An EOQ model under trade credit and conditional cash discount

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Abstract
This paper tries to extend Goyal (1985) to develop the retailer’s inventory model under trade credit and conditional cash discount. We assume that the retailer can obtain the cash discount with a large order quantity under trade credit policy. Otherwise, the retailer will just obtain the trade credit with a small order quantity. In addition, we modify the assumption that the unit purchasing price and unit selling price were equal. Under these conditions, we want to investigate the retailer’s optimal ordering policy within the EOQ framework. Mathematical models have been derived for obtaining the optimal cycle time for item so that the annual total relevant cost is minimized. Furthermore, numerical examples are given to illustrate the results developed in this paper.

Keywords: EOQ, inventory, trade credit, cash discount.

1. Introduction
The supplier offers the trade credit and the cash discount policy to stimulate the demand of the retailer. The supplier can use the marketing alternatives of the trade credit or the cash discount to promote his/her

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products. Recently, the research area of trade credit has been considered. At first, Goyal (1985) derived an EOQ model under the condition of trade credit. Huang and Chung (2003) extended Goyal’s model (1985) to cash discount policy for early payment. The retailer can obtain the cash discount when the payment is paid within the cash discount period offered by the supplier. Otherwise, the retailer will pay full payment within the trade credit period. Huang (2004a) relaxed the assumption that the selling price was equal to the purchasing price in Huang and Chung (2003). Many articles related to the inventory policy under trade credit and cash discount can be found in Huang (2004b, 2004c, 2005).

Above published articles assumed that the retailer could obtain the cash discount when the payment was paid before the cash discount period offered by the supplier. Otherwise, the retailer would pay full payment within the trade credit period. What the above statement describes is just one way of various credits offered by supplier. The main purpose of the supplier’s credits wants to promote his/her commodities. Hence, this paper wants to modify above credits offered by supplier to develop the retailer’s inventory model. We assume that the retailer can obtain the cash discount with a large order quantity under trade credit policy. Otherwise, the retailer will just obtain the trade credit with a small order quantity. In this policy, the retailer will be attracted to order a large quantity to obtain the cash discount. In addition, we modify the assumption that the unit purchasing price and unit selling price were equal. Under these conditions, we want to investigate the retailer’s optimal ordering policy within the EOQ framework.

2. Model formulation

For convenience, we adopt the same notation and assumptions as in Huang and Chung (2003).

Notation:

\( D \) = annual demand

\( A \) = cost of placing one order

\( W \) = quantity at which the cash discount permitted per order

\( c \) = unit purchasing price per item

\( s \) = unit selling price per item

\( h \) = unit stock holding cost per item per year excluding interest charges
AN EOQ MODEL

\[ I_c = \text{interest which can be earned per } $ \text{ per year} \]
\[ I_p = \text{interest charges per } $ \text{ investment in inventory per year} \]
\[ r = \text{cash discount rate, } 0 < r < 1 \]
\[ M = \text{the period of the trade credit in years} \]
\[ T = \text{the cycle time in years} \]
\[ TRC(T) = \text{the annual total relevant cost} \]
\[ T^* = \text{the optimal cycle time of } TRC(T) \]
\[ Q^* = \text{the optimal order quantity} = DT^*. \]

Assumptions:

(1) Demand rate is known and constant.
(2) Shortages are not allowed.
(3) Time horizon is infinite.
(4) Replenishments are instantaneous with a known and constant lead time.
(5) \( I_p \geq I_c \) and \( s > c \).
(6) If \( Q \geq W \), i.e. \( T \geq W/D \), supplier offers the cash discount under the trade credit, otherwise just offers the trade credit.
(7) When the trade credit offered, a deposit is made of the unit selling price of generated sales revenue into an interest bearing account during the time the account is not settled. At the end of this period, the retailer pays off all items sold, keeps profits, and starts paying for the interest charges on the items in stocks. We assume that the interest earned from profits is too small to neglect it.

The annual total relevant cost consists of the following elements. There are two cases to occur: (1) \( M \geq W/D \), (2) \( M < W/D \).

Case I. Suppose that \( M \geq W/D \).

(1) Annual ordering cost = \( \frac{A}{T} \).
(2) Annual stock holding cost (excluding interest charges) = \( \frac{DTh}{2} \).
(3) Annual purchasing cost:

(i) \( W/D \leq T \).

In this case, since the supplier offers the cash discount, the annual purchasing cost = \( c(1 - r)D \).
(ii) \( W/D > T \).

In this case, since the supplier does not offer the cash discount, the annual purchasing cost = \( cD \).

(4) Annual cost of interest charges for the items kept in stock:

(i) \( M \leq T \).

Annual interest payable = \( \frac{c(1-r)IpD(T-M)^2}{2T} \).

(ii) \( T \leq M \).

Annual interest payable = 0.

(5) Annual interest earned:

(i) \( M \leq T \).

Annual interest earned = \( slc\left(\frac{DM^2}{2}\right)/T \).

(ii) \( T \leq M \).

Annual interest earned = \( slc\left[\frac{DT^2}{2} + DT(M-T)\right]/T \)

\[ = \frac{slcDT(M-T^2)}{2T} \] .

From the above arguments, the annual total relevant cost for the retailer can be expressed as:

\[ TRC(T) = \text{ordering cost} + \text{stock-holding cost} \]
\[ + \text{purchasing cost} + \text{interest payable} \]
\[ - \text{interest earned}. \]

We show that the annual total relevant cost is given by

\[ TRC(T) = \begin{cases} 
TRC_1(T) & \text{if } M \leq T \\
TRC_2(T) & \text{if } \frac{W}{D} \leq T < M \\
TRC_3(T) & \text{if } 0 < T < \frac{W}{D} 
\end{cases} \]  

where

\[ TRC_1(T) = \frac{A}{T} + \frac{hDT}{2} + c(1-r)D + \frac{c(1-r)IpD(T-M)^2}{2T} \]
\[ - \frac{slcDM^2}{2T} \] .
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\[ TRC_2(T) = \frac{A}{T} + \frac{hDT}{2} + cM - rD - sDIe \left( M - \frac{T}{2} \right) \]  

(3)

and

\[ TRC_3(T) = \frac{A}{T} + \frac{hDT}{2} + cD - sDIe \left( M - \frac{T}{2} \right) . \]  

(4)

Then, we find \( TRC_1(M) = TRC_2(M) \) and \( TRC_2(T) < TRC_3(T) \) for \( T > 0 \).

Case II. Suppose that \( M < W/D \).

If \( M < W/D \), equations 1(a, b, c) will be modified as

\[
TRC(T) = \begin{cases} 
TRC_1(T) & \text{if } \frac{W}{D} \leq T \\
TRC_4(T) & \text{if } M \leq T < \frac{W}{D} \\
TRC_3(T) & \text{if } 0 < T < M.
\end{cases}
\]  

(5a)  

(5b)  

(5c)

When \( M < W/D \), the annual total relevant cost, \( TRC_4(T) \), consists of the following elements.

1. Annual ordering cost = \( \frac{A}{T} \).

2. Annual stock holding cost = \( \frac{DTh}{2} \).

3. In this case, since the supplier does not offer the cash discount, the annual purchasing cost = \( cD \).

4. Annual interest payable = \( \frac{cIpD(T-M)^2}{2T} \).

5. Annual interest earned = \( sIe \left( \frac{DM^2}{2} \right) / T \).

Combined above elements, we get

\[ TRC_4(T) = \frac{A}{T} + \frac{hDT}{2} + cD + \frac{cIpD(T-M)^2}{2T} - \frac{sIeDM^2}{2T} . \]  

(6)

Then, we find \( TRC_1(T) < TRC_4(T) \) for \( T > 0 \) and \( TRC_4(M) = TRC_3(M) \).

3. Decision rule of the optimal cycle time \( T^* \)

The main purpose of this section is to develop a solution procedure to determine the optimal cycle time \( T^* \).

Case I. Suppose that \( M \geq W/D \).
From equations (2)-(4) yield

\[ TRC'_1(T) = \frac{-[2A + DM^2(c(1-r)I_p - sI_e)]}{2T^2} + \frac{D[h + c(1-r)I_p]}{2}, \]  

(7)

\[ TRC'_1(T) = \frac{2A + DM^2[c(1-r)I_p - sI_e]}{T^3}, \]  

(8)

\[ TRC'_2(T) = TRC'_3(T) = \frac{-A}{T^2} + \frac{D(h + sI_e)}{2}, \]  

(9)

and

\[ TRC''_2(T) = TRC''_3(T) = \frac{2A}{T^2} > 0. \]  

(10)

Equation (10) implies that both \( TRC_2(T) \) and \( TRC_3(T) \) are convex on \( T > 0 \). However, equation (8) implies that \( TRC_1(T) \) is convex on \( T > 0 \) if \( 2A + DM^2[c(1-r)I_p - sI_e] > 0 \).

Let \( TRC'_i(T) = 0 \), for all \( i = 1 \sim 3 \). Then we can obtain

\[ T^*_1 = \sqrt{\frac{2A + DM^2[c(1-r)I_p - sI_e]}{D[h + c(1-r)I_p]}} \quad \text{if} \quad 2A + DM^2[c(1-r)I_p - sI_e] > 0 \]  

(11)

and

\[ T^*_2 = T^*_3 = \sqrt{\frac{2A}{D(h + sI_e)}}. \]  

(12)

Equation (11) implies that the optimal value of \( T \) for the case of \( M \leq T \), that is \( M \leq T^*_1 \). We substitute equation (11) into \( M \leq T^*_1 \), then we can obtain that

if and only if \( -2A + DM^2(h + sI_e) \leq 0 \).

Likewise, equation (12) implies that the optimal value of \( T \) for the case of \( W/D \leq T < M \), that is \( W/D \leq T^*_2 < M \). We substitute equation (12) into \( W/D \leq T^*_2 < M \), then we can obtain that

if and only if \( -2A + DM^2(h + sI_e) > 0 \)

and

if and only if \( -2A + \frac{W^2}{D}(h + sI_e) \leq 0 \).

Finally, equation (12) implies that the optimal value of \( T \) for the case of \( T < W/D \), that is \( T^*_3 < W/D \). We substitute equation (12) into \( T^*_3 < W/D \), then we can obtain that

if and only if \( -2A + \frac{W^2}{D}(h + sI_e) > 0 \).
Furthermore, we let
\[ \Delta_1 = -2A + DM^2(h + s I_e) \]  
(13)
and
\[ \Delta_2 = -2A + \frac{W^2}{D}(h + s I_e). \]  
(14)
Equations (13)-(14) imply that \( \Delta_1 \geq \Delta_2 \). In addition, we know \( TRC_3(T) > TRC_2(T) \) for all \( T > 0 \) from equations (3)-(4). From above arguments, the optimal cycle time \( T^* \) can be obtained as follows.

**Theorem 1.** When \( M \geq W/D \).

(A) If \( \Delta_1 \leq 0 \), then \( TRC(T^*) = TRC_1(T^*_1) \) and \( T^* = T^*_1 \).

(B) If \( \Delta_1 > 0 \) and \( \Delta_2 \leq 0 \), then \( TRC(T^*) = TRC_2(T^*_2) \) and \( T^* = T^*_2 \).

(C) If \( \Delta_2 > 0 \), then \( TRC(T^*) = \min \{TRC_2(W/D), TRC_3(T^*_3)\} \) and \( T^* \) is \( W/D \) or \( T^*_3 \) is associated with least cost.

**Case II.** Suppose that \( M < W/D \).

From equation (6) yields
\[ TRC_4' = \frac{-[2A + DM^2(c I_p - s I_e)]}{2T^2} + \frac{D(h + c I_p)}{2} \]  
(15)
and
\[ TRC_4''(T) = \frac{2A + DM^2(c I_p - s I_e)}{T^3}. \]  
(16)
Equation (16) implies that \( TRC_4(T) \) is convex on \( T > 0 \) if \( 2A + DM^2(c I_p - s I_e) > 0 \).

Let \( TRC_4'(T) = 0 \), then we can obtain
\[ T^*_4 = \sqrt[3]{\frac{2A + DM^2(c I_p - s I_e)}{D(h + c I_p)}} \text{ if } 2A + DM^2(c I_p - s I_e) > 0. \]  
(17)
Similar as above procedure in Case I. We substitute equation (11) into \( T^*_1 \geq W/D \), then we can obtain that
if and only if \(-2A + D \left[ \left( \frac{W}{D} \right)^2 h + M^2 s I_e \right] + Dc(1 - r) I_p \left[ \left( \frac{W}{D} \right)^2 - M^2 \right] \leq 0 \).

Substitute equation (17) into \( M \leq T^*_4 < W/D \), then we can obtain that
if and only if \(-2A + D \left[ \left( \frac{W}{D} \right)^2 h + M^2 s I_e \right] + Dc I_p \left[ \left( \frac{W}{D} \right)^2 - M^2 \right] > 0 \).
and
\[ -2A + DM^2(h + s) \leq 0. \]
Substitute equation (12) into \( T^*_3 < M \), then we can obtain that
\[ -2A + DM^2(h + s) > 0. \]
Furthermore, we let
\[ \Delta_3 = -2A + D \left( \frac{W}{D} \right)^2 h + M^2 s I_e + Dc(1-r)I_p \left( \frac{W}{D} \right)^2 - M^2 \] (18)
and
\[ \Delta_4 = -2A + D \left( \frac{W}{D} \right)^2 h + M^2 s I_e + DcI_p \left( \frac{W}{D} \right)^2 - M^2 \] . (19)
Equations (18)-(19) and (13) imply that \( \Delta_4 > \Delta_3 > \Delta_1 \). In addition, we know \( TRC_4(T) > TRC_3(T) \) for all \( T > 0 \) from equations (2) and (6). From above arguments, the optimal cycle time \( T^* \) can be obtained as follows.

**Theorem 2.** When \( M < W/D \).

(A) If \( \Delta_4 \leq 0 \), then \( TRC(T^*) = TRC_1(T^*_1) \) and \( T^* = T^*_1 \).
(B) If \( \Delta_4 > 0 \) and \( \Delta_3 \leq 0 \), then \( TRC(T^*) = TRC_1(T^*_1) \) and \( T^* = T^*_1 \).
(C) If \( \Delta_3 > 0 \) and \( \Delta_1 \leq 0 \), then \( TRC(T^*) = \min \{ TRC_1(W/D), \)
\[ TRC_4(T^*_4) \} \) and \( T^* \) is \( W/D \) or \( T^*_4 \) associated with least cost.
(D) If \( \Delta_1 > 0 \), then \( TRC(T^*) = TRC_3(T^*_3) \) and \( T^* = T^*_3 \).

4. **Numerical examples**

To illustrate the results, let us apply the proposed method to solve the following numerical examples. The optimal cycle time is summarized in Table 1.

5. **Conclusions**

The main purpose of the supplier’s credits wants to promote his/her commodities. Excepted the trade credit policy, the supplier can adopt the cash discount with a large order quantity to strengthen his/her promotion. This paper investigates the retailer’s inventory policy that the retailer can obtain the cash discount with a large order quantity under trade credit policy. In addition, we provide a very efficient solution procedure to determine the optimal cycle time \( T^* \).
### Table 1

Optimal cycle time with various $r$

Let $A = $120/order, $D = 1000$ units/year, $c = $20/unit, $s = $25/unit, $h = $3/unit/year, $I_e = $0.13$/year, $I_p = $0.15$/year, $M = 0.12/year

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