

Results for the series of the resolution of the partition (11,8,3)

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Abstract

By applying Capelli identities, place polarization and diagrams for a free $\mathcal{R}$ -module $\mathcal{F}$ where $\mathcal{R}$ has identity and $\mathcal{DF}_i$ be the divided power algebra of degree i . We prove the complexness sequence of the complex elements for the partition (11,8,3).

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1. Introduction

$\mathcal{K}_{\lambda/\mu}\mathcal{F} = \text{Im}(\mathcal{DF} \rightarrow \wedge \mathcal{F})$ is a Weyl module for the partition λ/μ , we depended this work for Capelli identities, place polarization and diagrams, [1, 2]. This research employs the same idea in [3, 4] for (11,8,3).

We can apply that for $\mathcal{SL}(2, \mathcal{U})$, $\mathcal{U} = 81, 729, 125, 3125, 121, 196, \text{PG}(3,8)$, [5, 6], some finite groups and spaces [7-10]. Also, for the studies by authors in [11-15]. We write $\mathcal{DF}(\mathcal{p} \otimes q \otimes r)$ instead of $\mathcal{DF}_{\mathcal{p}} \otimes \mathcal{D}_{\mathcal{p}}\mathcal{F} \otimes \mathcal{D}_{\mathcal{p}}\mathcal{F}$.

2. The Main Result

The characteristic zero resolution in the case of the partition (11,8,3) has the formulation:

$$\begin{aligned} \mathcal{DF}(13 \otimes 8 \otimes 1) &\rightarrow \mathcal{DF}(13 \otimes 7 \otimes 2) \oplus \mathcal{DF}(12 \otimes 9 \otimes 1) \rightarrow \\ \mathcal{DF}(12 \otimes 7 \otimes 3) &\oplus \mathcal{DF}(11 \otimes 9 \otimes 2) \rightarrow \mathcal{DF}(11 \otimes 8 \otimes 3) \end{aligned}$$

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See the pursue diagram:

$$\begin{array}{ccccc} \mathcal{DF}(13 \otimes 8 \otimes 1) & \xrightarrow{d_1} & \mathcal{DF}(13 \otimes 7 \otimes 2) & \xrightarrow{d_2} & \mathcal{DF}(12 \otimes 7 \otimes 3) \\ \downarrow \hbar_1 & \mathcal{Q} & \downarrow \hbar_2 & \mathcal{T} & \downarrow \hbar_3 \end{array}$$

$$\mathcal{DF}(12 \otimes 9 \otimes 1) \xrightarrow{\mathcal{G}_1} \mathcal{DF}(11 \otimes 9 \otimes 2) \xrightarrow{\mathcal{G}_2} \mathcal{DF}(11 \otimes 8 \otimes 3)$$

If we define $d_1(m) = \partial_{32}(v)$; where $m \in \mathcal{DF}(13 \otimes 8 \otimes 1)$,

$\hbar_1(v) = \partial_{21}(m)$; where $m \in \mathcal{DF}(13 \otimes 8 \otimes 1)$ and

$\hbar_2(v) = \partial_{21}^{(2)}(m)$; where $m \in \mathcal{DF}(13 \otimes 7 \otimes 2)$

Now, we need to define the map

$$\mathcal{G}_1: \mathcal{DF}(12 \otimes 9 \otimes 1) \rightarrow \mathcal{DF}(11 \otimes 9 \otimes 2)$$

Which fabricate the diagram $\mathcal{Q}$ commutative, i.e. $\mathcal{G}_1 \circ \hbar_1 = \hbar_2 \circ d_1$

Which implies that $\mathcal{G}_1 \circ \partial_{21} = \partial_{21}^{(2)} \circ \partial_{32}$

By employing Capelli identity we gain:

$$\partial_{21}^{(2)} \circ \partial_{32} = \partial_{32} \circ \partial_{21}^{(2)} - \partial_{31} \circ \partial_{21} = \left(\frac{1}{2} \partial_{32} \circ \partial_{21} - \partial_{31}\right) \circ \partial_{21}.$$

Thus, $\mathcal{G}_1 = \frac{1}{2} \partial_{32} \circ \partial_{21} - \partial_{31}$

Define $\mathcal{G}_2(m) = \partial_{32}(m)$; where $m \in \mathcal{DF}(12 \otimes 9 \otimes 1)$ and $\hbar_3(v) = \partial_{21}(m)$; where $m \in \mathcal{DF}(13 \otimes 7 \otimes 2)$

We require acquainting d_2 to fabricate the diagram $\mathcal{T}$ commute:

$$d_2: \mathcal{DF}(13 \otimes 7 \otimes 2) \rightarrow \mathcal{DF}(12 \otimes 7 \otimes 3) \text{ such that}$$

$$\hbar_3 \circ d_2 = \mathcal{G}_2 \circ \hbar_2 \text{ i.e. } \partial_{21} \circ d_2 = \partial_{32} \circ \partial_{21}^{(2)}$$

By employing Capelli identity we gain:

$$\partial_{32} \circ \partial_{21}^{(2)} = \partial_{21}^{(2)} \circ \partial_{32} + \partial_{21} \circ \partial_{31} = \partial_{21} \circ \left(\frac{1}{2} \partial_{21} \circ \partial_{32} + \partial_{31}\right)$$

Thus, $d_2 = \frac{1}{2} \partial_{21} \circ \partial_{32} + \partial_{31}$.

Now take the pursue diagram:

$$\begin{array}{ccccc} \mathcal{DF}(13 \otimes 8 \otimes 1) & \xrightarrow{d_1} & \mathcal{DF}(13 \otimes 7 \otimes 2) & \xrightarrow{d_2} & \mathcal{DF}(12 \otimes 7 \otimes 3) \\ \downarrow \hbar_1 & \mathcal{H} & \downarrow \hbar_2 & \nearrow w & \downarrow \hbar_3 \\ \mathcal{DF}(12 \otimes 9 \otimes 1) & \xrightarrow{\mathcal{G}_1} & \mathcal{DF}(11 \otimes 9 \otimes 2) & \xrightarrow{\mathcal{G}_2} & \mathcal{DF}(11 \otimes 8 \otimes 3) \end{array}$$

Acquaint $w: \mathcal{DF}(12 \otimes 9 \otimes 1) \rightarrow \mathcal{DF}(12 \otimes 7 \otimes 3)$ by

$$w(m) = \partial_{32}^{(2)}(m); \text{ where } m \in \mathcal{D}_{12} \otimes \mathcal{D}_9 \otimes \mathcal{D}_1$$

Proposition 2.1: The diagram $\mathcal{H}$ is commutative.

Proof: To prove the diagram $\mathcal{H}$ is commutative, we require proving

$$\begin{aligned}
 d_2 \circ d_1 &= w \circ h_1 \\
 d_2 \circ d_1 &= \left(\frac{1}{2}\partial_{21} \circ \partial_{32} + \partial_{31}\right) \circ \partial_{32} \\
 &= \partial_{32}^{(2)} \circ \partial_{21} - \partial_{32} \circ \partial_{31} + \partial_{31} \circ \partial_{32} = \partial_{32}^{(2)} \circ \partial_{21} = w \circ h_1
 \end{aligned}$$

Proposition 2.2: The diagram $\mathcal{N}$ is commutative.

Proof: $\mathcal{G}_2 \circ \mathcal{G}_1 = \partial_{32} \circ \left(\frac{1}{2}\partial_{32} \circ \partial_{21} - \partial_{31}\right)$
 $= \partial_{21} \circ \partial_{32}^{(2)} + \partial_{32} \circ \partial_{31} - \partial_{32} \circ \partial_{31} = \partial_{21} \circ \partial_{32}^{(2)} = h_3 \circ w$

Eventually, we acquaint the maps σ_1, σ_2 and σ_3 as pursue:

- $\sigma_3(m) = (d_1(m), h_1(m)); \forall m \in \mathcal{DF}(13 \otimes 8 \otimes 1)$
- $\sigma_2((m_1, m_2)) = (d_2(m_1) - w(m_2), \mathcal{G}_1(m_2) - h_2(m_1));$
 $\forall m \in \mathcal{DF}(13 \otimes 7 \otimes 2) \oplus \mathcal{DF}(12 \otimes 9 \otimes 1)$
- $\sigma_1((m_1, m_2)) = (h_3(m_1) + \mathcal{G}_2(m_2));$
 $\forall m \in \mathcal{DF}(12 \otimes 7 \otimes 3) \oplus \mathcal{DF}(11 \otimes 9 \otimes 2),$

where $\sigma_3: \mathcal{DF}(13 \otimes 8 \otimes 1) \rightarrow \mathcal{DF}(13 \otimes 7 \otimes 2) \oplus \mathcal{DF}(12 \otimes 9 \otimes 1)$

$\sigma_2: \mathcal{DF}(13 \otimes 7 \otimes 2) \oplus \mathcal{DF}(12 \otimes 9 \otimes 1) \rightarrow \mathcal{DF}(12 \otimes 7 \otimes 3) \oplus \mathcal{DF}(11 \otimes 9 \otimes 2)$

And $\sigma_1: \mathcal{DF}(12 \otimes 7 \otimes 3) \oplus \mathcal{DF}(11 \otimes 9 \otimes 2) \rightarrow \mathcal{DF}(11 \otimes 8 \otimes 3)$

Proposition 2.3: The following sequence is complex.

$$\begin{aligned}
 0 \rightarrow \mathcal{DF}(13 \otimes 8 \otimes 1) &\xrightarrow{\sigma_3} \mathcal{DF}(13 \otimes 7 \otimes 2) \oplus \mathcal{DF}(12 \otimes 9 \otimes 1) \xrightarrow{\sigma_2} \\
 \mathcal{DF}(12 \otimes 7 \otimes 3) \oplus \mathcal{DF}(11 \otimes 9 \otimes 2) &\xrightarrow{\sigma_1} \mathcal{DF}(11 \otimes 8 \otimes 3)
 \end{aligned}$$

Proof: From the acquaint, it is known that ∂_{21} and ∂_{32} are injective [1], then we get σ_3 is injective by Capelli identities.

$$\begin{aligned}
 (\sigma_2 \circ \sigma_3)(s) &= \sigma_2(d_1(s), h_1(s)) \\
 &= \sigma_2(\partial_{32}(s), \partial_{21}(s)) \\
 &= (d_2(\partial_{32}(s)) - w(\partial_{21}(s)), \mathcal{G}_1(\partial_{21}(s)) \\
 &\quad - h_2(\partial_{32}(s))) \\
 &= \left(\frac{1}{2}\partial_{21}\partial_{32} + \partial_{31}\right)\partial_{32}(s) - \partial_{32}^{(2)}\partial_{21}(s) = 0 \\
 \mathcal{G}_1(\partial_{21}(s)) - h_2(\partial_{32}(s)) &= \left(\frac{1}{2}\partial_{32}\partial_{21} - \partial_{31}\right)\partial_{21}(s) - \partial_{21}^{(2)}\partial_{32}(s) = 0
 \end{aligned}$$

We reign $(\sigma_2 \circ \sigma_3)(s) = 0$ and

$$\begin{aligned}
 (\sigma_1 \circ \sigma_2)(s_1, s_2) &= \sigma_1(d_2(s_1) - w(s_2), \mathcal{G}_1(s_2) - h_2(s_1)) \\
 &= \sigma_1\left(\left(\frac{1}{2}\partial_{21}\partial_{32} + \partial_{31}\right)(s_1) - \partial_{32}^{(2)}(s_2),\right.
 \end{aligned}$$

$$\begin{aligned}
& (\tfrac{1}{2}\partial_{32}\partial_{21} - \partial_{31})(s_2) - \partial_{21}^{(2)}(s_1)) \\
& = \partial_{21}(\tfrac{1}{2}\partial_{21}\partial_{32} + \partial_{31})(s_1) - \partial_{32}^{(2)}(s_2)) \\
& \quad + \partial_{32}((\tfrac{1}{2}\partial_{32}\partial_{21} - \partial_{31})(s_2) - \partial_{21}^{(2)}(s_1)) \\
& = (\partial_{21}^{(2)}\partial_{32} + \partial_{21}\partial_{31} - \partial_{32}\partial_{21}^{(2)})(s_1) \\
& \quad + (\partial_{32}^{(2)}\partial_{21} - \partial_{32}\partial_{31} - \partial_{21}\partial_{32}^{(2)})(s_2) = 0
\end{aligned}$$

Conclusion

From the above work by proving the contracting homotopy for the maps define for the partition (11,8,3) we gain that the seq. of the elements of W.re. of characteristic-free re. is exact.

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