Effect of repair cost on the rework decision-making in an economic production quantity model

Singa Wang Chiu ∗

Department of Business Administration
Chaoyang University of Technology
168, Gifeng E. Rd.
Wufeng, Taichung County
Taiwan 413
R.O.C.

Abstract

This paper studies effects of repair cost on the rework decision-making in an imperfect quality Economic Production Quantity (EPQ) model. The optimal lot sizes and the expected annual inventory costs for the cases of EPQ model with a rework process and without any reworking of defective items are compared and analyzed. With straightforward numerical derivations, this note proposes a set of mathematical equations to assist in determining on whether it is beneficial or not to rework. Sensitivity analysis and numerical example are provided to demonstrate their practical usages.

Keywords: Manufacturing, production, rework-or-scrub, EPQ, defective rate.

1. Introduction

The Economic Order Quantity (EOQ) model was first introduced several decades ago, it balances the inventory holding and setup costs and derives an optimal order quantity that minimizes total inventory costs. Regardless of its simplicity, the EOQ model is still applied industry-wide today [1,10,11,13]. In the manufacturing sector, when items are produced internally instead of being obtained from an outside supplier,

∗E-mail: swang@mail.cyut.edu.tw
the Economic Production Quantity (EPQ) model is often used to determine the optimal production lot size.

The classic EPQ model assumes that the manufacturing process will function perfectly at all times. But in reality, the production of defective items during a production run is inevitable. Sometimes, the imperfect quality items can be reworked and fixed with extra repairing cost. For instances, Printed Circuit Board Assembly (PCBA) in the PCBA manufacturing, plastic goods in the plastic injection molding process, etc., sometimes employ rework as an acceptable process in terms of level of quality. A considerable amount of research has been carried out to address the imperfect quality finite production model [4, 5, 7, 8, 9, 14, 15] and additional examples are surveyed as follows. Rosenblatt and Lee [12] proposed an EPQ model that deals with imperfect quality. They assumed that at some random point in time the process might shift from an in-control to an out-of-control state, and a fixed percentage of defective items are produced. Approximate solutions for obtaining an optimal lot size were developed in their paper. Cheng [2] formulated inventory as a geometric program and obtained closed-form optimal solutions for an EOQ model with demand-dependent unit production cost and imperfect production processes. Chung [6] investigated bounds for production lot sizing with machine breakdown conditions. This paper employs the optimal production lot-sizing results from the works of Chiu and Chiu [3, 5] and considers the decisions on whether or not to rework the repairable defective items in an imperfect quality EPQ model with backordering not permitted.

2 Nomenclature

\[ \lambda = \text{demand rate (items per unit time)}, \]
\[ P = \text{production rate (items per unit time)}, \]
\[ P_1 = \text{rate of rework of defective items per unit time}, \]
\[ x = \text{a random defective rate, } x \text{ is a random variable with known probability density function}, \]
\[ \theta = \text{the percentage of defective items that cannot be reworked (rate of scrap items produced)}, \]
\[ d = \text{the steady production rate of defective items per unit time}, \]
\[ Q_1 = \text{production lot size per cycle for the EPQ model with the reworking of defective items}, \]
$Q_2 = \text{production lot size per cycle for the EPQ model without the rework process},$

$C = \text{production cost per item ($/item, inspection cost per item is included)},$

$C_R = \text{repair cost for each imperfect quality item reworked ($/item),}$

$C_S = \text{disposal cost for each scrap item produced ($/item),}$

$H_1 = \text{the maximum level of on-hand inventory in units, when regular production process stops},$

$H = \text{the maximum level of on-hand inventory in units, when rework process ends},$

$K = \text{setup cost for each production run;}$

$h = \text{holding cost per item per unit time ($/item/unit time),}$

$h_1 = \text{holding cost for each reworked items per unit time ($/item/unit time),}$

$b = \text{shortage cost per item per unit time (i.e. $/item/unit time),}$

$TCU(Q_1) = \text{the total production-inventory costs per unit time for the EPQ model with the reworking of defective items},$

$TCU(Q_2) = \text{the total production-inventory costs per unit time for the EPQ model without the rework process.}$

### 3. Mathematical modelling and analysis

This paper studies the decisions on rework-or-scrap the defective items in an imperfect quality EPQ model. We reconsider the model studied by [3, 4] where the assumptions of the production rate ‘$P$’ is constant and is much larger than the demand rate ‘$\lambda$’, and ‘$x$’ percent of defective items were randomly generated by an imperfect process. Among the defective items, there is a portion $\theta$ of the imperfect quality items that cannot be repaired, are scrap items. The production rate ‘$d$’ of the defective items could be expressed as the production rate times the defective percentage, i.e., $d = P \cdot x$, and if the decision is to rework the repairable defective items then they are to be done at a rate of ‘$P_1$’ when each regular production ends. Additional notations used are displayed in the previous section.

The EPQ model with the reworking of defective items has the optimal production lot-size and the optimal expected annual costs [4] as shown in Eqs. (1) and (2).
S. W. CHIU

\[ Q^*_1 = \sqrt{\frac{2K\lambda}{h(1 - \frac{\lambda}{P}) + \frac{\lambda(1-\theta)^2}{P_1}(h_1 - h)E[x^2] - 2h\theta\left(1 - \frac{\lambda}{P}\right)E[x] + h\theta^2E[x^2]}}, \quad (1) \]

\[ E[TCU(Q_1)] = \lambda \left[ C_1 \frac{1}{1 - \theta \cdot E[x]} + C_\kappa (1 - \theta) \frac{E[x]}{1 - \theta \cdot E[x]} + C_\gamma \frac{E[x]}{1 - \theta \cdot E[x]} \right] \]
\[ + \frac{K\lambda}{Q_1} \frac{1}{1 - \theta \cdot E[x]} + \frac{\lambda Q_1(1 - \theta)^2}{2P_1} \]
\[ \times \left(1 - h\right) \frac{E[x^2]}{1 - \theta \cdot E[x]} + h\theta Q_1 \left(1 - \frac{\lambda}{P}\right) \]
\[ - hQ_1 \left(1 - \frac{\lambda}{P}\right) \frac{1}{1 - \theta \cdot E[x]} + \frac{hQ_1\theta^2 E[x^2]}{2} \frac{1}{1 - \theta \cdot E[x]} \]. \quad (2) \]

When the rework process is not considered, the optimal production lot-size and the optimal expected annual costs \([3]\) are as shown in Eqs. (3) and (4).

\[ Q^*_2 = \sqrt{\frac{2K\lambda}{h(1 - \frac{\lambda}{P}) - 2h\left(1 - \frac{\lambda}{P}\right)E[x] + hE[x^2]}}, \quad (3) \]

\[ E[TCU(Q_2)] = \lambda \left[ C_1 \frac{1}{1 - E[x]} + C_\gamma \frac{E[x]}{1 - E[x]} \right] \]
\[ + \frac{K\lambda}{Q_2} \frac{1}{1 - E[x]} + \frac{hQ_2^2}{2} \]
\[ \times \left(1 - \frac{\lambda}{P}\right) \frac{1}{1 - E[x]} + hQ_2 \left(1 - \frac{\lambda}{P}\right) \]
\[ - hQ_2 \left(1 - \frac{\lambda}{P}\right) \frac{1}{1 - E[x]} + \frac{hQ_2\theta^2 E[x^2]}{2} \frac{1}{1 - E[x]} \]. \quad (4) \]

The decision on whether to rework or to scrap defective items in such an imperfect quality EPQ model, can be determined by selecting the smaller values between Eqs. (2) and (4). Let \( C_\gamma \) represents the breakeven value of unit repair cost that makes the optimal expected annual costs \( E[TCU(Q_1)] = E[TCU(Q_2)] \). Hence from Eqs. (2) and (4), we obtain \( C_\gamma \) as:
The economic production quantity (EPQ) model is a variation of the economic order quantity (EOQ) model that incorporates the cost of carrying inventory. The model is used to determine the optimal order quantity that minimizes the total inventory costs over a period. The total cost function in the EPQ model includes the setup cost, holding cost, and shortage cost.

The cost function in the EPQ model is given by:

\[
C_\gamma = C \frac{1}{1 - E(x)} + C_S \frac{1}{1 - E(x)} + \frac{K}{Q_1 E(x)} \frac{1}{1 - \theta} \times \left( \frac{1 - \theta E(x)}{1 - E(x)} \right) \left( \frac{1}{W} - 1 \right) + \frac{hQ_1}{2\lambda} \frac{1}{E(x)} \frac{1}{1 - \theta} \left( 1 - \frac{\lambda}{P} \right) \times \left( W \frac{1 - \theta E(x)}{1 - E(x)} - 1 \right) - \frac{Q_1(1 - \theta)^2}{2P_1} \left( h_1 - h \right) \frac{E(x)^2}{E(x)} \frac{1}{1 - \theta} \right.

\[
- \frac{hQ_1}{\lambda} \left( 1 - \frac{\lambda}{P} \right) \frac{1}{1 - \theta} \left( W \frac{1 - \theta E(x)}{1 - E(x)} - \theta \right) + \frac{hQ_1}{2\lambda} \frac{E(x)^2}{E(x)} \frac{1}{1 - \theta} \left( W \frac{1 - \theta E(x)}{1 - E(x)} - \theta^2 \right)
\]

(5)

where

\[
W = \sqrt{\frac{h \left( 1 - \frac{\lambda}{P} \right) + \frac{\lambda(1 - \theta)^2}{P_1} (h_1 - h) E(x^2)}{-2h \theta \left( 1 - \frac{\lambda}{P} \right) E(x) + h \theta^2 E(x^2)}}.
\]

If we let

\[
\alpha = \frac{1}{Q_1} \frac{1}{E(x)} \frac{1}{1 - \theta} \left[ 1 - \theta E(x) \right] - \frac{W[1 - E(x)]}{W}
\]

(6)

and

\[
\epsilon = \frac{hQ_1}{\lambda} \frac{1}{1 - \theta} \frac{1}{1 - E(x)} \left( \frac{1}{2 E(x)} \left( 1 - \frac{\lambda}{P} \right) \left[ W(1 - \theta E(x)) - (1 - E(x)) \right] \right) - \left( 1 - \frac{\lambda}{P} \right) \left[ W(1 - \theta E(x)) - \theta(1 - E(x)) \right] + \frac{1}{2 E(x)} \left[ W(1 - \theta E(x)) - \theta^2(1 - E(x)) \right]
\]

\[
- \frac{Q_1(1 - \theta)^2}{2P_1} \left( h_1 - h \right) \frac{E(x^2)}{E(x)} \frac{1}{1 - \theta}.
\]

(7)

Substituting \( \alpha \) and \( \epsilon \) in Eq. (5), we obtain:

\[
C_\gamma = (C + C_S + K\alpha) \frac{1}{1 - E(x)} + \epsilon.
\]

(8)

The decisions on either to rework or to scrap the defective items in an imperfect quality EPQ model can be made by comparing \( C_R \) and \( C_\gamma \). That is, for instance, if \( C_R < C_\gamma \) then it will be better off to rework the repairable defective items.
In Eq. (8), suppose we would like to find an approximation to $Cr$ without involving the complex computation of $\varepsilon$; then let $\beta_1$ satisfies the following:

$$C_Y = \beta_1 \left[ (C + C_S + K\alpha) \frac{1}{1 - E(x)} \right]. \quad (9)$$

Let

$$Z_1 = (C + C_S + K\alpha) \frac{1}{1 - E(x)}$$

$$\therefore \beta_1 = 1 + \left( \frac{\varepsilon}{Z_1} \right). \quad (10)$$

Hence, if an approximation to $\left( \frac{\varepsilon}{Z_1} \right)$ is obtainable; then the computation of $Cr$ can be simplified as shown in Eq. (9).

Furthermore, let $Z_2 = (C + C_S) \frac{1}{1 - E(x)}$ and $\varepsilon_x = \frac{K\alpha}{1 - E(x)}$, if we would like to find an approximation to $Cr$ without involving the complex computations of $\varepsilon_x$ and $\varepsilon$; then let $\beta_2$ satisfies the following:

$$C_Y = \beta_2 \left[ (C + C_S) \frac{1}{1 - E(x)} \right] \quad (11)$$

$$\therefore \beta_2 = 1 + \left( \frac{\varepsilon_x + \varepsilon}{Z_2} \right). \quad (12)$$

Hence, it follows that if an approximation of $\left( \frac{\varepsilon_x + \varepsilon}{Z_2} \right)$ is obtainable, the computation of $Cr$ can be simplified as shown in Eq. (11).

Numerical examples are provided in the following section to demonstrate its practical usage. Sensitivity analysis of $C_Y$ in regard to various cost related parameters and the estimated ranges for $\beta_1$ and $\beta_2$ are presented in Section 4.

4. Numerical example

A firm manufactures a product which has experienced a relatively flat demand of 4,000 units per year and the production rate of the item is 10,000 units per year. The percentage of defective items produced is assumed to follow the uniform distribution over the interval $[0, 0.1]$, and not all of the imperfect quality items are repairable; there is a proportion $\theta = 0.1$ of them are scrap. Other parameters are given below:

- $P_1 =$ reworking rate at 600 units per year,
- $K =$ $450 for each production run,
C = $2 per item,
CR = $0.5 repaired cost for each item reworked,
CS = $0.3 disposal cost for each scrap item,
h = $0.6 per item per unit time,
h1 = $0.8 per item reworked per unit time.

For the decision on either to rework or to scrap the repairable defective items, one can use Eq. (8) and find the breakeven value of repairing cost Cγ = $2.40. In this example, ∴ CR = $0.5 < Cγ (see Figure 1); therefore “to rework” is a better choice.

\[ E[TCU(Q*)] \]

\[ E[TCU(Q_{1})] \]

\[ E[TCU(Q_{2})] \]

Figure 1

Decisions on to rework or to scrap the repairable defective items

To verify the above decision, for the case of reworking the repairable defective items, one can use equations (1) and (2) and obtain \( Q^*_1 = 3,162 \) and the optimal annual expected costs \( E[TCU(Q^*_1)] = $9,281 \). On the other hand, suppose all defective items (whether they are repairable or not) are treated as scrap items and are discarded; by using Eqs. (3) and (4), one obtains \( Q^*_2 = 3,323 \) and \( E[TCU(Q^*_2)] = $9,625 \). Obviously, \( E[TCU(Q^*_1)] < E[TCU(Q^*_2)] \), the above result is confirmed.

5. Sensitivity analysis

From the example and Eq. (8), the breakeven value of \( Cγ \), is determined by a set of parameters \( S = \{x, \theta, C, CS, K, h, h1\} \). For \( x \) ranges from 0.1 to 0.2 and \( \theta \) falls within the range \([0.1, 0.3]\); for \( CS/C \) limits within \([0.1, 0.5]\) and \( h1/h \) falls within the interval of \([1,1.5]\); and for \( K/h \) ranges from 200 to 1000, the impact of these variations on the
components \( \varepsilon \) and \( \varepsilon_x \) of \( C_{\gamma} \) are analyzed and the results are presented in Table 1 (in Appendix). One notices that \( \varepsilon \) and \( K\alpha/[1 - E[x]] \) are both negative and they are relatively small in comparison with the value of \( (C + Cs)/[1 - E[x]] \).

The behavior of the optimal expected annual cost with respect to the defective rate \( x \) is depicted in Figure 2. One notices that as \( x \) increases, \( E[T\text{CU}(Q_1)] \) and \( E[T\text{CU}(Q_2)] \) both increase, and the difference between the costs increases too.

![Figure 2](image_url)

The behavior of the optimal expected annual costs with respect to the defective rate \( x \)

The behavior of \( C_{r} \) with respect to the defective rate \( x \) and scrap rate \( \theta \) is presented in Figure 3. One notices that as \( x \) increases, the value of \( C_{r} \) increases; and as \( \theta \) decreases, \( C_{r} \) decreases slightly.

![Figure 3](image_url)

The behavior of \( C_{r} \) with respect to the defective rate \( x \) and the scrap rate \( \theta \)
5.1 Comments on the sensitivity analysis

From the analytical results of Table 1, one realizes that \( \left( \frac{\varepsilon}{Z_1} \right) \in [-8.6\%, 2.4\%] \), hence from Eq. (10), we obtain \( \beta_1 \in [0.884, 1.024] \). One notices that as \( K/h \) ratio increases, the range of \( \beta_1 \) becomes narrower; and as \( C_s/C \) decreases, the range of \( \beta_1 \) is thinner too. Since the approximation to \( \left( \frac{\varepsilon}{Z_1} \right) \) is obtained, one can use Eq. (13) to determine whether or not “to rework”.

If \( C_R < \left\{ \text{the lower bound of} \left( \frac{\beta_1 (C + C_s + K\alpha)}{1 - E(x)} \right) \right\} \) then “rework”,

else if \( C_R > \left\{ \text{the upper bound of} \left( \frac{\beta_1 (C + C_s + K\alpha)}{1 - E(x)} \right) \right\} \) then “scrap”,

else if \( C_R < C_y \) then rework, otherwise scrap. (13)

Also, from the example and analytical results of Table 1, one realizes that the range of \( \left( \frac{\varepsilon x + \varepsilon}{Z_2} \right) \in [-13.8\%, 10.9\%] \). Hence, from Eq. (12), we obtain \( \beta_2 \in [0.862, 1.109] \).

One notices that as \( C_s/C \) decreases, the range of \( \beta_2 \) becomes narrower; and as \( K/h \) ratio increases, the range of \( \beta_2 \) is thinner too. For example, if \( K/h \geq 400 \) then \( \beta_2 \in [0.903, 1.077] \). Once \( \beta_2 \) is available, one can use Eq. (14) for the rework-or-scrap decision making.

If \( C_R < \left\{ \text{the lower bound of} \left( \frac{\beta_2 (C + C_s)}{1 - E(x)} \right) \right\} \) then “rework”,

else if \( C_R > \left\{ \text{the upper bound of} \left( \frac{\beta_2 (C + C_s)}{1 - E(x)} \right) \right\} \) then “scrap”,

else, use Eq. (13) for decision making. (14)

For instance, if we take advantage of Eq. (14) to resolve the example stated in Section 4, since all parameters fall within the anticipated ranges and \( K/h \geq 400 \):

\[
C_R = \$0.5 < \left\{ \text{the lower bound of} \left( \frac{0.903 + 1.077}{1 - 0.05} \right) \right\} = \$2.19,
\]

\[
\therefore \text{decision is to rework}.
\]

6. Conclusions

This note studies effects of repair cost on the rework decision-making in an imperfect quality Economic Production Quantity (EPQ) model. With straightforward numerical derivations, this note proposes a set of mathematical equations including the exact breakeven point of repair cost
and its approximated forms, to assist in determining whether it is beneficial to rework the defective items or not. Sensitivity analysis and numerical example are provided to demonstrate their practical usages. For the future research, one interesting consideration among others will be that when shortages are allowed and backordered.

Appendix

Table 1

| Variations of parameters in $S$ effects on the components of $C_Y$, when $x = 0.2$ |
|---|---|---|---|---|---|---|
| $x$ | $\theta$ | $C_S/C$ | $h_1/h$ | $K/h$ | $\varepsilon$ | $(\varepsilon/\varepsilon_x)^\%$ | $(\varepsilon_x+X)^\%$
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>1</td>
<td>200</td>
<td>0.023</td>
<td>0.6%</td>
<td>0.023</td>
<td>1.2%</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>400</td>
<td>0.016</td>
<td>0.4%</td>
<td>0.016</td>
<td>0.9%</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>600</td>
<td>0.013</td>
<td>0.4%</td>
<td>0.013</td>
<td>0.7%</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>800</td>
<td>0.011</td>
<td>0.3%</td>
<td>0.011</td>
<td>0.6%</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1000</td>
<td>0.010</td>
<td>0.3%</td>
<td>0.010</td>
<td>0.6%</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>200</td>
<td>-0.069</td>
<td>-1.9%</td>
<td>-0.069</td>
<td>-3.8%</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>400</td>
<td>-0.049</td>
<td>-1.3%</td>
<td>-0.049</td>
<td>-2.7%</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>600</td>
<td>-0.040</td>
<td>-1.1%</td>
<td>-0.040</td>
<td>-2.2%</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>800</td>
<td>-0.034</td>
<td>-0.9%</td>
<td>-0.034</td>
<td>-1.9%</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1000</td>
<td>-0.031</td>
<td>-0.8%</td>
<td>-0.031</td>
<td>-1.7%</td>
<td></td>
</tr>
<tr>
<td>0.3</td>
<td>0.1</td>
<td>200</td>
<td>0.023</td>
<td>1.5%</td>
<td>0.023</td>
<td>3.1%</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>400</td>
<td>0.016</td>
<td>1.1%</td>
<td>0.016</td>
<td>2.2%</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>600</td>
<td>0.013</td>
<td>0.9%</td>
<td>0.013</td>
<td>1.8%</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>800</td>
<td>0.011</td>
<td>0.8%</td>
<td>0.011</td>
<td>1.6%</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1000</td>
<td>0.010</td>
<td>0.7%</td>
<td>0.010</td>
<td>1.4%</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>200</td>
<td>-0.069</td>
<td>-5.0%</td>
<td>-0.069</td>
<td>-9.5%</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>400</td>
<td>-0.049</td>
<td>-3.5%</td>
<td>-0.049</td>
<td>-6.7%</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>600</td>
<td>-0.040</td>
<td>-2.8%</td>
<td>-0.040</td>
<td>-5.5%</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>800</td>
<td>-0.034</td>
<td>-2.4%</td>
<td>-0.034</td>
<td>-4.8%</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1000</td>
<td>-0.031</td>
<td>-2.2%</td>
<td>-0.031</td>
<td>-4.3%</td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>0.1</td>
<td>200</td>
<td>0.023</td>
<td>2.2%</td>
<td>0.023</td>
<td>4.5%</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>400</td>
<td>0.016</td>
<td>1.6%</td>
<td>0.016</td>
<td>3.2%</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>600</td>
<td>0.013</td>
<td>1.3%</td>
<td>0.013</td>
<td>2.6%</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>800</td>
<td>0.011</td>
<td>1.1%</td>
<td>0.011</td>
<td>2.3%</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1000</td>
<td>0.010</td>
<td>1.0%</td>
<td>0.010</td>
<td>2.0%</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>200</td>
<td>-0.069</td>
<td>-7.4%</td>
<td>-0.069</td>
<td>-13.8%</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>400</td>
<td>-0.049</td>
<td>-5.1%</td>
<td>-0.049</td>
<td>-9.7%</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>600</td>
<td>-0.040</td>
<td>-4.1%</td>
<td>-0.040</td>
<td>-7.9%</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>800</td>
<td>-0.034</td>
<td>-3.6%</td>
<td>-0.034</td>
<td>-6.9%</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1000</td>
<td>-0.031</td>
<td>-3.2%</td>
<td>-0.031</td>
<td>-6.2%</td>
<td></td>
</tr>
</tbody>
</table>

(Contd. Table 1)
## ECONOMIC PRODUCTION QUANTITY MODEL

<table>
<thead>
<tr>
<th>$x$</th>
<th>$C_S/C$</th>
<th>$h_1/h$</th>
<th>$K/h$</th>
<th>$\epsilon$</th>
<th>$\left[\frac{\epsilon}{\epsilon_{X}}\right]_{%}$</th>
<th>$\epsilon_{X}$</th>
<th>$\left[\frac{\epsilon_{X}}{\epsilon_{Z}}\right]_{%}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>1</td>
<td>200</td>
<td>0.026</td>
<td>0.7%</td>
<td>0.082</td>
<td>3.0%</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>400</td>
<td>0.019</td>
<td>0.5%</td>
<td>0.058</td>
<td>2.1%</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>600</td>
<td>0.015</td>
<td>0.4%</td>
<td>0.048</td>
<td>1.7%</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>800</td>
<td>0.013</td>
<td>0.4%</td>
<td>0.041</td>
<td>1.5%</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1000</td>
<td>0.012</td>
<td>0.3%</td>
<td>0.037</td>
<td>1.3%</td>
<td></td>
</tr>
<tr>
<td>1.5</td>
<td>1</td>
<td>200</td>
<td>-0.088</td>
<td>-2.4%</td>
<td>0.026</td>
<td>-1.7%</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>400</td>
<td>-0.062</td>
<td>-1.7%</td>
<td>0.018</td>
<td>-1.2%</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>600</td>
<td>-0.051</td>
<td>-1.4%</td>
<td>0.015</td>
<td>-1.0%</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>800</td>
<td>-0.044</td>
<td>-1.2%</td>
<td>0.013</td>
<td>-0.8%</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1000</td>
<td>-0.039</td>
<td>-1.1%</td>
<td>0.011</td>
<td>-0.8%</td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>0.3</td>
<td>1</td>
<td>200</td>
<td>0.026</td>
<td>1.7%</td>
<td>0.082</td>
<td>7.5%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>400</td>
<td>0.019</td>
<td>1.2%</td>
<td>0.058</td>
<td>5.3%</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>600</td>
<td>0.015</td>
<td>1.0%</td>
<td>0.048</td>
<td>4.3%</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>800</td>
<td>0.013</td>
<td>0.9%</td>
<td>0.041</td>
<td>3.8%</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1000</td>
<td>0.012</td>
<td>0.8%</td>
<td>0.037</td>
<td>3.4%</td>
<td></td>
</tr>
<tr>
<td>1.5</td>
<td>1</td>
<td>200</td>
<td>-0.088</td>
<td>-6.0%</td>
<td>0.026</td>
<td>-4.3%</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>400</td>
<td>-0.062</td>
<td>-4.2%</td>
<td>0.018</td>
<td>-3.0%</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>600</td>
<td>-0.051</td>
<td>-3.5%</td>
<td>0.015</td>
<td>-2.5%</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>800</td>
<td>-0.044</td>
<td>-3.0%</td>
<td>0.013</td>
<td>-2.2%</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1000</td>
<td>-0.039</td>
<td>-2.7%</td>
<td>0.011</td>
<td>-1.9%</td>
<td></td>
</tr>
<tr>
<td>0.3</td>
<td>0.3</td>
<td>1</td>
<td>200</td>
<td>0.026</td>
<td>2.4%</td>
<td>0.082</td>
<td>10.9%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>400</td>
<td>0.019</td>
<td>1.8%</td>
<td>0.058</td>
<td>7.7%</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>600</td>
<td>0.015</td>
<td>1.5%</td>
<td>0.048</td>
<td>6.3%</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>800</td>
<td>0.013</td>
<td>1.3%</td>
<td>0.041</td>
<td>5.4%</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1000</td>
<td>0.012</td>
<td>1.1%</td>
<td>0.037</td>
<td>4.9%</td>
<td></td>
</tr>
<tr>
<td>1.5</td>
<td>1</td>
<td>200</td>
<td>-0.088</td>
<td>-8.6%</td>
<td>0.026</td>
<td>-6.2%</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>400</td>
<td>-0.062</td>
<td>-6.1%</td>
<td>0.018</td>
<td>-4.4%</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>600</td>
<td>-0.051</td>
<td>-5.0%</td>
<td>0.015</td>
<td>-3.6%</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>800</td>
<td>-0.044</td>
<td>-4.3%</td>
<td>0.013</td>
<td>-3.1%</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1000</td>
<td>-0.039</td>
<td>-3.9%</td>
<td>0.011</td>
<td>-2.8%</td>
<td></td>
</tr>
</tbody>
</table>

### References


*Received January, 2006*