma) \oplus \varepsilon^*(b) = \varepsilon^*(a') \oplus \beta^*(\gamma') \oplus \varepsilon^*(b')$ , thus  $\varepsilon^*(a) \oplus \beta^*(\gamma) \oplus \varepsilon^*(b)$  is singleton.  $\square$

**Proposition 3.5 :** If  $(H, \odot)$  is fuzzy  $\Gamma$ -hypergroup, then  $H / \varepsilon^*$  is  $\Gamma / \beta^*$ -group.

**Proof :** It is clear that  $\varepsilon^*(0_H)$  is the zero element of  $(H / \varepsilon^*, \oplus)$  and  $\varepsilon^*(-a)$  is the opposite element of  $\varepsilon^*(a)$  such that  $\varepsilon^*(a) \oplus \varepsilon^*(-a) = \varepsilon^*(0_H)$ .

For any  $\varepsilon^*(x), \varepsilon^*(y), \varepsilon^*(z) \in H / \varepsilon^*$  and  $\beta^*(\alpha), \beta^*(\gamma) \in \Gamma / \beta^*$ , we should prove:

$$\varepsilon^*(x) \oplus \beta^*(\alpha) \oplus [\varepsilon^*(y) \oplus \beta^*(\gamma) \oplus \varepsilon^*(z)] = [\varepsilon^*(x) \oplus \beta^*(\alpha) \oplus \varepsilon^*(y)] \oplus \beta^*(\gamma) \oplus \varepsilon^*(z)$$

Applying the associative property of  $H$ , we have:

$$\begin{aligned} \varepsilon^*(x) \oplus \beta^*(\alpha) \oplus [\varepsilon^*(y) \oplus \beta^*(\gamma) \oplus \varepsilon^*(z)] &= \{\varepsilon^*(d) \mid [x \odot \alpha \odot (y \odot \gamma \odot z)](d) > 0\} \\ &= \{\varepsilon^*(d) \mid [(x \odot \alpha \odot y) \odot \gamma \odot z](d) > 0\} \\ &= [\varepsilon^*(x) \oplus \beta^*(\alpha) \oplus \varepsilon^*(y)] \oplus \beta^*(\gamma) \oplus \varepsilon^*(z) \end{aligned}$$

$\square$

**Theorem 3.6 :**  $\varepsilon^*$  is the smallest fuzzy strongly regular equivalence so that  $H / \varepsilon^*$  is  $\Gamma / \beta^*$ -group.

**Proof :** Let  $\lambda$  be a fuzzy strongly regular relation such that  $H / \lambda$  is a  $\Gamma / \beta^*$ -group. Let  $\varphi : H \rightarrow H / \lambda$  be the canonical projection.  $\varphi$  is a good homomorphism and besides if  $x \varepsilon y$ , then there exists  $u \in U; u = \prod_{i=1}^n (a_i \odot \gamma_i \odot b_i)$ , then  $u(x) > 0, u(y) > 0$ . By 3.2,

$$\varphi(u) = \{\varphi(x) \mid u(x) > 0\}, \varphi(u) = \{\varphi(y) \mid u(y) > 0\}$$

This yields to  $\varphi(x) = \varphi(y)$  and  $x\lambda y$ . Therefore  $x\varepsilon y \Rightarrow x\lambda y$ .

At last, if  $x\varepsilon^*y$ , then  $\exists m \in \mathbb{N}$  and  $(w_0 = x, w_1, \dots, w_m = y) \in H^{m+1}$  such that  $x = w_0\varepsilon w_1\varepsilon \dots \varepsilon w_m = y$ . Therefore,  $x = w_0\lambda w_1\lambda \dots \lambda w_m = y$ , and the transitivity of  $\lambda$  indicates  $x\lambda y$ , thus  $\varepsilon^* \subset \lambda$ .  $\square$

Suppose  $(H_1, \odot)$  be a fuzzy  $\Gamma_1$ -hypergroup and  $(H_2, \oplus)$  be a fuzzy  $\Gamma_2$ -hypergroup. Then  $(f, \varphi): (H_1, \Gamma_1) \rightarrow (H_2, \Gamma_2)$  is called a homomorphism (good homomorphism) if  $f: H_1 \rightarrow H_2$  and  $\varphi: \Gamma_1 \rightarrow \Gamma_2$  are homomorphism (good homomorphism) and  $f(x \odot \gamma \odot y) \subseteq f(x) \oplus \varphi(\gamma) \oplus f(y)$  ( $f(x \odot \gamma \odot y) = f(x) \oplus \varphi(\gamma) \oplus f(y)$ ) for all  $x, y \in H$  and  $\gamma \in \Gamma$ .

**Theorem 3.7 :** Let  $(H, \odot)$  be a fuzzy  $\Gamma$ -hypergroup and  $\varepsilon^*$  the fundamental relation on  $H$  and  $\beta^*$  the fundamental relation on  $(\Gamma, \otimes)$ . Then  $(\pi_H, \pi_\Gamma): (H, \Gamma) \rightarrow (H/\varepsilon^*, \Gamma/\beta^*)$  is a good epimorphism, where  $\pi_H: H \rightarrow H/\varepsilon^*$  is defined as  $\pi_H(x) = \varepsilon^*(x)$ , and,  $\pi_\Gamma: \Gamma \rightarrow \Gamma/\beta^*$  is defined as  $\pi_\Gamma(\gamma) = \beta^*(\gamma)$ .

**Proof:**  $\pi_H$  and  $\pi_\Gamma$  are well-defined and onto, as  $\pi_H(x) = \varepsilon^*(x)$ ,  $\pi_\Gamma(\gamma) = \beta^*(\gamma)$  for  $x \in H, \gamma \in \Gamma$ . To prove that  $(\pi_H, \pi_\Gamma)$  is a good homomorphism, we should verify that  $\pi_H(x \odot y) = \pi_H(x) \oplus \pi_H(y)$  and  $\pi_\Gamma(\alpha \otimes \gamma) = \pi_\Gamma(\alpha) \oplus \pi_\Gamma(\gamma)$  for all  $x, y \in H$  and  $\alpha, \gamma \in \Gamma$ , and prove  $\pi_H(x \odot \gamma \odot y) = \pi_H(x) \oplus \pi_\Gamma(\gamma) \oplus \pi_H(y)$ .

We just prove  $\pi_H(x \odot \gamma \odot y) = \pi_H(x) \oplus \pi_\Gamma(\gamma) \oplus \pi_H(y)$ , and the rests are similar. Let  $(x \odot \gamma \odot y)(d) > 0$  then  $\varepsilon^*(d) \in \varepsilon^*(x) \oplus \beta^*(\gamma) \oplus \varepsilon^*(y)$ . As  $|\varepsilon^*(x) \oplus \beta^*(\gamma) \oplus \varepsilon^*(y)| = 1$ , we have  $\varepsilon^*(x) \oplus \beta^*(\gamma) \oplus \varepsilon^*(y) = \varepsilon^*(d)$ . Via 3.2,  $\pi_H(x \odot \gamma \odot y) = \{\pi_H(d) : (x \odot \gamma \odot y)(d) > 0\}$ , is equal to  $\varepsilon^*(x \odot \gamma \odot y) = \{\varepsilon^*(d) : (x \odot \gamma \odot y)(d) > 0\}$ . We can write  $\varepsilon^*(x \odot \gamma \odot y) = \varepsilon^*(d)$ , therefore  $\varepsilon^*(x) \oplus \beta^*(\gamma) \oplus \varepsilon^*(y) = \varepsilon^*(d) = \varepsilon^*(x \odot \gamma \odot y)$ . So,

$$\pi_H(x) \oplus \pi_\Gamma(\gamma) \oplus \pi_H(y) = \varepsilon^*(x) \oplus \beta^*(\gamma) \oplus \varepsilon^*(y) = \varepsilon^*(x \odot \gamma \odot y) = \pi_H(x \odot \gamma \odot y).$$

$\square$

**Proposition 3.8 :** Let  $H$  be a fuzzy  $\Gamma$ -hypergroup. If  $u \in U$  and  $u = \bigcap_{i=1}^n (a_i \odot \gamma_i \odot b_i)$ , and if  $u(z) > 0$ , then we have

$$\varepsilon^*(z) = \varepsilon^*(u) = \bigcup_{i=1}^n \varepsilon^*(a_i) \oplus \beta^*(\gamma_i) \oplus \varepsilon^*(b_i).$$

**Proof :** If  $u(z) > 0$  and  $u = \prod_{i=1}^n (a_i \odot \gamma_i \odot b_i)$ , then  $(\prod_{i=1}^n a_i \odot \gamma_i \odot b_i)(z) > 0$ , and we can write  $\varepsilon^*(z) \in \biguplus_{i=1}^n \varepsilon^*(a_i) \oplus \beta^*(\gamma_i) \oplus \varepsilon^*(b_i)$ . From the singleton property, we have the equality  $\varepsilon^*(z) = \biguplus_{i=1}^n \varepsilon^*(a_i) \oplus \beta^*(\gamma_i) \oplus \varepsilon^*(b_i)$ . Besides by 3.2,  $\varepsilon^*(z) = \varepsilon^*(u)$ .  $\square$

**Theorem 3.9 :** Let  $H_1$  be a fuzzy  $\Gamma_1$ -hypergroups,  $H_2$  be fuzzy  $\Gamma_2$ -hypergroup and  $\varepsilon_1^*, \beta_1^*, \varepsilon_2^*$  and  $\beta_2^*$  the fundamental relations on  $H_1, \Gamma_1, H_2$  and  $\Gamma_2$  respectively. If  $(f, g) : (H_1, \Gamma_1) \rightarrow (H_2, \Gamma_2)$  is a good homomorphism (isomorphism) and  $f(0) = 0$ , then there is a unique good homomorphism (isomorphism)  $(f^*, g^*) : (H_1 / \varepsilon_1^*, \Gamma_1 / \beta_1^*) \rightarrow (H_2 / \varepsilon_2^*, \Gamma_2 / \beta_2^*)$  so that the next diagram has commutative property.

$$\begin{array}{ccc} (H_1, \Gamma_1) & \xrightarrow{(f, g)} & (H_2, \Gamma_2) \\ \downarrow (\pi_{H_1}, \pi_{\Gamma_1}) & & \downarrow (\pi_{H_2}, \pi_{\Gamma_2}) \\ (H_1 / \varepsilon_1^*, \Gamma_1 / \beta_1^*) & \xrightarrow{(f^*, g^*)} & (H_2 / \varepsilon_2^*, \Gamma_2 / \beta_2^*) \end{array}$$

**Definition 3.10 :** Consider a functor from category  $F\text{-}\Gamma\text{-Hg}$  of fuzzy  $\Gamma$ -hypergroup with homomorphism into category  $\Gamma / \beta^*\text{-g}$ , of  $\Gamma$ -groups that is,  $F : F\text{-}\Gamma\text{-Hg} \rightarrow \Gamma / \beta^*\text{-g}$ . We call  $F$  a fundamental functor.

**Theorem 3.11 :** The functor  $F : F\text{-}\Gamma\text{-Hg} \rightarrow \Gamma / \beta^*\text{-g}$  such that  $(H, \odot) \mapsto (H / \varepsilon^*, \oplus)$  with  $F(H) = H / \varepsilon^*$ , and  $F(f, g) = (f^*, g^*)$  is covariant.

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