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Metric dimension of linear polyomino chain

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Abstract

Metric dimension has applications in real life including robots, fiber optics, network designs, chemical structures etc. Metric dimension of graph is minimum amount of vertices required to construct resolving set by independently determining each other's distances. The set is known as graph's resolving set if each pair of vertices in G has unique representation. In this work, we find metric dimension of linear polyomino chain.

Subject Classification: *05C12, 05C76, 05C90, 68R10.*

Keywords: *Resolving sets, Metric dimension, Metric basis, Linear polyomino chain.*

1. Introduction

In recent years, demand of mathematical applications has increased. Graph theory has proven to be extremely relevant to an enormous domain. Graph theory has rapidly grown in both theoretical results and real-world applications. Distance is a fundamental concept in graph theory. Several characteristics linked to distances in graphs are capturing the interest of many researchers. The metric dimension has received special attention in various studies.

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The ordered pair G=(V,E) is made up of non-empty set of vertices V(G) and edges E(G). If every vertex pair has unique valve, then set S is resolving set of G. A resolving set of G has metric dimension, which is its least cardinality. A minimum number of vertices in resolving set is called basis of G.

Slater [1] introduced idea of metric dimension for the first time in 1975. Harary and Melter [2] in 1976 further discussed Slater's concept. In work of Blumenthal [22] "Theory and Applications of Distance Geometry" shows history of resolving set and metric dimension. These early works opened the way to investigating of metric dimension.

According to research on metric dimension, it's computing is NP-hard and has difficulties for different graph forms, such as split and interval graphs. Significant progress has been made by researchers in the field of graph theory. Because of its applicability to many domains, including network discovery, geographical routing protocols and combinatorial optimization, research on metric dimension is particularly interesting. Feng, Xu, and Wang [3] In 2013, computed metric dimension of line graph. Numerous studies examined the metric dimension, providing details on its applicability, approximation methods and computing complexity on both random and deterministic graphs. Behtoei, Payrovi, and Batool [4], Bollobás, Mitsche, and Prałat [5] determined the precise value of metric dimension of Andrasfai graphs.

Researchers made significant contribution in this field. Hernando et al. [6] determined metric dimension of graph families. In 2016, Ali, Laila, and Ali [7] also proposed metric dimension of two graph families. Metric dimension can be used to compute molecular descriptors, which are numerical values that represent the structural features of molecules. These descriptors are useful for activities such as virtual screening, drug creation and chemical similarity comparison.

In 2000, metric dimension of connected graphs and resolvability are discussed by Chartrand et al. [8]. Okamoto, Phinezy, and Zhang [9] also discussed local metric dimension of connected graph. Metric dimension is also used in several scientific fields including communications and topographic steering convention determination [10] and robot routes in mechanical autonomy [11]. Imran et al. [12] determined circulant graph's metric dimension.

Ahmed et al. [13] determined metric dimension of Petersen graphs. Petersen graphs are famous and well-studied example of graph theory, recognized for its exquisite structure, non-planarity and symmetry, making it a significant subject of study for both mathematicians and

computer scientists. Bollob´as found the metric dimension of ramdom graphs [10]. Mohamed and Amin [14] determined the metric dimension of several graphs such as Tadpole, Lilly and specific tree [16-22].

2. Polyomino Linear Chain

A sequence or arrangement of polyominoes-shapes created by attaching squares edge to edge-arranged in a straight line is called a polyomino linear chain. For example, A single square is called a monomino. Two squares linked to form a domino (see Figure 1). Three squares linked to make up a tromino. Four squares linked on all sides make up a tetromino. The polyominoes can be orientated or connected in different ways, but the main feature is that they form a linear sequence when aligned along a single axis. When polyominoes are arranged in a line, they can be placed in various configurations.

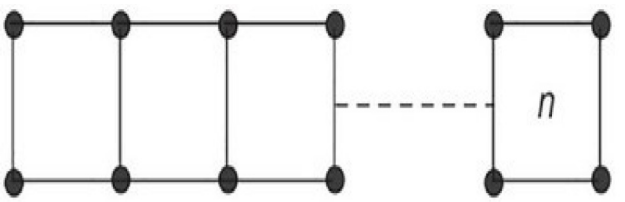

Figure 1

General figure of polyomino linear chain.

The arrangement might include different types of polyominoes (like a domino followed by a tromino) or just one type repeated. The primary property is the linearity-the polyominoes all lie in a straight line, as opposed to forming complex 2D shapes or tiling patterns. They are also used in Mathematical problems. In combinatorial mathematics, polyomino chains are examined. For example one can investigate the number of possible ways to arrange polyominoes in a line to create a chain that spans a specific region. In certain puzzles, players must employ various polyominoes to fill a linear space or grid, with certain shapes being prohibited. For instance, filling a 2x8 grid with solely dominoes and tetrominoes.

Applications of polyomino linear chain

Polyomino linear graphs have applications in image processing and computational geometry. The ability of these graphs to model pixel-based structures aids in the segmentation and recognition of shapes in digital image processing. In optical character recognition software, polyomino linear graphs can be used by character recognition systems to encode and differentiate between alphanumeric symbols according to their geometric form. Likewise, these graphs aid in the modeling of environments as grids in robotics and path planning, where each node denotes an area that the robot can inhabit or travel, hence increasing the efficiency of navigation algorithms.

Polyomino linear graphs are used in combinatorics and tiling theory to examine domino placements and tiling difficulties, especially when examining how specific forms may fill an area without gaps or overlaps. Polyomino linear graphs have real world applications, these graphs can also be used in bioinformatics to describe the linear folding patterns of biological structures, including proteins or RNA strands, providing information about their stability and functionality. Polyomino linear graphs are a useful tool in many scientific and technical fields because of their ease of use and adaptability.

In chemistry, linear polyomino chains are used to simulate linear molecules or polymeric chains. For example, in simplified grid-based molecular modeling, they can depict DNA strands, polypeptides or alkanes. One of the classic combinatorial problems is the enumeration of polyomino chains of a given length or shape. Utilized in the study of generating functions, path enumeration and recursive structures. The vibrational modes of molecules and the characteristics of polymers can be studied by analyzing their adjacency matrices and Laplacian spectra. Assist in investigating eigenvalue patterns in graphs that resemble regular grids.

3. Main Results

We determined metric dimension of Polyomino Linear Chains in this section.

Theorem 1: *For all n, metric dimension of Polyomino Linear Chains is 2.*

Proof: The divisions of vertices of graph are as follows $\{c_{1\prime1}, c_{1\prime2}, c_{1\prime3}, c_{1\prime4},..,c_{1,n+1,} c_{2\prime1}, c_{2\prime2}, c_{2\prime3}, c_{2\prime4},\ldots,c_{2,n+1}\}$. For case $n = 2$ see Table 1, for case $n = 3$ see Table 2, for case $n = 4$ see Table 3 and for case $n = 4$ see Table 4. Let S = $\{c_{1,1},c_{1,n+1}\}$ is resolving set of polyomino linear chain.

Table 1
Representation of distances of n=2.

Vertices	Distances
$r(c_{1\prime 1} \mid S)$	(0, 2)
$r(c_{1\prime 2} \mid S)$	(1, 1)
$r(c_{1\prime 3} \mid S)$	(2, 0)
$r(c_{2\prime 1} \mid S)$	(1, 3)
$r(c_{2\prime 2} \mid S)$	(2, 2)
$r(c_{2\prime 3} \mid S)$	(3, 1)

Table 2
Representation of distances of n=3.

Vertices	Distances	Vertices	Distances
$r(c_{1\prime 1} \mid S)$	(0, 3)	$r(c_{2\prime 1} \mid S)$	(1, 4)
$r(c_{1\prime 2} \mid S)$	(1, 2)	$r(c_{2\prime 2} \mid S)$	(2, 3)
$r(c_{1\prime 3} \mid S)$	(2, 1)	$r(c_{2\prime 3} \mid S)$	(3, 2)
$r(c_{1\prime 4} \mid S)$	(3, 0)	$r(c_{2\prime 4} \mid S)$	(4, 1)

Table 3
Representation of distances of n=4.

Vertices	Distances	Vertices	Distances
$r(c_{1\prime 1} \mid S)$	(0, 4)	$r(c_{2\prime 1} \mid S)$	(1, 5)
$r(c_{1\prime 2} \mid S)$	(1, 3)	$r(c_{2\prime 2} \mid S)$	(2, 4)
$r(c_{1\prime 3} \mid S)$	(2, 2)	$r(c_{2\prime 3} \mid S)$	(3, 3)
$r(c_{1\prime 4} \mid S)$	(3, 1)	$r(c_{2\prime 4} \mid S)$	(4, 2)
$r(c_{1\prime 5} \mid S)$	(4, 0)	$r(c_{2\prime 5} \mid S)$	(5, 1)

Table 4
Representation of distances of n=5.

Vertices	Distances	Vertices	Distances
$r(c_{1\prime 1} \mid S)$	(0, 5)	$r(c_{2\prime 1} \mid S)$	(1, 6)
$r(c_{1\prime 2} \mid S)$	(1, 4)	$r(c_{2\prime 2} \mid S)$	(2, 5)

Contd...

$r(c_{1'3} \mid S)$	(2, 3)	$r(c_{2'3} \mid S)$	(3, 4)
$r(c_{1'4} \mid S)$	(3, 2)	$r(c_{2'4} \mid S)$	(4, 3)
$r(c_{1'5} \mid S)$	(4, 1)	$r(c_{2'5} \mid S)$	(5, 2)
$r(c_{1'6} \mid S)$	(5, 0)	$r(c_{2'6} \mid S)$	(6, 1)

After generalization, we have

Case 1 Case 2

For i=1 For i=2

$$r(c_{1,m} \mid S) = \begin{cases} (0, n+1) \ if & m = 1, n \geq 2 \\ (1, n-1) \ if & m = 2, n \geq 2 \\ (2, n-2) \ if & m = 3, n \geq 2 \\ (3, n-3) \ if & m = 4, n \geq 3 \\ (4, n-4) \ if & m = 5, n \geq 4 \\ (5, n-5) \ if & m = 6, n \geq 5 \\ (6, n-6) \ if & m = 7, n \geq 6 \\ (7, n-7) \ if & m = 8, n \geq 7 \\ (8, n-8) \ if & m = 9, n \geq 8 \\ \vdots & \\ (m-1, n-(m-1) \ if & m = n+1, n \geq m-1 \end{cases}$$

$$r(c_{2,m} \mid S) = \begin{cases} (1, n+1) \ if & m = 1, n \geq 2 \\ (2, n) \ if & m = 2, n \geq 2 \\ (3, n-1) \ if & m = 3, n \geq 2 \\ (4, n-2) \ if & m = 4, n \geq 3 \\ (5, n-3) \ if & m = 5, n \geq 4 \\ (6, n-4) \ if & m = 6, n \geq 5 \\ (7, n-5) \ if & m = 7, n \geq 6 \\ (8, n-6) \ if & m = 8, n \geq 7 \\ (9, n-7) \ if & m = 9, n \geq 8 \\ \vdots & \\ (m, n-(m-2) \ if & m = n+1, n \geq m-1 \end{cases}$$

We know that only paths have metric dimension 1 [15] and polyomino linear chains is not path graph so its metric dimension is 2.

4. Applications

There are numerous applications for metric dimension across various fields. In Robotics, metric dimension is useful for designing localization and navigation algorithms. Metric dimension can help robots to find their way around a space by measuring distances to landmarks. Robots also use metric dimension to calculate distances to objects, modify their trajectories in real time and improving their ability to travel safely. This can be essential in developing effective communication networks.

In Geographic Information Systems, the metric dimension plays important for optimizing geographic data analysis and improving decision-making processes. Metric dimension improves geographic queries more efficient and enhances the accuracy of distance-based studies by calculating the lowest number of landmarks in a geographical area. For example when planning the placement of infrastructure such as roads, utilities and emergency services understanding the concept of metric dimension enables urban planners to strategically allocate resources to ensure optimal coverage and accessibility. This method not only improves efficiency but it also helps in identifying crucial places for environmental monitoring or tragedy response.

In biology, the concept of metric dimension can be used to examine biological networks. Researchers can concentrate on vital biomarkers or genes for diseases research by identifying a small number of key nodes that can distinguish all other nodes. In evolutionary biology, metric dimension can be used to investigate evolutionary relationships between species by finding key taxa with unique distances from other species in a phylogenetic tree.

Metric dimension can be used in social network analysis to comprehend how nodes or individuals in a network can be identified by their connections or relationships. It can assist in the development of algorithms that concentrate on locating distinct people or groups inside a network by calculating their distances from one another. Metric dimension can be used to diagnose or discover faults, especially in complex systems like computer networks or electrical grids. A basic selection of monitoring locations (or sensors) can be chosen and the distances (or performance variations) noted at these places can be used to locate or localize the defect.

Identification of fault-tolerant networks is aided by the metric dimension. Because it resolves distances between remaining nodes, a network with a limited resolving set can function well even in the event that some nodes fail or become inactive. This concept is also helpful for

evaluating the robustness of different networks. It helps in the design of networks that can continue to function even in the event of attacks or node failures.

Algorithms that are intended to determine if two graphs are structurally identical, or isomorphic, can make use of the metric dimension. These algorithms can concentrate on resolving sets that provide an efficient way of graph comparison. Metric dimension helps in unique identification of patterns in graphs, which makes matching and recognition procedures in complex datasets more effective.

5. Conclusions

In this study, we determine that linear polyomino chain's metric dimension is 2. Analyzing metric dimension also provides information including traffic flow in urban planning, medicines delivery systems, material science, network design, optimization etc. These applications highlight the importance of metric dimension in promoting innovation and advancement.

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