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Low strength connectivity of fuzzy graphs by using edge-cut sets

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Abstract

A edge-cut set Q of edges in fuzzy graph $R = (V, E, \kappa, \tau)$ is called minimum edge-cut(MEC) if the number of this set is smaller than the others. We consider $ST(e) = \max\{\kappa(r) \mid r \in e \cap f, f \in E(R)\backslash Q\}$, $e \in Q$. The strength of Q is defined to be $max\{ST(e) \mid e \in Q\}$ and denoted by $S(Q)$. The minimum value of the strength all MEC sets of $R = (V, E, \kappa, \tau)$ is called low strength connectivity of R. This article investigates the minimal edge-cut sets within well-known fuzzy graphs, establishing their low strength properties. Additionally, it presents an algorithm designed to evaluate low strength connectivity in such graphs, utilizing Sage software for implementation. Practical examples demonstrating the application of low strength connectivity are provided, specifically in the contexts of hospital management and network analysis.

Subject Classification (2020): *05C72.*

Keywords: *Edge-cut, Minimum edge-cut, Strength, Low strength connectivity.*

1. Introduction

A graph is a formal construct composed of vertices (nodes) and edges (connections) that depict relationships and interactions between these vertices. Vertices serve as the fundamental units within a graph, representing entities or points in a network. Edges, on the other hand, are the lines or links that establish

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connections between pairs of vertices, showing how they are connected or related. The configuration of nodes and links within a graph graphically represents how various components are interconnected and interact with each other. Graph theory is a field within mathematics focused on analyzing the features and attributes of graphs, including ideas such as how elements are linked, routes, loops, and other related concepts. In a graph, vertices are commonly denoted as circles or points, while edges are represented as lines or arcs connecting these vertices. Edges in a graph can be either directed or undirected, showing whether the connections have a particular direction or not. Directed edges imply one-way relationships between vertices, while undirected edges reflect bidirectional associations. Graphs can be categorized based on various properties, such as being acyclic (without cycles) or cyclic (with cycles) and connected (where all vertices are reachable) or disconnected (containing isolated components). The degree of a vertex in a network indicates how many connections it has, offering information about its level of interaction and significance within the overall system.

A fuzzy graph expands upon classical graph theory by integrating degrees of uncertainty, allowing for the modeling of ambiguous or imprecise relationships between nodes and connections. In a fuzzy graph, the connections between vertices are not merely binary (either present or absent) but are assigned degrees of membership or truth values, reflecting the strength or degree of association. This degree of association in fuzzy graphs allows for a more nuanced representation of relationships, accommodating imprecise or vague information in the graph structure. Fuzzy logic, developed by Lotfi Zadeh in the 1960s [28]. Fuzzy graphs, an extension of fuzzy logic to graph theory was introduced by Rosenfeld [24] in 1975, incorporate the notion of uncertainty into graph structures. [17] Fuzzy graphs are particularly useful in modeling real-world systems where relationships may not be clearly defined and can exhibit varying levels of strength. The fuzzy adjacency matrix is a common representation used in fuzzy graphs, where each entry represents the degree of connection between vertices.Fuzzy graphs [21] allow for the examination of data that is uncertain or partial, supporting applications across multiple domains like decision processes and recognizing patterns. Fuzzy graphs are classified according to various fuzzy relationships, including fuzzy routes, fuzzy loops, and fuzzy links, offering a broad structure for studying systems with fuzzy characteristics [13]. [22] The theory of fuzzy graphs introduces new concepts like fuzzy degrees of vertices, fuzzy neighborhoods, and fuzzy closures, enriching traditional graph theory with fuzzy logic principles. Fuzzy graphs find applications in various domains, including social networks, transportation systems, image processing, and database management, where uncertainty and imprecision are prevalent. Fuzzy graph algorithms and techniques have been developed to handle fuzzy relationships efficiently, ensuring accurate analysis and decision-making in uncertain environments. Shrinivas, Vertrivel, and Elango [25] provided an overview of how graph theory is applied within computer science. Mordeson and Chang-Shyh [20] examined various operations related to fuzzy graphs. Additionally, Bhattacharya [4] explored certain observations concerning fuzzy graphs.

A set of edges in a graph that, when removed, divides into distinct components is known as an edge-cut set. Such sets are pivotal in understanding graph connectivity and structure. The removal of these edges can fragment or isolate parts of the graph, impacting its overall connectivity. Edge-cut sets are integral to various graph algorithms and network analysis, aiding in tasks like network optimization and flow control. Their identification is crucial for evaluating network vulnerability and integrity. The size and composition of an edge-cut set can significantly influence the graph's resilience to disruptions. Edge-cut sets play a vital role in network design, communication systems, and social network analysis. Understanding and analyzing edge-cut sets are essential for pinpointing critical links and ensuring network robustness. Researchers leverage edge-cut sets to devise strategies for enhancing network security and reliability. Edge-cut sets are particularly relevant in transportation networks, where they can disrupt traffic flow when certain edges are removed. In computer networks, edge-cut sets represent potential failure points that need to be addressed for continued network operation. By manipulating edge-cut sets, one can explore graph partitioning and connectivity enhancement techniques. Detecting minimal edge-cut sets offers insights into the essential edges required for maintaining graph connectivity. Edge-cut sets provide a lens into the interrelationships between different graph components and their interactions. Applications of edge-cut sets span diverse disciplines such as biology, sociology, and computer science. Identifying edge-cut sets is essential for optimizing network performance and resource allocation strategies. These sets are utilized in clustering algorithms to group nodes based on their connectivity properties. Efficient computation of edge-cut sets enhances the efficiency and scalability of graph algorithms. The study of edge-cut sets aids in evaluating network robustness against faults and disturbances. Analyzing edge-cut sets unveils critical network dependencies and relationships. The concept of edge-cut sets is beneficial for assessing the resilience of infrastructure networks like power grids. Designers leverage edge-cut sets to create fault-tolerant networks and enhance network robustness. Edge-cut sets help in identifying bottleneck edges that hinder information flow in a graph. Their analysis is pivotal in understanding the overall structure and dynamics of a network. Researchers use edge-cut sets to explore network fragmentation and connectivity patterns. Edge-cut sets are instrumental in evaluating the impact of edge removal on network integrity and connectivity. Understanding edge-cut sets leads to insights into network behavior under different failure scenarios. The concept of edge-cut sets is closely linked to graph partitioning algorithms that aim to divide graphs into disjoint subsets. Efficient computation of edge-cut sets aids in improving network efficiency and performance. Bauer et al. [2] explored problems related to the extremal properties of connectivity and the development of dependable probabilistic network models. Biedl and Murali examined the vertex connectivity characteristics of 1-plane graphs [5]. The concepts of conditional connectivity, the connectivity of Cartesian products of graphs, circulant graphs, and their roles in combinatorial optimization and directed connectivity enhancement were discussed in [6, 7, 9,

11]. Manzano et al. [16] analyzed the connectivity aspects of data center network architectures. Esfahanian and Hakimi [8] investigated the conditional edge-connectivity of graphs, while J and Ji introduced a form of restricted edge connectivity in graphs [19]. Mathew and Sunitha focused on node and arc connectivity within fuzzy graphs [18]. Over the past decade, extensive research has been conducted on graph connectivity, resulting in numerous publications in this area [1, 12, 23, 26]. Ma examined fuzzy average edge connectivity and its applications in communication networks [15]. Gong and Hua analyzed edge connectivity in bipolar fuzzy networks [10].

In this article, low strength connectivity of the edge-cut set in famous fuzzy graphs was determined. An algorithm was introduced to calculate low strength connectivity in these graphs by using Sage software [3]. An instance of utilizing an edge-cut set within a network can be observed in social media platforms. Additionally, the use of low strength connectivity is demonstrated in the management of hospitals and network systems.

2. Definitions and Notations

This part introduces essential ideas from both graph theory and fuzzy graph theory, as referenced in [14, 27].

Consider a finite, non-empty set denoted by V. A graph $R = (V, E)$ defined on V includes a collection of vertices V and a set of edges E, where each edge in E is an unordered pair of distinct vertices from V. We will write (t, h) to represent an edge. When (t, h) is an edge, the vertices t and h are said to be connected or adjacent. A graph is termed complete if every pair of vertices is directly connected by an edge. In the context of graph theory, a path is a sequence of vertices where each consecutive pair is linked by an edge. A graph is connected if there exists a path between any two vertices, allowing traversal from any vertex to any other via the edges. A cycle is a path that begins and ends at the same vertex. An edge whose removal results in the graph becoming disconnected or increasing the number of separate components is called a bridge.

Definition 2.1: A bipartite graph is a graph where the set of vertices can be split into two separate groups, V_1 and V_2, such that no two vertices within the same group are directly connected. A bipartite graph graph is called complete bipartite graph if every vertex in V_1 is connected to all vertices in V_2. This type of graph is represented as $X_{r,z}$, where $r = |V_1|$ and $z = |V_2|$.

Definition 2.2: A star graph is a special case of a complete bipartite graph, specifically $X_{1,z}$, where one group contains a single central vertex connected to all other vertices in the second group.

Definition 2.3: A cycle graph, also called a circular graph, is a structure where vertices are arranged in a closed loop, with each vertex connected to exactly two neighbors, forming a single cycle.

Definition 2.4: A path graph is a linear arrangement of vertices connected sequentially by edges, forming a straight chain without any branches or cycles.

A fuzzy graph constructed on a finite, non-empty set V is defined as a tuple $R = (V, E, \kappa, \tau)$ where $\kappa: V \to [0,1]$ assigns a membership degree to each vertex, representing a fuzzy subset of V. The relation $\tau: V \times V \to [0,1]$ is a symmetric fuzzy relation on V, satisfying the condition that for any $r, z \in V$, $\tau(r, z) \leq \min\{\kappa(r), \kappa(z)\}$. The crisp (or classical) graph associated with the fuzzy graph R is denoted as $R^* = (\kappa^*, \tau^*)$ where κ^* is the set of vertices with positive membership degrees, i.e., $\kappa^* = \{r \in V | \kappa(r) > 0\}$, and τ^* is the set of vertex pairs with positive relation values, i.e., $\tau^* = \{(r, z) \in V \times V | \tau(r, z) > 0\}$, the vertices r and z are considered neighbors.

Definition 2.5: Let $\kappa: V \to [0,1]$ be a fuzzy membership function assigned to the set V. The complete fuzzy graph, abbreviated $CO - FG$ based on κ is represented as $R = (V, E, \kappa, \tau)$, where for any vertices $r, z \in V$, the relation $\tau(r, z)$ is defined as the minimum of $\kappa(r)$ and $\kappa(z)$. A fuzzy star graph, labeled as $X_{1,z}$, is a specific type of fuzzy graph where the underlying crisp star graph is $X_{1,z}$.

Definition 2.6: A fuzzy graph $R = (V, E, \kappa, \tau)$ is classified as bipartite if its vertex set V can be divided into two non-empty subsets, V_1 and V_2, such that the membership function τ assigns a value of zero to any pair of vertices both belonging to the same subset (either V_1 or V_2). Additionally, if for every r in V_1 and z in V_2, the membership $\tau(r, z)$ equals the minimum of $\kappa(r)$ and $\kappa(z)$, then R is called a complete bipartite fuzzy graph, abbreviated $COB - FG$. This graph is represented by $X_{n,m}$, where $n = |V_1|$ and $m = |V_2|$.

Definition 2.7: A path fuzzy graph, abbreviated $P - FG$ $R = (V, E, \kappa, \tau)$ with s vertices is a path, with starting vertex r_0 and ending vertex r_{s-1}. In a $P - FG$, $E(R) = \{(r_i, r_{i+1})\}$ such that $\tau(r_i, r_{i+1}) \leq \kappa(r_i) \wedge \kappa(r_{i+1})$, where $0 \leq i < s - 1$. A circle (cycle) fuzzy graph, abbreviated $CI - FG$ $R = (V, E, \kappa, \tau)$ with s vertices is a cycle, which is a closed path where the starting and ending vertices r_0 and r_s are the same. In a $CI - FG$, $E(R) = \{(r_i, r_{i+1})\}$ such that $\tau(r_i, r_{i+1}) \leq \kappa(r_i) \wedge \kappa(r_{i+1})$, where $0 \leq i < s$. A fuzzy graph $R = (V, E, \kappa, \tau)$ is a wheel, abbreviated $W - FG$ if and only if $R = (V, E, \kappa^*, \tau^*)$ is a wheel.

3. Low strength connectivity of fuzzy graph

Definition 3.1: An edge-cut in a connected fuzzy graph $R = (V, E, \kappa, \tau)$ refers to a collection of edges that, when removed, cause the graph to become either disconnected or reduce to a single vertex.

Definition 3.2: The smallest possible set of edges whose removal disconnects a connected fuzzy graph $R = (V, E, \kappa, \tau)$ is called a minimum edge-cut set, abbreviated as MEC.

Definition 3.3: Consider a connected fuzzy graph $R = (V, E, \kappa, \tau)$, with Q representing a MEC of R. For each edge e in Q, define $ST(e) = max\{\kappa(r)|r \in e \cap f, f \in E(R)\backslash Q\}$. The strength of Q is defined to be $max\{ST(e)|e \in Q\}$ and denoted by $S(Q)$.

Definition 3.4: The minimum value of the strength all MEC sets of $R = (V, E, \kappa, \tau)$ is called low strength connectivity of graph R and denoted by $LS(R)$.

Note: A MEC set Q that exhibits low strength connectivity is considered the weakest if $\sum_{e \in Q} \tau(e)$ is the smallest between all MEC sets that share the lowest strength value. This particular MEC set is represented as $WLS(R)$.

Note: A cardinality of MEC set does not exceed the smallest number of neighbors$(N(r), r \in V(R))$ among all vertices in the graph, i.e., it is less than or equal to $min|\{N(r)|r \in V(R)\}|$.

Proposition 3.5: Let $R = (V, E, \kappa, \tau)$ be a connected fuzzy graph of order $p = \sum_{r \in V(R)} \kappa(r)$. Then $LS(R) \leq p$.

Proof: Let Q be a $WLS(R)$. For all $e = (r, z) \in Q$, we have $\tau(e) \leq \kappa(r) \wedge \kappa(z) \leq \kappa(r) \leq \sum_{r \in V(R)} \kappa(r)$ and hence $LS(R) \leq p$.

In this context, each vertex r is assigned a pair of labels: the first indicating its label and the second being $\kappa(r)$. Edges (r, z) are also labeled with $\tau(r, z)$. The symbol $\vee$ stands for maximum. The computation of low strength connectivity was performed using Sage software to assist the process.

Example 3.6: Examine the connected fuzzy graph $R = (V, E, \kappa, \tau)$ illustrated in Figure 1 with $V(R) = \{(0,0.3), (1,0.4), (2,0.6), (3,0.7), (4,0.8), (5,0.7), (6,0.9), (7,0.8)\}$. For this fuzzy graph, the subset $Q = \{((2,0.6), (3,0.7))\}$ is MEC set. In this fuzzy graph $LS(R) = 0.7$ and $WLS(R) = Q$.

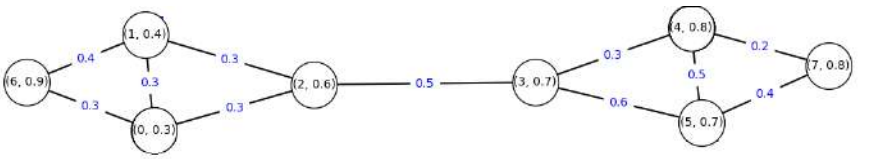


Figure 1
The graph with $LS(R) = 0.7$

Proposition 3.7: Let $R = (V, E, \kappa, \tau)$ be a connected fuzzy graph and $m = max\{\kappa(r)|r \in V(R)\}$. Then $LS(R) \leq m$.

Proof: Let Q be a MEC set of R and $m = max\{\kappa(r)|r \in V(R)\}$. For any $e = (r, z) \in Q$, $ST(e) \leq \kappa(r) \vee \kappa(z) \leq m$ and hence $LS(R) \leq m$.

Proposition 3.8: Let $R = (V, E, \kappa, \tau)$ be a $CI - FG$ with $V(R) = \{r_i | 0 \leq i < s = |V(R)|\}$. Then $LS(R) = min\{\kappa(r_{i-1}) \vee \kappa(r_{i+1}), \kappa(r_i) \vee \kappa(r_{i+1}) | 0 \leq i < s\}$.

Proof: Suppose that $R = (V, E, \kappa, \tau)$ is a $CI - FG$ with $V(R) = \{r_i | 0 \leq i < s\}$ and $E(R) = \{(r_i, r_{i+1}) | 0 \leq i < s\}$. Every two-element subset of $E(R)$ is a MEC set. The set of all two-element subset of $E(R)$ is $\{(r_{i-1}, r_i), (r_i, r_{i+1})\}$. Assume that $Q_i = \{(r_i, r_{i-1}), (r_i, r_{i+1})\}$, $1 \leq i \leq s-1$ and $Y_{i,j} = \{(r_i, r_{i+1}), (r_j, r_{j+1})\}$, $0 \leq i, j < s$, $\{(r_i, r_{i+1})\} \cap \{(r_j, r_{j+1})\} = \emptyset$. The set of all two-element subset of $E(R)$ is $\{Q_i | 1 \leq i \leq s-1\} \cup \{Y_{i,j} | 0 \leq i, j < s\}$. For any i, $S(Q_i) = \kappa(r_{i-1}) \vee \kappa(r_{i+1})$. Also we can see that $min\{\kappa(r_i) \vee \kappa(r_{i+1}) | 0 \leq i < s\}$ equals $min\{S(Y_{i,j}) | 0 \leq i, j < s\}$. Therefore $LS(R) = min\{\kappa(r_{i-1}) \vee \kappa(r_{i+1}), \kappa(r_i) \vee \kappa(r_{i+1}) | 0 \leq i < s\}$.

Example 3.9: Examine the $CI - FG$ $R = (V, E, \kappa, \tau)$ depicted in Figure 2 with $V(R) = \{(0,0.9), (1,0.7), (2,0.6), (3,0.8), (4,0.5)\}$. For R, $MEC = \{((2,0.6), (3,0.8)), ((3,0.8), (4,0.5))\}$ and $LS(R) = 0.6$.

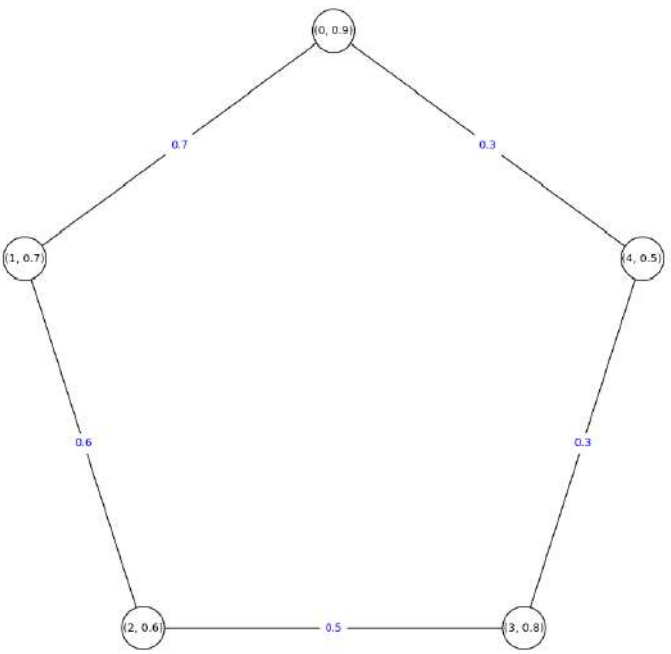


Figure 2
The $CI - FG$ with $LS(R) = 0.6$

As the number of vertices grows, Sage software and its programming environment facilitate the straightforward computation of low strength connectivity. For instance, in the subsequent in the Figure 3, an arbitrary graph

with 20 vertices-each with specified weights is generated, and its low strength connectivity is determined. The commands used to perform this calculation are provided below.

```
R = graphs.CycleGraph(20)
k=R.order()
List=[]
for vertex in R.vertices(sort=True):
       R.delete_vertices([vertex])
for p in range(k):
       R.add_vertex(n(random(),digits=3))
V= R.vertices()
for p in range(k):
       R.add_edge(V[p],V[(p+1).mod(k)])
       R.set_edge_label(V[p], V[(p+1).mod(k)],
RR.random_element(0,min(V[p],V[(p+1).mod(k)])))
Test=[]
for p in range(k):
       Test.append((max(R.neighbors(V[p])),V[p]))
print(min(Test))
```

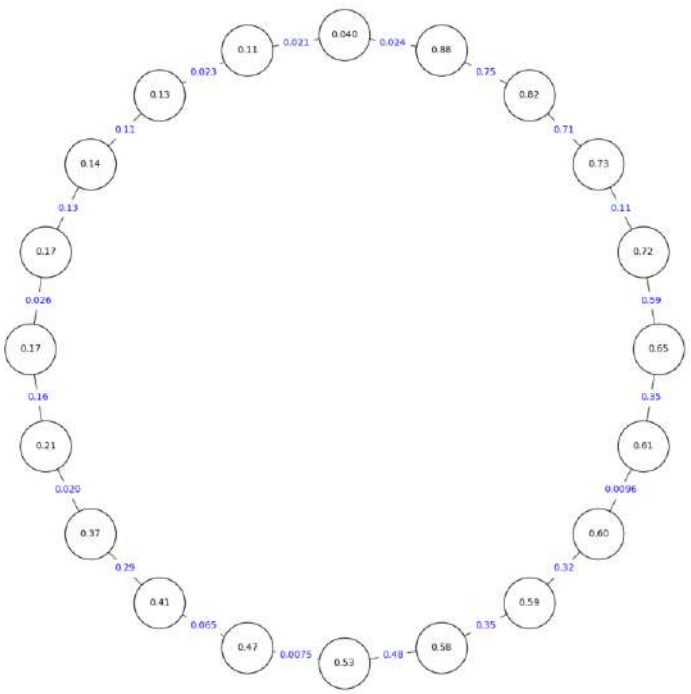


Figure 3
The $CI-FG$ with $LS(R)=0.13$ and $MEC=\{(0.13,0.11),(0.11,0.04)\}$

Proposition 3.10: Let $R = (V, E, \kappa, \tau)$ be a $W - FG$ with distinguished vertex r. Then $LS(R) \geq \kappa(r)$.

Proof: Analyze the $W - FG$ $R = (V, E, \kappa, \tau)$. where each MEC set comprises three edges linked to the vertices on the cycle. If the weight of a particular vertex r is the highest among all, then by the definition of low strength connectivity, $LS(R) = \kappa(r)$. Additionally, because the edges in the MEC set intersect with other edges at vertex r, which has weight $\kappa(r)$, it follows that $LS(R)$ is at least $\kappa(r)$.

Example 3.11: Using Sage software, the $W - FG$ $R = (V, E, \kappa, \tau)$ depicted in the Figure 4 has an LS value of 0.29.

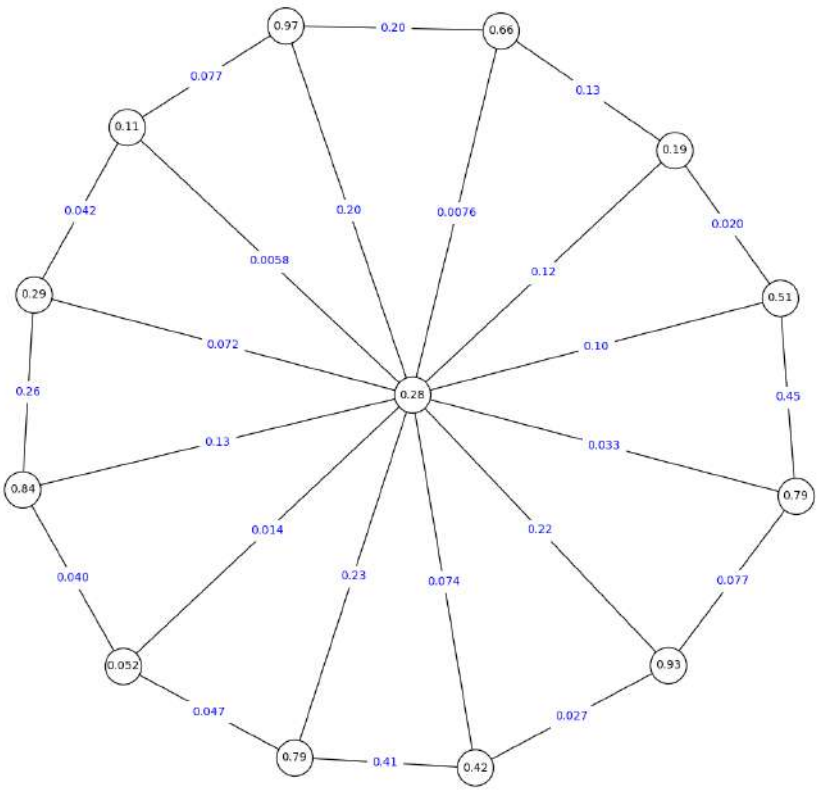


Figure 4
The $W - FG$ with $LS(R) = 0.29$

```
R = graphs.WheelGraph(13)
k=R.order()
List=[]
for vertex in R.vertices(sort=True):
       R.delete_vertices([vertex])
for j in range(k):
       R.add_vertex(n(random(),digits=2))
V= R.vertices()
R.add_edge(V[k-1],V[1])
R.set_edge_label(V[k-1], V[1],
RR.random_element(0,min(V[k-1],V[1])))
for j in range(1,k-1):
       R.add_edge(V[j],V[j+1])
       R.set_edge_label(V[j], V[j+1],
RR.random_element(0,min(V[j],V[j+1])))
Test=[]
for r in V:
       Test.append(max(R.neighbors(r)))
print(min(Test))
```

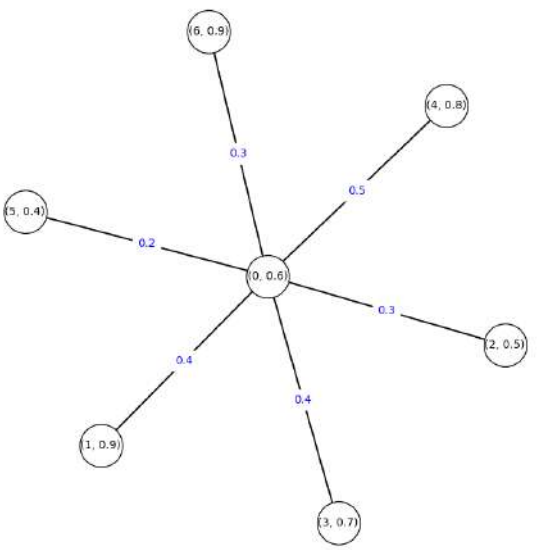


Figure 5
The star fuzzy graph $X_{1,6}$ with $LS(X_{1,6}) = 0.6$

Proposition 3.12: Let $R = (V, E, \kappa, \tau)$ be a star fuzzy graph with central vertex r then $LS(R)$ is $\kappa(r)$ and $WLS(R)$ contains (r, z) with the least edge weight.

Proof: Clearly, by removing every edge in star fuzzy graph R, it is disconnected and hence the MEC set is obtained. Since intersection edges $e \in MEC$ and

$f \in E(R)\backslash e$ is central vertex r then $LS(R) = \kappa(v)$. Now, we select the edge (r, z) with the least weight in $E(R)$. Therefore $WLS(R) = \{(r, z)\}$.

Example 3.13: The star fuzzy graph $X_{1,6} = (V, E, \kappa, \tau)$, illustrated in Figure 5, has a vertex set $V(X_{1,6})$ consisting of the elements $\{(0,0.6), (1,0.9), (2,0.5), (3,0.7), (4,0.8), (5,0.4), (6,0.9)\}$. In this graph, $LS(X_{1,6})$ is 0.6, and the set $WLS(X_{1,6})$ includes the edge $\{(0,0.6), (5,0.4)\}$.

By using programming in the Sage software environment, we can calculate the low strength connectivity in the Figure 6.

```
R=graphs.StarGraph(30)
k=R.order()
List=[]
for vertex in R.vertices(sort=True):
      R.delete_vertices([vertex])
for p in range(k):
      R.add_vertex((p,n(random(),digits=3)))
W= R.vertices()
for p in range(1,k):
      R.add_edge((0,W[0][1]),(p,W[p][1]))
      R.set_edge_label((0,W[0][1]), (p,W[p][1]),
RR.random_element(0,min(W[0][1], W[p][1])))
print("LS(R)=",W[0])
```

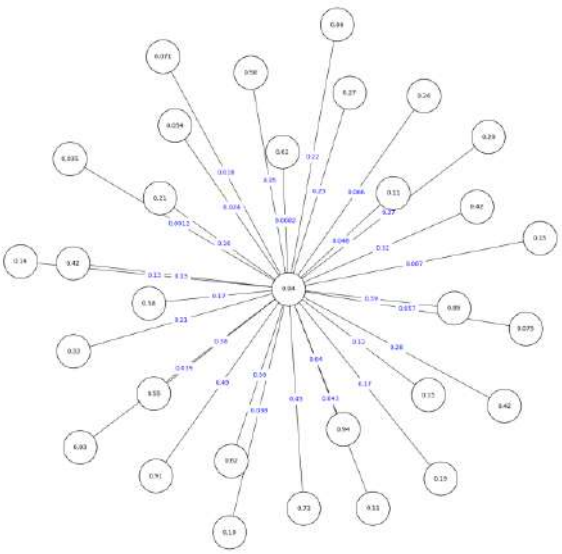


Figure 6
The star fuzzy graph $X_{1,30}$ with $LS(X_{1,30}) = 0.94$

Proposition 3.14: Let $R = (V, E, \kappa, \tau)$ be a $CO - FG$ with s vertices. Then a MEC set of R contain $s - 1$ edges connected to a vertex $r \in V(R)$. Suppose that r is a vertex of maximum weight. Then $WLS(R) = \{(r, r_i) | r \neq r_i \in V(R)\}$ and $LS(R)$ is $max\{\kappa(r_i) | r_i \neq r \in V(R)\}$.

Proof: Suppose that $R = (V, E, \kappa, \tau)$ is a $CO - FG$ with $|V(R)| = s$. Removing the $s - 1$ edges incident to a specific vertex r_i disconnects the graph, thereby forming a MEC set. For any i, $1 \leq i \leq s$, let $Q_i = \{(r_1, r_i), (r_2, r_i), \ldots, (r_{i-1}, r_i), (r_{i+1}, r_i), \ldots, (r_s, r_i)\}$ be a MEC set contains of all edges with an end point r_i. Then $S(Q_i) = max\{\kappa(r_j) | j \neq i\}$. For finding minimum value of the strength all MEC sets of R, consider a vertex r with maximum weight, then $LS(R) = max\{\kappa(r_i) | r_i \neq r \in V(R)\}$ and $WLS(R) = \{(r, r_i) | r \neq r_i \in V(R)\}$.

Example 3.15: Consider $CO - FG$ $R = (V, E, \kappa, \tau)$ depicted in Figure 7, where the vertex set V(R) includes {(0,0.7), (1,0.7), (2,0.4), (3,0.8), (4,0.6)}. For this fuzzy graph, we have $LS(R) = 0.7$, $MEC(R)$ = ((3,0.8), (4,0.6)), ((2,0.4), (3,0.8)), ((1,0.7), (3,0.8)), ((0,0.7), (3,0.8)).

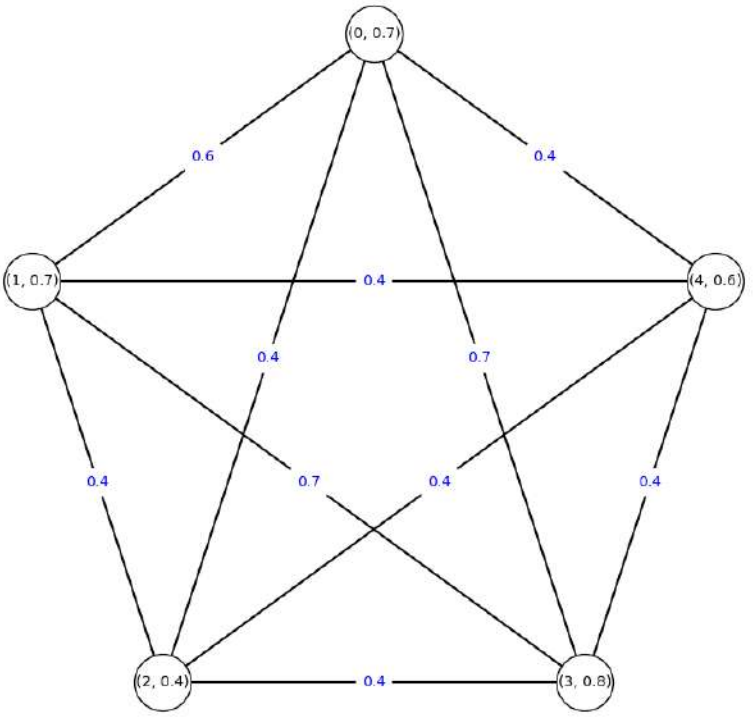


Figure 7
The $CO - FG$ with $LS(R) = 0.7$

Example 3.16: Consider the $CO - FG$ with random weight for vertices in Figure 8.

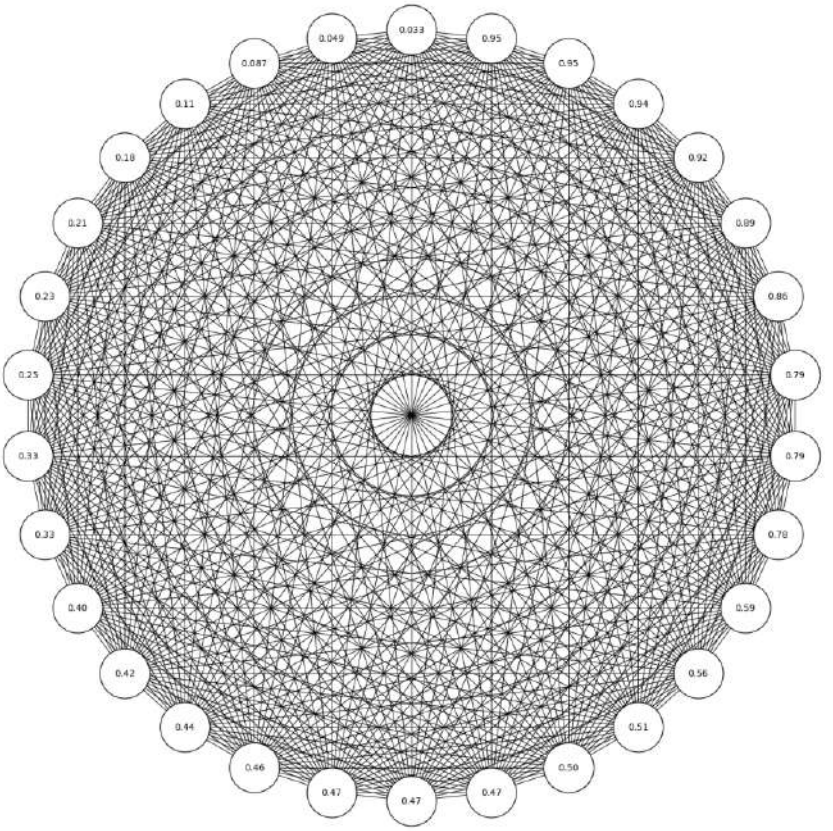


Figure 8
The complete fuzzy graph with $LS(R) = 0.95$

```
R=graphs.CompleteGraph(30)
k=R.order()
for vertex in R.vertices(sort=True):
      R.delete_vertices([vertex])
for j in range(k):
      R.add_vertex(n(random(),digits=2))
for r in R.vertices():
for z in R.vertices():
      if r!=z:
      R.add_edge(r,z)
```

```
        R.set_edge_label(r, z, min(r,z))
W= R.vertices()
print(sorted(W))
print("LS(R)=",W[k-2])
```

Proposition 3.17: Let $X_{n,m} = (V, E, \kappa, \tau)$ be a $COB - FG$ contains two parts $U = \{H_i | 1 \leq i \leq n\}$ and $V = \{G_j | 1 \leq j \leq m\}$ with n and m vertices, respectively where $m > n$. For any j, $\{(h_i, g_j) | 1 \leq i \leq n\}$ is a MEC set and $LS(X_{n,m}) = max\{\kappa(h_i) | 1 \leq i \leq n\}$. If $n = m$ then $LS(X_{n,m}) = min\{max\{\kappa(h_i)\}, max\{\kappa(g_j)\}\}$.

Proof: Let $m \geq n$. If Q is a MEC set then $Q = \{(h_i, g_j) | 1 \leq i \leq n\}$ for a j, $1 \leq j \leq m$. By the definition of the $S(Q)$, we have $S(Q) = max\{\kappa(h_i) | 1 \leq i \leq n\}$ and hence $LS(X_{n,m}) = max\{\kappa(h_i) | 1 \leq i \leq n\}$. If $n = m$ then all MEC sets Q are as $\{(h_i, g_j) | 1 \leq i \leq n\}$, for a j, $1 \leq j \leq m$ or $\{(h_i, g_j) | 1 \leq j \leq n\}$ for a j, $1 \leq j \leq n$. If a MEC set Q is as $\{(h_i, g_j) | 1 \leq i \leq n\}$, for a j, then as the above case $S(Q) = max\{\kappa(h_i) | 1 \leq i \leq n\}$ and if Q is as $\{(h_i, g_j) | 1 \leq j \leq n\}$, for a i then $S(Q) = max\{\kappa(v_j) | 1 \leq j \leq n\}$. Therefore $LS(X_{n,m}) = min\{max\{\kappa(h_i) | 1 \leq i \leq n\}, max\{\kappa(g_j) |\ 1 \leq j \leq n\}\}$.

The subsequent examples demonstrate the computation of low strength connectivity in the Figures 9 and 10 (I and II).

Example 3.18: Analyze the $COB - FG$s $X_{n,m} = (V, E, \kappa, \tau)$ illustrated in Figure 9 to determine their low strength connectivity.

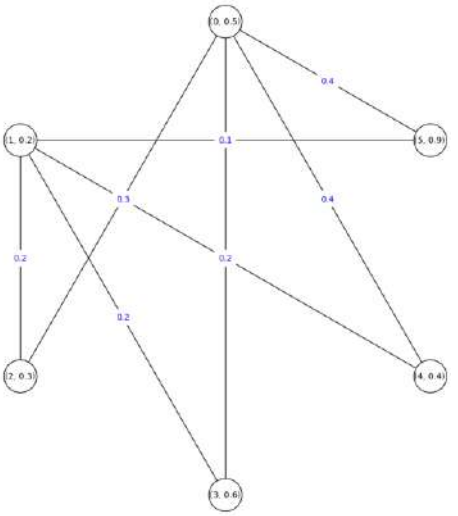


Figure 9
The $COB - FG$h with $LS(X_{2,4}) = 5/10$ and $WLS(X_{2,4}) = \{((3, 0.6), (0, 0.5)), ((1, 0.2), (3, 0.6))\}$

```
R = BipartiteGraph
    (graphs.CompleteBipartiteGraph(10,15))
k=len(R.left)
h=len(R.right)
for vertex in R.vertices(sort=True):
    R.delete_vertices([vertex])
for i in range(k):
    R.add_vertex(n(random(),digits=2),left=True)
for j in range(h):
    R.add_vertex(n(random(),digits=2),right=True)
for r in R.left:
    for z in R.right:
        if r!=z:
        R.add_edge(u,v)
        R.set_edge_label(r, z, min(r,z))
if k==h:
        print("LS(R)=",min(max(R.left),max(R.right)))
if k<h:
        print("LS(R)=",max(R.left))
```

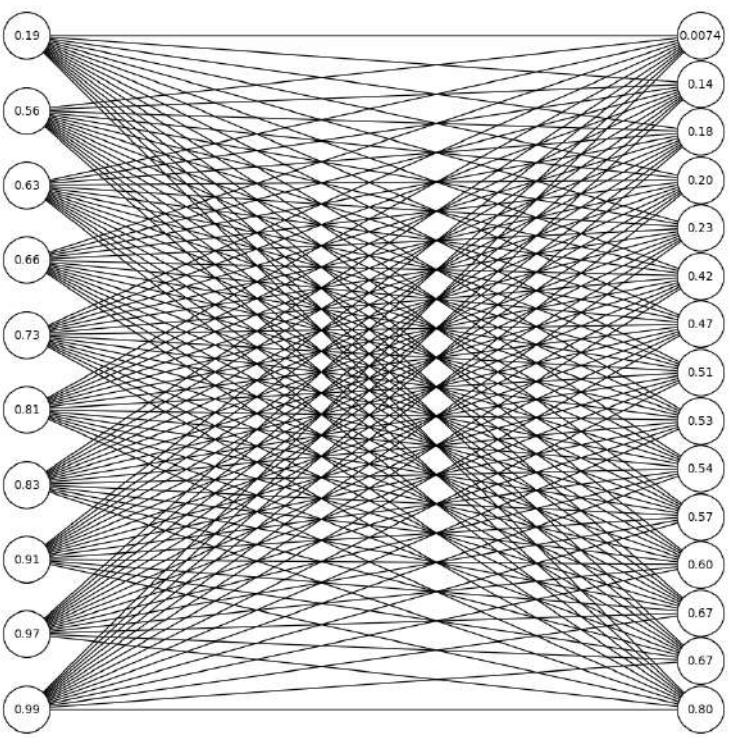


Figure 10(I)
The $COB - FG$ with $LS(X_{10,15}) = 0.99$

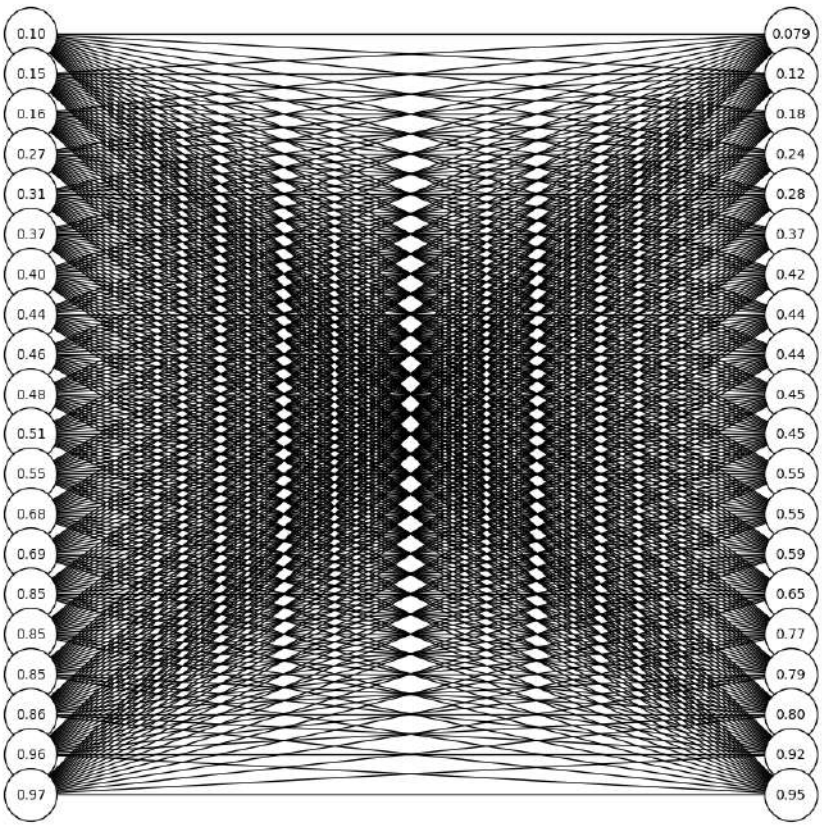


Figure 10(II)
The $COB-FG$ with $LS(X_{20,20})=0.95$

Proposition 3.19: Let $R=(V,E,\kappa,\tau)$ be a $P-FG$ with s vertices $\{r_i|0\leq i\leq s-1\}$. We have the following cases:

Case I: if $s=2$ then $LS(R)=0$.

Case II: if $s=3$ then $LS(R)=\kappa(r_1)$.

Case III: if $s>3$ then $LS(R)=min\{\kappa(r_1),\kappa(r_{n-2}),max\{\kappa(r_i),\ \kappa(r_{i+1})|1\leq i\leq s-3\}\}$.

Proof: Suppose that $R=(V,E,\kappa,\tau)$ is a $P-FG$ with s vertices $\{r_i|0\leq i\leq s-1\}$. Clearly, in the $P-FG$, by removing each of edges, the graph becomes disconnected. The following situations are presented below:

Case I: if $s=2$ then there exist just one MEC $\{(r_0,r_1)\}$. So intersection (r_0,r_1) and $E(R)\backslash\{(r_0,r_1)\}$ is the empty set and hence $LS(R)=0$.

Case II: if $s = 3$ then there are exact two MEC sets $Q_1 = \{(r_0, r_1)\}$ and $Q_2 = \{(r_1, r_2)\}$. We can easily see that $S(Q_1) = S(Q_2) = \kappa(r_1)$ and hence $LS(R) = \kappa(r_1)$.

Case III: assume that $s > 3$. All MEC sets Q are as $\{(r_i, r_{i+1})\}$, $0 \leq i \leq s - 1$. Then we have $S(\{(r_0, r_1)\}) = \kappa(r_1)$, $S(\{(r_{n-2}, r_{n-1})\}) = \kappa(r_{s-2})$ and for $1 \leq i \leq s - 3$, $S(\{(r_i, r_{i+1})\}) = max\{(\kappa(r_i), \kappa(r_{i+1}))$. Therefore $LS(R)$ $=min\{\kappa(r_1), \kappa(r_{s-2}), max\{\kappa(r_i), \kappa(r_{i+1})|1 \leq i \leq s - 3\}\}$.

Utilizing Sage software, the low strength connectivity was determined for a $P - FG$ consisting of 30 vertices, with vertex and edge weights illustrated in Figure 11. The resulting $LS(R)$ value is 0.11.

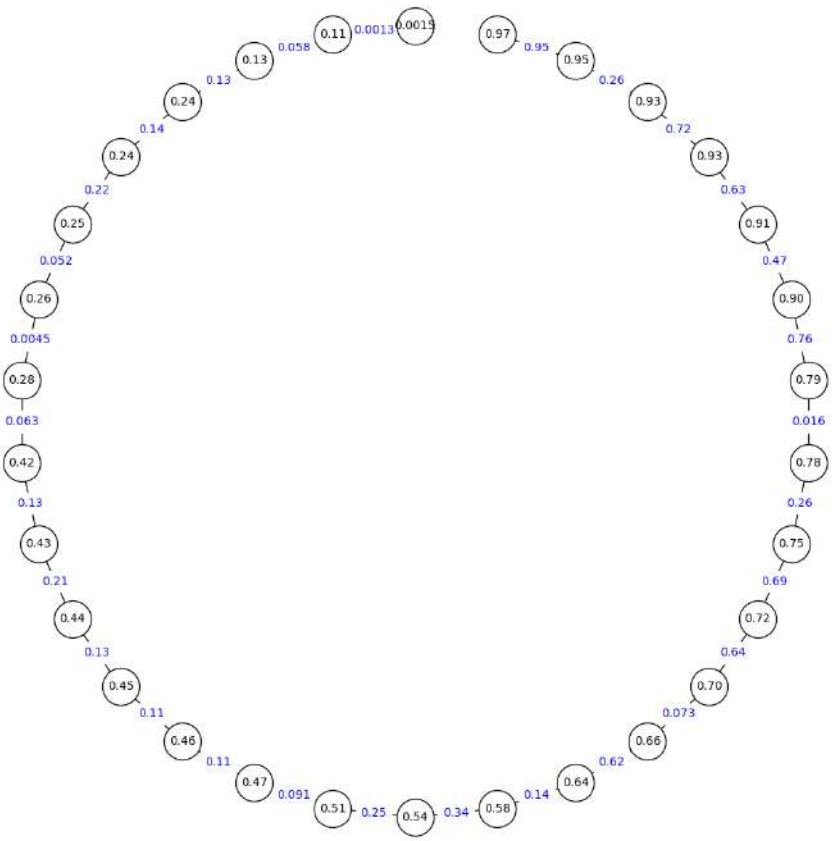


Figure 11
The $P - FG$ with $LS(R) = 0.11$, $WLS(R) = \{(0.11, 0.0015)\}$

```
R = graphs.PathGraph(30)
k=R.order()
List=[]
for vertex in R.vertices(sort=True):
       R.delete_vertices([vertex])
for p in range(k):
       R.add_vertex(n(random(),digits=3))
V= R.vertices()
for p in range(0,k-1):
       R.add_edge(V[p],V[p+1])
       R.set_edge_label(V[p], V[p+1],
RR.random_element(0,min(V[p],V[p+1])))
for p in range(k):
       List.append(min(R.edge_label(V[p],V[p+1]),
R.edge_label(V[p-1],V[p])))
E= R.edges(labels=False)
Test=[V[1],V[k-2]]
F=Set(E).difference(Set([E[0],E[k-2]]))
for e in F:
       Test.append(max([e[0],e[1]]))
print(min(Test))
```

Proposition 3.20: Let $R = (V, E, \kappa, \tau)$ be a $CI - FG$ with vertex set $\{r_0, r_1, \ldots r_{n-1}\}$ and edge set $\{e_i = (r_i, r_{i+1}) | 0 \leq i \leq s-2\}$ such that $\kappa(r_i) < \kappa(r_{i+1})$ where $0 \leq i \leq s-1$. Then we have:

$$\begin{cases} LS(R\backslash e) = LS(R) = \kappa(r_2), & e = e_0 \\ LS(R\backslash e) = \kappa(r_0) < LS(R) = \kappa(r_2), & e = e_1 or e_{s-2} \\ LS(R\backslash e) = \kappa(r_1) < LS(R) = \kappa(r_2), & e \neq e_0, e_1, e_{s-2}. \end{cases}$$

Proof: Let $R = (V, E, \kappa, \tau)$ be a $CI - FG$. Then by Proposition 3.8, we have:

$LS(R) = min\{max\{\kappa(x) | x \in N(r_i)\}\},\ 0 \leq i \leq s-1$

$= min\{max\{\kappa(r_1), \kappa(r_{s-1})\}, max\{\kappa(r_0), \kappa(r_{s-2})\}, max\{\kappa(r_i), \kappa(r_{i+1})\}\}$, $0 \leq i \leq s-3$

$= min\{\kappa(r_{s-1}), \kappa(r_{s-2}), \kappa(r_2), \ldots, \kappa(r_{s-3})\} = \kappa(r_2)$.

For any edge e, $R\backslash e$ is a $P - FG$. By Proposition 3.19, we have

$LS(R\backslash e) = min\{\kappa(r_2), \kappa(r_{s-1}), max\{\kappa(r_i), \kappa(r_{i+1})\}, 2 \leq i \leq s-2\}$

$= min\{\kappa(r_2), \kappa(r_{s-1}), \kappa(r_3), \ldots, \kappa(r_{s-1})\} = \kappa(r_2)$.

Similarity, we can see that $LS(R\backslash e_1) = LS(R\backslash e_{s-2}) = \kappa(r_0)$. Also if $e \neq e_0, e_1, e_{s-2}$ then $LS(R\backslash e) = \kappa(r_1)$. Thus complete the proof.

Proposition 3.21: Let $R = (V, E, \kappa, \tau)$ be a $CI - FG$ with s vertices $\{r_0, r_1, \ldots r_{n-1}\}$ such that $\kappa(r_i) < \kappa(r_{i+1})$ where $0 \leq i \leq s-1$. If we remove an arbitrary vertex r in R, $LS(R\backslash r) \geq LS(R)$.

Proof: Let $R = (V, E, \kappa, \tau)$ be a $CI - FG$. Then by Proposition 3.8, $LS(R) = r_1$. Suppose that r is a vertex in R. Then $R\backslash r$ is a $P - FG$. If $r = r_0$ then by Proposition 3.19, we have $LS(R\backslash r) = min\{max\{(\kappa(r_i), \kappa(r_{i+1}))|i = 2, \ldots, s - 2\}, \kappa(r_2), \kappa(r_{s-2})\}$

$= \kappa(r_2) \geq LS(R)$. If $r = r_{s-1}$ then by Proposition 3.19, we have $LS(R\backslash r) = min\{max\{(\kappa(r_i), \kappa(r_{i+1}))|i = 1, \ldots, s - 2\}, \kappa(r_1), \kappa(r_{s-3})\}$

$= \kappa(r_1) = LS(R)$. If $r = r_i$, where $0 \leq i \leq s - 1$ then by Proposition 3.19, we have $LS(R\backslash r) =$

$min\{max\{(\kappa(r_i), \kappa(r_{i+1}))|i = 1, \ldots, s - 2\}, \kappa(r_1), \kappa(r_{s-3})\} = \kappa(r_1) = LS(R)$.

Proposition 3.22: Let $R = (V, E, \kappa, \tau)$ be a $CO - FG$ with s vertices $\{r_0, r_1, \ldots r_{s-1}\}$ such that $\kappa(r_i) < \kappa(r_{i+1})$ where $0 \leq i \leq s - 1$. If we remove an arbitrary edge e in R then for calculating low strength connectivity, we have the following cases:

Case I: if we remove an edge $e = (r_i, r_j)$ where $1 \leq i, j < s - 1, i \neq j$ then $LS(R) > LS(R\backslash e)$.

Case II: if we remove an edge $e = (r_{s-2}, r_{s-1})$ then $LS(R) < LS(R\backslash e)$.

Case III: if we remove an edge $e = (r_i, r_{s-1})$ where $1 \leq i < s - 2$ then $LS(R) = LS(R\backslash e)$.

Proof: By Proposition 3.14 the MEC sets of R with $|V(R)| = s$ consist of $s - 1$ edges $\{e_1, e_2, \ldots e_{s-1}\}$ and $LS(R)$ is $\kappa(r_{s-2})$. Let we remove an edge $e = (r_i, r_j)$ where $1 \leq i, j \leq s, i \neq j$. The MEC sets of $R\backslash(r_i, r_j)$ consist of $s - 2$ edges. In case I, by removing (r_i, r_j) intersection $s - 2$ edges connected to r_i or r_j and the other edges contains r_{s-1} and hence $LS(R) > LS(R\backslash e)$. In case II, by removing (r_{s-2}, r_{s-1}) intersection $s - 2$ edges connected to r_{s-1} or r_{s-2} and the other edges contains r_{s-3} and hence $LS(R) < LS(R\backslash e)$. In case III, by removing (r_i, r_{s-1}) where $1 \leq i < s - 2$ intersection $s - 2$ edges connected to r_i or r_{s-1} and the other edges contains r_{s-2} and hence low strength does not change.

Example 3.23: Analyze the $CO - FG$ $R = (V, E, \kappa, \tau)$ shown in Figure 12, where the initial $LS(R)$ value is 0.82. Eliminating the edge $(0.16,0.22)$ raises $LS(R)$ to 0.84. Removing the edge $(0.84,0.82)$ results in a decrease in $LS(R)$. If the edge $(0.16,0.84)$ is removed, $LS(R)$ remains at 0.82, indicating no impact on the low strength connectivity.

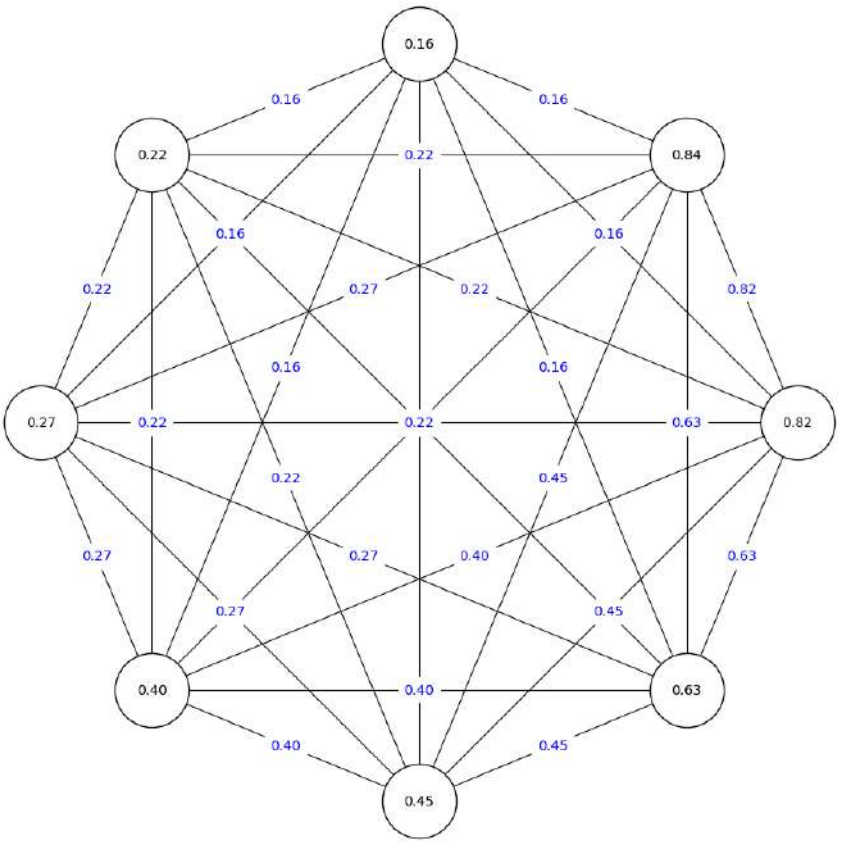


Figure 12
The graph with $LS(R) = 0.82$

Proposition 3.24: Let $R = (V, E, \kappa, \tau)$ be a $CO - FG$. If we remove an arbitrary vertex v in G, $LS(R) \leq LS(R \backslash v)$.

Proof: Let $R = (V, E, \kappa, \tau)$ be a v with s vertices $\{r_0, r_1, \dots v_{s-1}\}$ such that $\kappa(r_i) < \kappa(r_i + 1)$ where $0 \leq i \leq s - 1$. In a $CO - FG$, by removing a vertex, the graph is also complete. If we remove a vertex r_i where $0 \leq i < s - 2$ then MEC set contains $s - 2$ edges. Intersection $s - 2$ edges and $\frac{1}{2}(s^2 - 5s + 6)$ other edges contain $s - 2$ vertices of $\{r_0, r_1, \dots, r_{n-1}\} \backslash \{r_i\}$ and hence by Proposition 3.14, $LS(R \backslash r_i) < \kappa(r_{s-2})$. Therefore $LS(R) = LS(R \backslash r)$. If we remove a vertex r_{s-2} or r_{s-1} then intersection $s - 2$ edges and $\frac{1}{2}(s^2 - 5s + 6)$ other edges contain $s - 2$ vertices of $\{r_0, r_1, \dots, r_{s-1}\} \backslash \{r_{s-1}\}$ or $\{r_{s-2}\}$ and hence by Proposition 3.14, $LS(R \backslash r_{s-1}) = \kappa(r_{s-2})$ or $LS(R \backslash r_{s-2}) = \kappa(r_{s-3})$. $LS(R) < LS(R \backslash r)$.

Example 3.25: Examine the $CO-FG\ R=(V,E,\kappa,\tau)$ depicted in Figure 12, where $LS(R)=0.82$. Removing any vertex from the set {0.16, 0.22, 0.27, 0.40, 0.45, 0.63} does not affect the low strength value. However, deleting vertices 0.82 or 0.84 results in the new graph having a low strength of 0.63, indicating a reduction.

Note: In a star fuzzy graph consisting of s vertices, eliminating any vertex that is linked to the central node does not alter the low strength value.

Proposition 3.26: Let $X_{n,m}=(V,E,\kappa,\tau)$ be a $COB-FG$ contains two parts H and G with n and m vertices, respectively where $n\leq m$. If we remove an edge e in G, $LS(X_{n,m})\leq LS(X_{n,m}\backslash e)$.

Proof: Let $X_{n,m}=(V,E,\kappa,\tau)$ be a $COB-FG$ contains two parts $H=\{h_i|1\leq i\leq n\}$ and $G=\{g_j|1\leq j\leq m\}$. Suppose that $n<m$. By removing an edge (h,g), MEC contains $n-1$ edges connected to g. If $\kappa(h)<\kappa(h_i)$ where $1\leq i\leq n$ and $h\neq h_i$ then by definition low strength connectivity, $LS(X_{n,m}\backslash e)=max(\kappa(h_i))=LS(X_{n,m})$ and hence $LS(X_{n,m})=LS(X_{n,m}\backslash e)$. If $\kappa(h)=\kappa(h_i)$ where $1\leq i\leq n$ and $h\neq h_i$ then $LS(X_{n,m}\backslash e)=max(\kappa(h_i))<LS(X_{n,m})$ because intersection $n-1$ edges connected to g and $mn-(n-1)$ other edges have maximum weights less than $\kappa(h)$. Therefore $LS(X_{n,m})<LS(X_{n,m}\backslash e)$. Now, assume that $n=m$ and $max(\kappa(h_i))>max(\kappa(g_j))$. By removing edge (h,g), MEC contains $n-1$ edges connected to g or h. If $max(\kappa(N(g)-\{h\}))=max(\kappa(g_j))$ then low strength connectivity does not change. Otherwise low strength connectivity decreases.

Example 3.27: Consider the $COB-FG\ X_{4,5}=(V,E,\kappa,\tau)$ given in Figure 13, $LS(R)=0.87$. By removing edge an $(0.25,0.54)$, low strength connectivity does not change. By removing an edge $(0.87,0.25)$, low strength connectivity decreases to 0.54.

Note: Suppose that $X_{n,m}=(V,E,\kappa,\tau)$ is complete bipartite fuzzy graph contains two parts H and G with n and m vertices, respectively where $n\leq m$. If we remove an arbitrary vertex $h\in H$ then low strength connectivity does not change, because the new graph is complete bipartite fuzzy graph and by Proposition 3.26 low strength is maximum weight of vertices in part G. Assume that we remove an arbitrary vertex $g\in G$ with maximum weight. If $n<m-1$ then low strength connectivity decreases and if $n=m-1$ then $min\{max(\kappa(h_i)),\kappa(g)\}\leq\kappa(g)$ and hence low strength connectivity decreases. Now, suppose that $\kappa(g)$ is not maximum. If $n<m-1$ then low strength connectivity does not change. If $n=m-1$ then $min\{max(\kappa(h_i)),\kappa(g)\}\leq LS(X_{n,m})$.

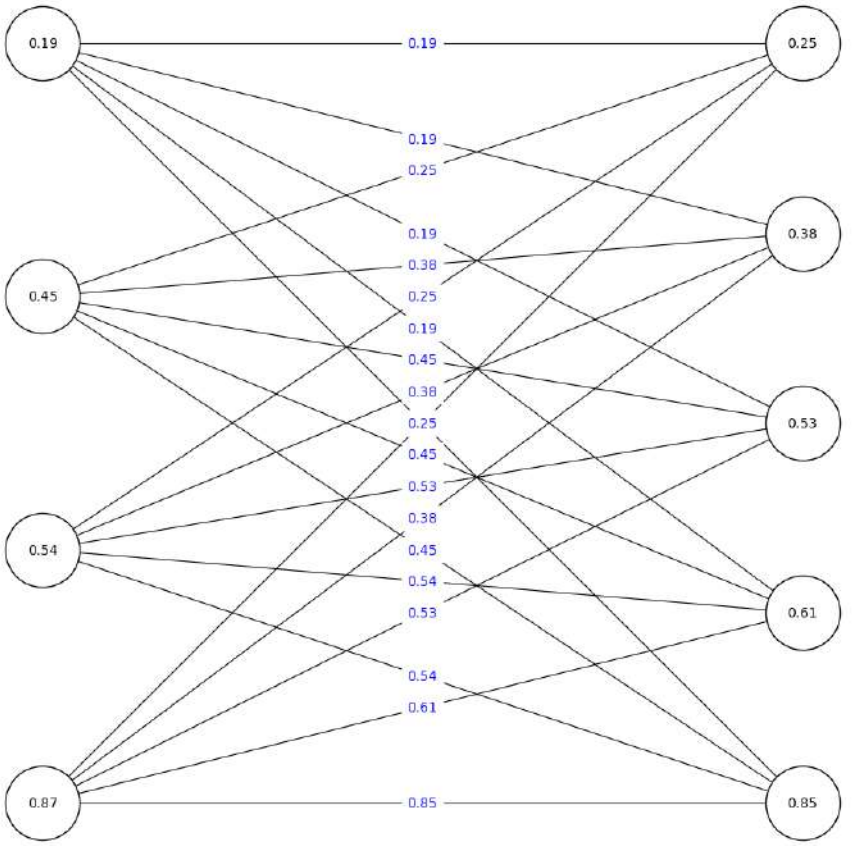


Figure 13
The complete bipartite fuzzy graph with $LS(X_{4,5}) = 0.87$

Proposition 1 Let $R = (V, E, \kappa, \tau)$ be a connected fuzzy graph contains only one bridge $e = (r, z)$. Then $MEC = \{(r, z)\}$ and $LS(R) = max\{\kappa(r), \kappa(z)\}$.

Proof: Clearly, by removing the bridge in R, it becomes disconnected. Intersection e and $E(R)\backslash e$ is vertices r, z and hence $LS(R) = max\{\kappa(r), \kappa(z)\}$.

Example 2 Consider the connected fuzzy graph $R = (V, E, \kappa, \tau)$ given in Figure 13 with $V(R) =$ {(0,0.5), (1,0.3), (2,0.6), (3,0.5), (4,0.4), (5,0.7)}.

The low strength of this graph can be computed using specific commands in Sage software.

```
from sage.graphs.connectivity import bridges
A=list(R.bridges(labels=False))
print("LS(R)=",max(A[0][0][1],A[0][1][1]))
```

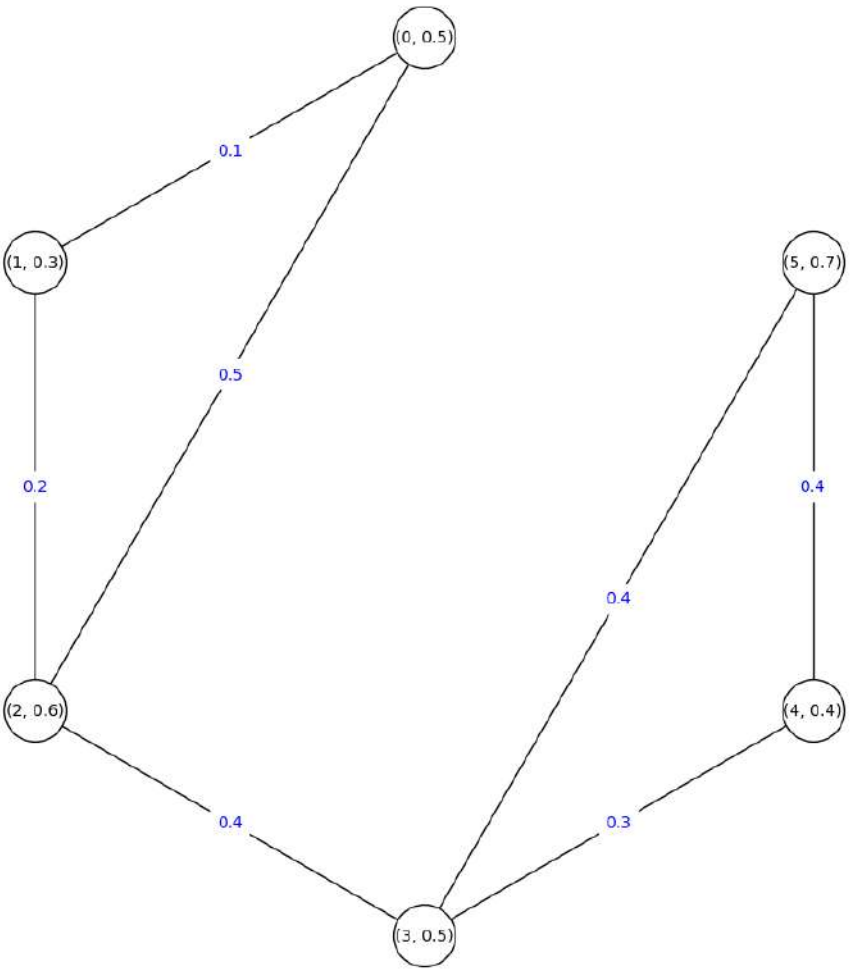


Figure 13
The fuzzy graph with $LS(R) = 0.6$

Corollary 3.30: Let $R = (V, E, \kappa, \tau)$ be a connected fuzzy graph with $s \geq 3$ vertices. If R contains a bridge e then $LS(R) > 0$.

4. Application

Edge-cut sets in graphs have various applications in network analysis and optimization. One common application is in the field of network security, where identifying critical edges that, when removed, can disrupt communication or isolate malicious nodes is crucial. In the study of social networks, edge-cut sets are useful for pinpointing critical links whose removal can disrupt or fragment significant groups or communities within the network. In transportation planning,

edge-cut sets can be used to optimize traffic flow by identifying roads whose closure would have the most significant impact on reducing congestion. In computer network design, edge-cut sets play a vital role in ensuring fault tolerance and resilience by isolating faulty components. Overall, edge-cut sets are essential tools for analyzing and optimizing the structure and connectivity of graphs in various real-world applications.

Application low strength connectivity in hospital management: In a hospital network represented by a fuzzy graph, edge-cut sets can be utilized to identify critical connections that, when disrupted, can impact the flow of resources, patient care, or communication between different departments or units within the hospital. Consider a fuzzy graph representing a hospital network with 10 vertices representing different departments or units within the hospital (such as Emergency, ICU, Surgery, Pediatrics, Pharmacy, Radiology, Laboratory, Administration, Obstetrics and Gynecology(OG) and Physical Therapy). The weight of the vertices is human power, equipment and facilities in that department(vertex of graph). The edges in this fuzzy graph R represent the strength of connections, interactions, or dependencies between these departments(vertices). The vertices of the graph and the weights assigned to them are provided in Table 1 and the edges and the weights of the edges are listed in Table 2. The relationship between departments in hospital is given in the Figure 14. Overall, communication between departments in a hospital is essential for promoting patient safety, enhancing care coordination, improving outcomes, and ensuring a positive experience for patients. By prioritizing effective communication strategies and fostering a culture of collaboration, hospitals can deliver high-quality care and meet the diverse needs of their patients.

Table 1
The vertices of R

u	Emergency	ICU	Pediatrics	Surgery	Pharmacy
$\kappa(u)$	0.95	0.8	0.75	0.7	0.8
u	Radiology	Administration	OG	Physical Therapy	Laboratory
$\kappa(u)$	0.8	0.9	0.7	0.6	0.8

Table 2
The edges of R

(Administration, Emergency)	0.85
(Administration , ICU)	0.8
(Administration , Laboratory)	0.6
(Administration , OG)	0.6
(Administration , Pediatrics)	0.6
(Administration , Pharmacy)	0.75
(Administration , Physical.T)	0.4
(Administration , Radiology)	0.5
(Administration , Surgery)	0.7

(Emergency , ICU)	0.8
(Emergency , Laboratory)	0.6
(Emergency , OG)	0.4
(Emergency , Pediatrics)	0.6
(Emergency , Pharmacy)	0.7
(Emergency , Physical.T)	0.4
(Emergency , Radiology)	0.5
(Emergency , Surgery)	0.7
(ICU , Laboratory)	0.6
(ICU , OG)	0.4
(ICU , Pediatrics)	0.6
(ICU , Pharmacy)	0.6
(ICU , Physical.T)	0.4
(ICU , Radiology)	0.5
(ICU , Surgery)	0.7
(Laboratory , Pharmacy)	0.6
(Laboratory , Radiology)	0.6
(Laboratory , Surgery	0.6
(OG , Pharmacy)	0.7
(OG , Surgery)	0.6
(Pediatrics , Physical.T)	0.4
(Pediatrics , Radiology)	0.5
(Pediatrics , Surgery)	0.6
(Pharmacy , Surgery)	0.6
(Physical.T , Radiology)	0.5
(Radiology , Surgery)	0.5

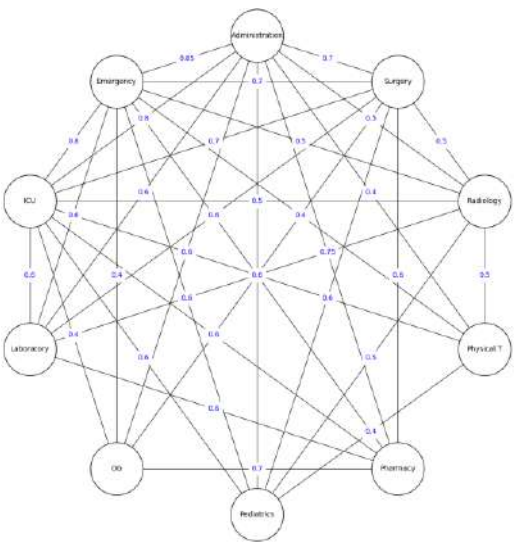


Figure 14
The graph relationship between departments in hospital

The management of a hospital wants to equip some departments in order to better solve the challenges related to patient care, work flow optimization and emergency response. For this purpose, he selects a set of edges that, if removed, would disrupt the hospital and put the lives of patients and staff at risk. In this graph MEC sets are

$$Q_1 = \{(Physical.T, ICU), (Physical.T, Administration), (Physical.T, Radiology), (Physical.T, Emergency), (Physical.T, Pediatrics)\}$$

and

$$Q_2 = \{(OG, Pharmacy), (OG, ICU), (OG, Surgery), (OG, Administration), (OG, Emergency)\}.$$

Now we calculate the strength of each of these two sets. We have $S(Q_1) = 0.95$ and $S(Q_2) = 0.95$. Therefore, we can strengthen the edges of these two cut sets to improve the hospital's performance quality in overcoming challenges. If the hospital does not have the ability to equip both sets, it can use a set whose weight is less. In this example $WLS(R) = Q_1$.

Application low strength connectivity in network: One example of the application of an edge-cut set in a network is in the context of a social media platform. Consider a social network where users are connected by edges representing friendships or interactions. By identifying and removing certain edges that act as bridges between different communities or groups of users, we can disrupt the flow of information or influence between these groups. This can be useful for controlling the spread of misinformation, preventing the formation of echo chambers, or targeting specific groups for advertising or content promotion. By strategically cutting edges in the network, we can influence the overall structure and dynamics of the social network, leading to various outcomes such as increased diversity of opinions, improved network resilience, or more targeted communication strategies.

In a social media platform network with 7 vertices and 10 edges, an edge-cut set can be used to identify connections that, when removed, would disconnect certain groups of users from each other. Let's consider a hypothetical scenario with 7 users (vertices) on a social media platform, labeled as A, B, C, D, E, F, and G. The edges represent connections or interactions between these users.

Here is a fuzzy graph with 7 vertices and 10 edges in a social media platform network. Suppose that 7 users A, B, C, G, E, F, and D respectively have power 0.8, 0.8, 0.7, 0.9, 0.8, 0.7 and 0.9 as vertex weight. The relationship between the countries is given in the Figure 15.

Now, let's consider edge-cut sets that could potentially disconnect the graph into two or more components. The MEC sets could be $Q_1 = \{(G, D), (G, F)\}$ and $Q_2 = \{(E, F), (B, E)\}$. Then $S(Q_1) = 0.9$ and $S(Q_2) = 0.8$ and hence low strength connectivity is 0.8. Given the weakening of communication interaction within edge-cut Q_2, the network should equip this set to prevent false news from easily infiltrating the network through this set.

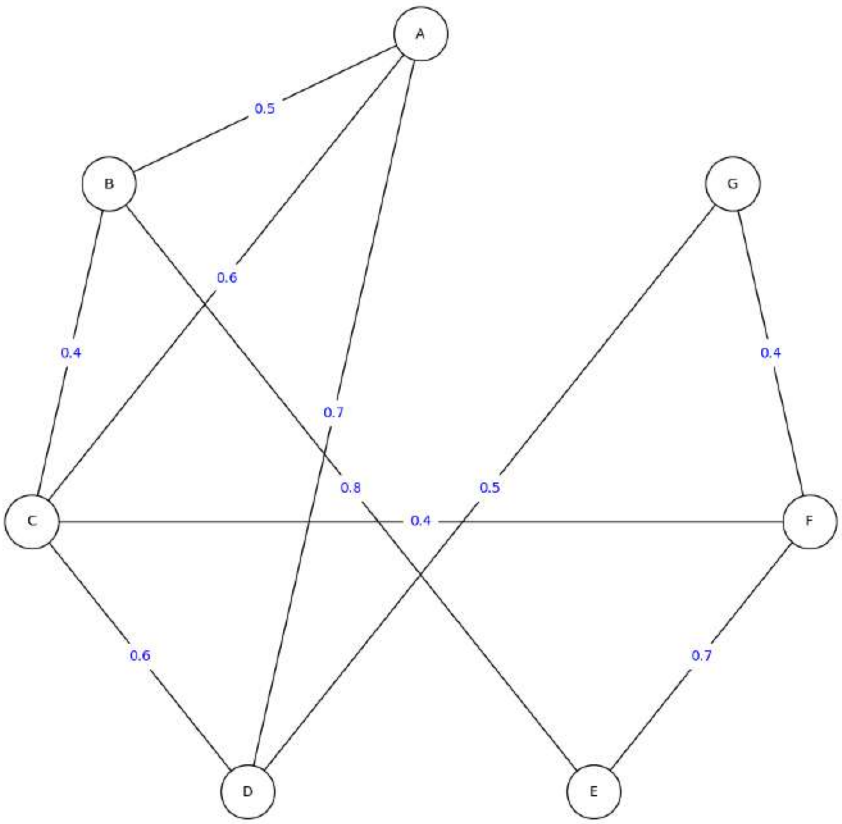


Figure 15
Social media platform network with 7 vertices and 10 edges

Conclusion

In this article, low strength connectivity of the connected fuzzy graph was introduced. Low strength connectivity was calculated in famous connected fuzzy graphs. For fuzzy graphs with a lot of vertices, low strength connectivity was calculated by using coding in the Sage software. The concept of the low strength connectivity in fuzzy graphs is crucial in network design, as it helps determine the resilience and reliability of a network. By analyzing edge connectivity, network engineers can identify critical links that, if disrupted, could potentially isolate parts of the network or lead to network failure. Understanding edge connectivity is essential for designing robust and fault-tolerant network architectures. In the future, we plan to focus on coding to obtain low strength in each connected fuzzy graph and its optimization.

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