Assessing the performance of various evaluation periods between the market model and the grey model in relation to event study

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Abstract

This investigation uses Event Study to research the predictive performances of the market model and the Grey forecasting GM(1, 1) model during a specific estimation period. A positive research was made and found the over 100 research objective data were modeled during the evaluation periods, the market model has more excellent examination ability, the abnormal return was calculated using the Grey forecasting GM(1, 1) model which involved a model forecasting bias. However, when fewer than 100 data were used for modeling during the evaluation period, the Grey forecasting GM(1, 1) can predict stock returns more accurately than the market model, demonstrating that the market model had forecasting bias. The results herein provide a valuable reference for using the Event Study method.

Keywords : Event study, market model, Grey forecasting GM(1, 1) model, abnormal return.

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1. Introduction

Dolly [8] was the first researcher to apply Event Study to analyze the price effect of stock splits. Subsequently, Fama et al [11] demonstrated this method in the CRSP data base of Chicago University and paying close attention to the other researchers. During the past 30 years, the Event Study is already one of the most important methodologies in financial and accounting researching field. The main objective of the Event Study is to measure how announcements of government policy information or the exposure of firm material message influence the stock markets in countries with effective capital markets. The researching operational processes of the Event Study includes recognizing the event and event day, seeking an appropriate estimation period and constructing a stock return forecasting model, calculating abnormal return (AR) at each certain time during the event period and examining whether or not remarkable AR exists during the event period. Among those processes, the construction of the forecasting model during estimation period is intended to anticipate normal stock returns assuming the event does not interfere with stock prices. Furthermore, compare the forecast and act returns during the event period to check the degree of abnormality and also measure the positive or negative abnormal change. The market model, mean adjusted returns model and market adjusted returns model can be used to construct a forecasting model.

Brown and Warner [3,4] found the market model superior to the mean adjusted returns model and the market adjusted returns model. The market model is used more generally and can be adopted by most investigators [13]. Dyckman et al [10] found the market model superior to other models, but it suffers the deviation problem of normal distribution. Pettengill and Clark [15] indicated using the Ordinary Least Squares method (OLS) that is based on the regression model to set the market model is superior to using other complex numerical models, but the estimation parameters of the regression forecasting model change following the time flow. The parameter values would vary owing to different estimation periods, producing variable forecasting results. Duffie and Pan [9] identified the regression forecasting model as having the limitations of requiring large amounts of value data and assuming this data conforms to a normal distribution. Those limitations destabilize the distribution of expected returns and cause overestimation or underestimation of AR. Furthermore,
degree of variation of stock prices can be incorrectly analyzed.

The major purpose of building the evaluation period parameter in the Event Study method is to utilize the stock prices to construct a model for measuring expected returns during the event period. The researchers calculate AR in the event period to measure the influencing degree of certain event be attack by of economic change or of the information connotation of stock prices implying AR [1]. The above description demonstrates that the researching performance is influenced by the length of the estimation period as when using Event Study. consequently, it is necessary to adopt an appropriate estimation period. Regardless of the type of numerical method used to construct the forecasting model during the estimation period, that method should have good ability to forecast the reasonable and normal parameter value for the event period. The shortcomings of the market model have always been important factors influences on the accuracy of the estimation parameter during the event period.

Ryngaert and Netter [16] considered the controversy regarding the Event Study method to involve determining the length of the estimation period. If a short-term estimation period is selected, the stock return forecasting model can obtain more accurate information regarding the degree of influence of certain events. Furthermore, when longer estimation periods are used to obtain situations involves excessively high estimation parameter error. Solibakke [17] considered the optimal length of the estimation period to be 100 to 300 days. But how many data should be selected from 100 to 300 are subjectively identified by the researcher. Whether such a large sampling period will influence the effective degree of the estimation parameter needs to be researched in depth. Furthermore, when the number of days in the sample period is too small or number of price data of new marketable stock does not exceed 100, the market model would result in estimation of systematical deviation. Hence, it is necessary to discuss the ability to construct an effective forecasting model since the number of samples in the estimation period is insufficient. The research motivation is attacked to seek a numerical method that capable of developing a forecasting model that only requires a small number of data to offset the shortcomings of the market model.

Grey forecast theory was developed in 1982 and later applied to various fields. The advantages of this theory are the ability to use original
data, the ability to build forecasting models based on just four pieces of data, the lack of any need for assumptions regarding data distribution before modeling and the easy of model operation [6]. Unlike the market model, which requires extensive known data to make an accurate forecast, Grey forecasting solves the problem of insufficient data in the modeling period. Lee et al [14] applied Grey theory to analyze the time series variation in financial data and found it feasible to apply this theory to rapidly observe the financial situation. Some investigations have also used the original model GM(1, 1) model of Grey forecasting theory to solve problems involving stock prices [5]. To conclude the above description and know the market model probably exists the estimation error and whether using the Grey forecasting GM(1, 1) model to estimate stock returns can eliminate the shortcomings of assuming a large sample and the parameter error of the estimation model is the focus of this paper. This paper selected the Taiwan government announcement of the Realty Securitization Regulations on July 9, 2003 as an event. Moreover, the event day was set as the first trading day after the announcement day, namely July 10. Since realty securitization can effectively solve the funding problems of manufacturing firms, the announcement event directly influenced the stock prices of manufacturing firms. Therefore, the stock prices of manufacturing firms were selected as the research objects. This study assumed the stock price values to be 100 pieces of data. The forecasting models were then constructed separately based on the market model and the Grey forecasting GM(1, 1) model. This study has three main purposes: first, determining whether the market model or the Grey forecasting GM(1, 1) model performed best when applied to a set of over 100 pieces of stock price data; second, determining whether the market model or the Grey forecasting GM(1, 1) model performed best when applied to a set containing less than 100 pieces of stock price data; third, analyzing and measuring the degree of model bias of the model with worse estimation ability in the situation involving less than 100 pieces of data. The research results presented in this investigation can provide a good reference for selecting another method for constructing a forecasting model.

2. Notations definitions and assumptions

Some notations were defined below for constructing a numerical model in this study:
\( t_1, t_2 \) : represents the first and last days of the estimation period.

\( t_2, t_3 \) : represents the first and last days of the model accuracy examination period.

\( N \) : number of sample firms.

\( t_3, t_4 \) : represents the first and last days of the event period.

\( T_i \) : is the length of the estimation period for the \( i \)th firm, that is \( t_2 - t_1 + 1 \).

\( W_i \) : is the length of the event period for the \( i \)th firm, that is \( t_4 - t_3 + 1 \).

\( R_{it} \) : is the \( i \)th firm’s actual return on trading day \( t \).

\( \hat{R}_{it} \) : is the \( i \)th firm’s expected return on trading day \( t \).

\( R_{mt} \) : is the market weighted average stock index return on trading day \( t \).

\( AR_{it} \) : is the \( i \)th firm’s abnormal return on trading day \( t \).

\( \bar{AR}_t \) : is the average abnormal return on trading day \( t \) of all of the sampled firms.

\( \alpha_i \) : is the intercept of the \( R_{it} \) regression model.

\( \beta_i \) : is the coefficient of the \( R_{it} \) regression model.

\( \epsilon_{it} \) : is the \( R_{it} \) regression error item of the \( i \)th firm on trading day \( t \).

\( \text{Cov}(\epsilon_{it}, \epsilon_{i\gamma}) \) : is the covariance of the \( R_{it} \) regression error items on any two trading days \( (\tau \text{ and } \gamma) \) for the \( i \)th firm.

\( \text{Cov}(\epsilon_{it}, R_{mt}) \) : is the covariance of the \( R_{it} \) regression error items and market return of the \( i \)th firm on trading day \( t \).

\( S_p^{(0)}(k), \hat{S}_p^{(0)}(k) \) : is the stock price and its expected value in period \( k \).

\( u_p \) : is the development coefficient of stock price in the Grey forecasting \( \text{GM}(1,1) \) model.

\( u_p \) : is the Grey controlling variable of stock price in the Grey forecasting \( \text{GM}(1,1) \) model.

\( \delta_p(k) \) : is the stock price forecasting accuracy degree in the Grey forecasting \( \text{GM}(1,1) \) model during period \( k \).

\( t_{AR} \) : is the \( t \)-test statistic of \( AR \) during estimation period.

\( \hat{\delta}_i \) : is the standard variant of difference in the Event Study.

The assumptions of this research are as follows:

(1) The government announcement of the Realty Securitization Regulations was assumed to have a positive or negative influence on the stock prices of markable manufacturing firms.
Abnormal returns were assumed to be the difference between the actual and expected stock returns.

3. Research method and design

This investigation compares the forecasting performance between the market model and Grey forecasting GM(1, 1) model during different estimation periods. To achieve this objective this study operates six steps using the Event Study method. The six steps are as follows: sample selection and deciding time parameter, constructing the return forecasting model using the market model, identifying the return forecasting model via Grey forecasting GM(1, 1), calculating AR, operating the statistic test of abnormal return and the hypotheses test. These steps are presented in detail below:

Step 1. Sample selection and time parameter decision

According to the selection of the Realty Securitization Regulations announcement for influencing the stock prices of manufacturing firms. This investigation selected 17 stock marketable manufacturing firms as researching objects. Stock price fluctuations were observed. The group of research objects was eventually narrowed down to just 17 firms. Meanwhile, the event day was July 7, 2003 \( t = 0 \), the period of 20 days before and after the event day been expressed as \( W(-20, 20) \) and the accuracy examination period of the model ran from the 21st day to the 40th before the event announcement day \([17]\). During the period from 140 \( t_3 = -140 \) to 41 \( t_4 = -41 \) days before the announcement data, a total of 100 data were taken as the estimation period of the market model. The actual market returns were taken as the dependent variable for constructing the market model. Subsequently, the viewing area of the estimation period was expanded to \( T(-41, -45) \), \( T(-41, -46) \) and \( T(-41, -140) \).

Step 2. Constructing the return forecasting model based on the market model

The market model assumes that the stock return and the market return exhibit a linear relationship. This relationship can be expressed numerically as:

\[
R_{it} = \alpha_i + \beta_i R_{mt} + \epsilon_{it}, \quad t = t_1, \ldots, t_2, \quad i = 1, 2, \ldots, N. \quad (1)
\]

The error item of the market model fits the following assumptions:

\[
E(\epsilon_{it}) = 0, \quad (2)
\]
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\[ \text{Cov}(\varepsilon_{it}, \varepsilon_{iy}) = \begin{cases} 0, & \tau \neq \gamma \\ \sigma_i^2, & \tau = \gamma \end{cases}, \quad \tau, \gamma \in [t_1, t_2] \]  

(3)

and

\[ \text{Cov}(\varepsilon_{it}, R_{mt}) = 0. \]  

(4)

Eqn. (2) assumes the average value of \( \varepsilon_{it} \) is zero on every trading day during the estimation period; Eqn. (3) assumes the \( \varepsilon_{it} \) of any two trading days, \( \tau \) and \( \gamma \), displays on correlation in \( T(t_1, t_2) \) and the variances of \( \varepsilon_{it} \) are identical regardless of the trading day. The variation of stock prices is \( \sigma^2 \). Eqn. (4) assumes that no correlation exists between the estimation period and the market return [12]. To construct the optimum estimation equation of Eqn. (1) for forecasting stock return, OLS can be performed to determine the optima evaluation values \( \hat{\alpha}_i \) and \( \hat{\beta}_i \), in which case the forecasting model of Eqn. (1) is:

\[ \hat{R}_{it} = \hat{\alpha}_i + \hat{\beta}_i R_{mt}, \quad t = t_3, \ldots, t_4, \quad i = 1, 2, \ldots, N. \]  

(5)

Eqn. (5) illustrates the stock return forecasting model based on the market model. This model can be used to forecast the reasonable returns of every firm during the event period. The forecast returns are the normal returns that are not influenced by the event.

**Step 3.** *Find the return forecasting model using the Grey forecasting GM(1,1) model*

After interpreting how to use the Grey forecasting GM(1,1) model, then to establish a stock price forecasting model based on the Event Study method. To design this model, stock price data during a certain period is gathered to make the original array and the Grey forecasting GM(1,1) model is applied to design a differential equation. The developed coefficient and grey controlled variable of the Grey forecasting GM(1,1) model are then calculated. When the Grey System develops a model, that model must apply a one order Accumulated Generating Operation (AGO) and a one order Inversed Accumulated Generating Operation (IAGO) to establish the stock price forecasting model [7], which is expressed as:

\[ S_P^{(0)}(k) = \left[ S_P^{(0)} (1) - \frac{\mu_P}{a_P} \right] e^{-(k-1)} \left(1 - e^{a_P} \right) \]  

(6)

Using the savage difference method to examine the accuracy of Eqn. (6), the savage difference method is to measure the error, \( \delta_P(k) \), between
the expected and actual values, for which the measurement formula is:

$$\delta_p(k) = \left| \frac{S_p^{(0)}(k) - \hat{S}_p^{(0)}(k)}{S_p^{(0)}(k)} \right| \times 100\%, \quad k = 2, 3, \ldots, n.$$  \hspace{1cm} (7)

The accuracy degree index is $1 - \delta_p(k)$ and the average accuracy degree exceeds 95% demonstrating that the stock price forecasting model has excellent forecasting efficiency [6].

Step 4. Calculating AR

AR indicates the difference between actual and expected stock returns. The AR of the $i$th firm in the $t$ the event period can be modeled as:

$$AR_{it} = R_{it} - \hat{R}_{it}, \quad t = t_3, \ldots, t_4, \ i = 1, 2, \ldots, N.$$  \hspace{1cm} (8)

If considering $N$ firms, the average AR of the $N$ firms is:

$$\overline{AR}_t = \frac{1}{N} \sum_{i=1}^{N} AR_{it}, \quad t = t_3, \ldots, t_4.$$  \hspace{1cm} (9)

Step 5. Operate the statistic test of AR

The statistic testing method of AR in the traditional Event Study method includes the estimation period $t$-test and the event period $t$-test. The main objective of the estimation period test is to examine the degree of accuracy of the market model, while the event period test aims to confirm the existence of AR [4]. The t-test statistic value of the estimation period is:

$$t_{AR} = \frac{AR}{1 \sqrt{\frac{1}{N} \sum_{i=1}^{N} S_i^2 \left( \frac{1}{N} \sum_{i=1}^{N} \frac{1}{T_i - 1} \sum_{t=t_1}^{t_2} (\hat{\epsilon}_{it} - \bar{\hat{\epsilon}}_{it})^2 \right)^2}} = \frac{AR}{1 \sqrt{\frac{1}{N} \sum_{i=1}^{N} S_i^2}}, \quad t \in [t_1, t_2],$$  \hspace{1cm} (10)

in Eqn. (10) $\hat{\epsilon}_{it} = R_{it} - E(\hat{R}_{it}), t \in [t_1, t_2], and$

$$\hat{S}_i = \sqrt{\frac{1}{n_1} \sum_{t=t_1}^{t_2} (\hat{\epsilon}_{it} - \bar{\hat{\epsilon}}_{it})^2}, \quad t \in [t_1, t_2].$$  \hspace{1cm} (11)

Owing to the estimation coefficients being used during the estimation period to assess the value of different item during the event period, the standard variances between the event period and the estimation period are different and the formula of the standard variance during the event
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\[ \sqrt{\text{Var}(AR_{it})} = \sqrt{\frac{\bar{S}_i^2}{1 + \frac{1}{T_i} + \frac{(R_{mt} - \bar{R}_{mi})^2}{\sum_{t=1}^{T} (R_{mT} - \bar{R}_{mi})^2}}}, \quad t \in W = [t_3, t_4]. \] (12)

Then put Eqn. (12) into Eqn. (10) and construct the \( t \)-test statistic value in the event period.

**Step 6. \( AR \) obvious hypotheses test**

This investigation defines hypotheses that during certain periods the average \( AR \) of stock equals zero, namely \( H_0: \bar{AR}_t = 0 \); moreover, the confronting position of hypothesize assumes that the average stock \( AR \) does not equal zero, namely \( H_1: \bar{AR}_t \neq 0 \). Whether these assumptions are accepted can be tested using the statistical \( t \)-test [2]. When the statistic \( t(\bar{AR}_t) \leq |t_{0.95, N-1}| \) exists; otherwise, reject \( H_0 \). Moreover, when \( P \)-value < 0.05 the stock prices obviously have average \( AR \).

4. Positive research

This section explains the research hypotheses, positive analysis and positive research results. Three hypotheses are proposed based on the research purpose.

**Hypothesis 1.** When the data number exceeds 100 during the estimation period, the market model has superior forecasting performance.

**Hypothesis 2.** When the data number is below 100, the Grey forecasting \( GM(1,1) \) model has the best forecasting performance.

**Hypothesis 3.** The forecasting model with worse forecasting efficiency generates more examination bias.

4.1 Positive analysis

(1) For the market model

Based on 100 data values, \( T(-140, -41) \) and applying 20 data values, \( T(-40, -21) \) during the examination period to test the accuracy of the market model. Observation showed that five data followed \( T(-41, -45) \) and six data followed \( T(-41, -46) \) \( et \ al \), continue increasing-length windows to 100 data of \( T(-41, -140) \) to use Eqns. (5), (8) and (9) to examine
the accuracy of the market model. Then show the trend of $P$-value that concerns the degree of accuracy of the increasing-length windows in Figure 1. Figure 1 reveals that the number of sample data in $T(t_1, t_2)$ is below 80, the forecasting model is biased and when the number of data is less than 84 the forecasting model with the highest accuracy degree. Figure 1 also showed that it is easy to reject the null hypothesis ($H_0 : AR_t = 0$) when the data number is below 80 during the estimation period; the forecasting parameters of the forecasting model are unsuitable for predicting the stock prices. Therefore, we know when the estimation period does not fall within the optimum length of 100 to 400 days, the market model is not a good method for forecasting in relation to the Event Study.

![Figure 1](image)

**Figure 1**

Examination of degree of accuracy during different estimation periods for the market model

Regarding the event study, this work applied the market model and the Grey forecasting GM(1, 1) model to conduct analyses of $AR$. The analytic results are shown in Figure 2 and 4. Observation of these figures can demonstrate whether adopt a large number of data during the estimation period to construct the forecasting model. The market model is more accurate brought the $\overline{AR_t}$ 0.5% when $W(-20, 20)$, if
based on the Grey forecasting GM(1, 1) model the $\overline{AR}$ was 2.77%, the error passed examination of these two forecasting model was 106.54% $((1.57\% - 0.76\%)/0.76)$. Observing the whole event period, the $\overline{AR}$ was more significant in $W(-1,0)$. Therefore, the $\overline{AR}$ of the, market model was 5.64% and the $\overline{AR}$ of the Grey forecasting GM(1, 1) model was 7.02% on the event day and the day before the event day, and the error passed examination value was 24.38% $((7.02\% - 5.64\%)/5.64\%)$. Following the above analysis, since the forecasting was based on large number of samples, the Grey forecasting GM(1, 1) model displayed less effective stock price forecasting performance, and obvious prediction bias. Consequently, hypotheses 1 and 3 are accepted.

Figure 2
Average abnormal returns of large numbers of data during the event period

(2) For the Grey forecasting GM(1, 1) model

Furthermore, examine the Grey forecasting GM(1, 1) model. Since applying the Grey forecasting GM(1, 1) model to construct the expecting model must consider the optimum number of data for the original array of the estimation period, which is how many the optimum $n$ value for accurate forecasting must be considered. To solve this problem, this investigation selected the 40th day before the announcement day as the day for examining accuracy. Furthermore, different estimation period, $n$-value, were adopted individually to find the forecasting model, and
Eqn. (7) was used to run the accuracy degree test. The evaluation found that the accuracy degree was 98.92% as the \( n \)-value (the estimation period) was 8, this \( n \)-value was the optimal \( n \)-value of the Grey forecasting GM\((1, 1)\) model. The accuracy degrees achieved with different \( n \)-values were shown in Figure 3.

**Figure 3**
Examination of the degree of accuracy during different estimation periods of the Grey forecasting GM\((1, 1)\) model

**Figure 4**
Average abnormal returns of a small number data during the event period
When constructing the forecasting model based on a small number of data, the Grey forecasting GM(1, 1) model had better prediction accuracy in comparison to the market model; the $\overline{AR}_t$ was 0.8% during the event period of the Grey forecasting GM(1, 1) model, compared to 0.68% for the market model, the error passed examination value between these two models was 15.9% ($(0.8\% - 0.68\%)/0.8\%$). These analytical results indicated that if limited data is available during the estimation period the Grey forecasting GM(1, 1) model is better than the Event Study method. From observing the $\overline{AR}_t$ of the event period ($-1, 1$) and finding that the $\overline{AR}_t$ of the market model was 5.16%, while that of Grey forecasting GM(1, 1) model was 18.5% ($(6.33\% - 5.16\%)/6.33\%$). Consequently, since the forecasting was based on small number of samples, the market model had less effective stock price forecasting performance and displayed clear prediction bias. Thus hypotheses 2 and 3 cannot be rejected.

4.2 Positive research results

From the positive analyses presented in Section 4.1, the positive research results that adopt the Grey forecasting GM(1, 1) model to construct the forecasting model for use with small data samples, when increasing-length windows over 84 days of the estimation periods forecasting is ineffective using this method. The analytical results indicate that the optimum data number for modeling is between 100 and 300 as adopting the market model. The positive research results also demonstrated that when the data number of the estimation period was only eight the Grey forecasting GM(1, 1) model was most accurate and moreover its accuracy increased with reducing data number. Thus, regarding differences in estimation period length, selecting a different and optimal forecasting model as with using the Event Study method, can increase the examination ability of $AR$ analysis.

5. Conclusions

When the Event Study method is adopted to investigate the extent to which the sample event influences stock prices, the market model is always adopted for constructing the forecasting model. However, the traditional market model requires a large sample of data in the estimation period. In reality, some research problems suffer the limitation of a shortage of data and thus the Event Study method cannot be applied. This investigation uses Event Study Theory to study the prediction performance
of the market model and the Grey forecasting $GM(1, 1)$ model during a specific estimation period. Stock prices of construction firms are adopted as examples in a discussion of how the announcement by the government of the Regulations on Real Estate. Securitization influenced stock prices. Three hypotheses were proposed, first when the data number is over 100 in the estimation period, the market model with more good forecasting performance. Second, when the data number is less than 100, the Grey forecasting $GM(1, 1)$ model has the best forecasting achievements. Third, the forecasting model with more bad forecasting efficiency would generate more bias of examination. However, when fewer than 100 data were used for modeling during the evaluation period, the Grey forecasting $GM(1, 1)$ model could be used to calculate the optimal number of data, $n$, for modeling during this period. This model can forecast stock returns more accurately than the Market Model, which suffered from forecasting bias. This study concluded that the Grey Forecasting $GM(1, 1)$ model saved calculating time because it only used a limited number of data to obtain the analytical results and moreover obtained better results than the market model. However, if researchers are concerned about the large number of data contained in the estimation period for the Event Study method, the market model has superior prediction performance. The results presented here supply valuable references for using the Event Study method to analyze abnormal stock returns when selected a suitable forecasting model for evaluation periods with various lengths. When the data of modeling is insufficient and short-term forecasts are required, the Grey Forecasting $GM(1, 1)$ model is the optimum choice.

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