Data envelopment analysis: an application in the transport sector

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Abstract
Passenger transportation is an important part of the overall development problem of the nation and affects in some way nearly all aspects of mobility in general. This paper deals with the problems caused due to a decline in the performance of the State Transport Undertakings with a special reference to Delhi Transport Corporation. A Linear Programming based technique called as Data Envelopment Analysis (DEA) is used to measure the efficiencies of various decision-making units. The analysis on this study is sought to provide a way to obtain a valid efficiency measure for each State Transport Undertaking.

Keywords: Integer programming problems, data envelopment analysis, transportation problem.

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1. Introduction

Our nation’s transportation system has a pervasive impact on each of us. The trade-off between the commercial objectives and social responsibility goals of State Transport Undertakings is an issue of serious concern. This paper deals with the problems caused due to a decline in the performance of the State Transport Undertakings with a special reference to Delhi Transport Corporation. The study in this paper has been discussed at three different phases. In the first phase, the performance of Delhi Transport Corporation has been compared with the State Transport Undertakings of other states of India. In the second phase, the performances of State Transport Undertakings of various metropolitan cities have been compared. Lastly, transport as a sector of Delhi Government has been compared with various other sectors with regard to the budgetary allocations and expenditures.

2. Mathematical formulation of the problem

Charnes, Cooper and Rhodes [5] proposed a non-parametric technique called Data Envelopment Analysis (DEA) in the year 1978. This technique measures the efficiencies of various decision-making units with multiple inputs and multiple outputs at constant returns to scale. This technique has been used in all the three phases to compare the efficiencies.

The technique of DEA is being used for measuring efficiencies of schools [2], hospital performance [10], research institutes, banking and insurance, ferry transport, restaurants etc. However, in the transport sectors, it has been used to compare the efficiencies of highways, combined transport terminals [4], Railroads [9], and motor carriers [1]. DEA is a suitable method for analysing efficiency and productivity in multiple input-output productions. The usual measure of efficiency i.e.

\[
\text{Efficiency} = \frac{\text{Output}}{\text{Input}}
\]

is often inadequate due to the existence of multiple inputs and outputs related to different resources, activities and environmental factors. Moreover different patterns of activity levels are supported by different amounts of resources making efficiency comparisons difficult. Data Envelopment Analysis (DEA) allows relative efficiency measures to be determined.
A common measure of relative efficiency is

\[
\text{Efficiency} = \frac{\text{Weighted sum of outputs}}{\text{Weighted sum of inputs}}.
\]

The efficiency derived for each unit is on a scale of 0-1, where a ‘0’ denotes an extremely inefficient unit and a score of ‘1’ represents an efficient unit. Efficiency scores are relative and are derived by comparison between the units in the data set being analysed. According to Charnes, Cooper and Rhodes, “100% efficiency is attained for a unit, only when:

(a) None of its outputs can be increased without either
   (i) increasing one or more of its inputs, or
   (ii) decreasing some of its other outputs.

(b) None of its inputs can be decreased without either
   (i) decreasing some of its outputs, or
   (ii) increasing some of its other inputs.”

Mathematically, using the appropriate notation, the efficiency of a unit, say “\(j\)”, using ‘\(p\)’ inputs in order to produce ‘\(s\)’ outputs is:

\[
\text{Efficiency of unit } j = \frac{u_1 y_{1j} + u_2 y_{2j} + \cdots + u_s y_{sj}}{v_1 x_{1j} + v_2 x_{2j} + \cdots + v_p x_{pj}}.
\]

Where

\[
\begin{align*}
u_r &= \text{the weight given to output } \text{‘}r\text{’} \\
y_{rj} &= \text{amount of output } \text{‘}r\text{’} \text{ from unit } \text{‘}j\text{’} \\
v_i &= \text{weight given to input } i \\
x_{ij} &= \text{amount of input } i \text{ to unit } j
\end{align*}
\]

where \(r = \{1, 2, \ldots, s\}\) and \(i = \{1, 2, \ldots, p\}\).

The initial assumption is that this measure of efficiency requires a common set of weights to be applied across all units. A single common set of weights is unsatisfactory. Each unit is allowed to adopt a set of weights that shows it in the most favourable light in comparison to the other units. Thus, efficiency of a target unit \(j_0\) can be obtained as a solution to the problem

Maximise the efficiency of unit \(j_0\)

Subject to the efficiency of all units being \(\leq 1\).
The variables of the above problem are the weights and the solution produces the weights most favourable to unit \( j_0 \) along with a measure of efficiency.

Let \( h_0 \) denote the efficiency of the target unit \( j_0 \),

\[
 j = J = \{1, 2, \ldots, n\} \quad \text{be the set of} \ n \ \text{units under study,}
\]

\[
r = R = \{1, 2, \ldots, s\} \quad \text{be the set of outputs, and}
\]

\[
i = I = \{1, 2, \ldots, p\} \quad \text{be the set of inputs.}
\]

The model of the problem thus becomes

\[
\text{(M1)} \quad \text{Max} \quad h_0 = \frac{\sum_{k} u_{rk} y_{rj_0}}{\sum_{l} v_{lj} x_{lj_0}}
\]

subject to

\[
\sum_{k} u_{rk} y_{rj} - \sum_{l} v_{lj} x_{lj} \leq 1 \quad \text{for each unit} \ j
\]

\[
u_{rk}, \ v_{lj} > \varepsilon.
\]

The \( u \)'s and \( v \)'s are variables of the problem and are constrained to be greater than or equal to small positive quantity \( \varepsilon \) in order to avoid any input or output being totally ignored in determining the efficiency. The model (M1) is a fractional linear program. To solve this model it is first necessary to convert it into linear form so that the methods of linear programming can be applied. The linear version of the constraints is given in model (M2).

\[
\text{(M2)} \quad \text{Max} \quad h_0 = \sum_{k} u_{rk} y_{rj_0}
\]

subject to

\[
\sum_{l} v_{lj} x_{lj_0} = 100 \quad \text{(say)}
\]

\[
\sum_{k} u_{rk} y_{rj} - \sum_{l} v_{lj} x_{lj} \leq 1, \ j \in J
\]

\[
u_{rk}, \ v_{lj} \geq \varepsilon > 0.
\]

The solution to the above problem gives a value of maximum \( h_0 \), that is the efficiency of the target unit \( j_0 \) along with the weights leading to that efficiency. The process of evaluation is a relative evaluation among the units under study. If \( h_0 = 1 \), then the unit is efficient relative to others. If \( h_0 < 1 \), then some other unit/units are more efficient than the target unit, even when the weights are chosen to maximise its efficiency. Thus, a unit may achieve an efficiency score of ‘1’ without being 100% efficient.
The solution to this linear program also provides a set of target inputs and outputs for the inefficient units. For each inefficient target unit, there would be units that would attain an efficiency score of ‘1’ with the same set of inputs and outputs as the target unit. These units are called as the peer units and their values of the inputs and outputs serve as the target values for the inefficient units. These target values of the inputs and the outputs form the basis for the potential improvement of the inefficient unit.

The potential improvements for the inefficient unit are:

\[
\text{Target value} - \frac{\text{Actual value}}{\text{Actual value}} \times 100.
\]

Thus, the goals of the study could be summarized as a solution to the problems of Public Transport Undertakings in providing management with information such as:

- An efficiency evaluation of individual Transport Undertakings.
- Targeted output goals and identification of needed input modifications.
- Identification of areas in which efficiency could be increased.

3. Data analysis (Phase I)

In Phase I of the analysis, the performance of State Transport Undertakings of six states of India namely, Uttar Pradesh, Karnataka, Delhi, Haryana, Maharashtra and Madhya Pradesh have been analysed. The analysis of the performance is based on both physical and financial parameters in order to facilitate national level comparison. As per the data available [7, 8] the various inputs and the outputs selected included the following:

<table>
<thead>
<tr>
<th>Inputs</th>
<th>Outputs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operating cost (I)</td>
<td>Road safety (O)</td>
</tr>
<tr>
<td>Taxes and interest (I)</td>
<td>Total revenue (O)</td>
</tr>
<tr>
<td>Average number of buses held (I)</td>
<td>Bus utilization (O)</td>
</tr>
<tr>
<td>Staff position (I)</td>
<td>Average number of buses on road (O)</td>
</tr>
</tbody>
</table>
Results. For the inputs and the outputs described above, the relative efficiencies were calculated. Of these six states, two were found to be inefficient. The efficiencies of these states is given in Table 1.

<table>
<thead>
<tr>
<th>States</th>
<th>Relative efficiencies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uttar Pradesh</td>
<td>1.00</td>
</tr>
<tr>
<td>Karnataka</td>
<td>1.00</td>
</tr>
<tr>
<td>Delhi</td>
<td>1.00</td>
</tr>
<tr>
<td>Haryana</td>
<td>1.00</td>
</tr>
<tr>
<td>Maharashtra</td>
<td>0.99</td>
</tr>
<tr>
<td>Madhya Pradesh</td>
<td>0.90</td>
</tr>
</tbody>
</table>

It can be observed from Table 1 that Delhi records an efficiency score of 1.00. The data for Delhi here include the data for the bus run by Delhi Transport Corporation as well as the buses run by the private operators. This means that the efficiency of DTC (Government owned) alone cannot be judged by this data. There are two inefficient units namely, Maharashtra and Madhya Pradesh. For both of these inefficient units, Delhi does not happen to be the peer unit. This shows that although Delhi has been projected as one of the efficient units but it was definitely not the unit to achieve its 100% efficiency at the first stage. The inputs and the outputs need to be improved further so that a better level of efficiency can be achieved. However, it can be concluded from the above analysis that as compared to the six mentioned states the transport undertaking of Delhi is one of the efficient units.

5. Data analysis (Phase II)

In Phase II of the analysis, the performance of transport undertakings of five metropolitan cities namely, Delhi (DTC), Mumbai (BEST), Bangalore, Kolkata and Chandigarh were analyzed. The inputs and the outputs defined for the purpose were same as in the Phase I of the analysis. For the city of Delhi, the data was collected for the Delhi Transport Corporation (State Transport) that includes the data of city buses and inter-state buses run, by the government and for Delhi Transport Corporation (Privately Owned Vehicles) that includes the data of city buses that are run by private owners.
**Results.** The relative efficiencies were calculated for the Transport Corporations of the five cities. Delhi (ST) had an efficiency of 0.81 units making it an inefficient unit. The efficiencies of all other cities are given in Table 2.

<table>
<thead>
<tr>
<th>States</th>
<th>Relative efficiencies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chandigarh</td>
<td>1.00</td>
</tr>
<tr>
<td>Delhi (POV)</td>
<td>1.00</td>
</tr>
<tr>
<td>BEST</td>
<td>1.00</td>
</tr>
<tr>
<td>Bangalore</td>
<td>1.00</td>
</tr>
<tr>
<td>Kolkata</td>
<td>1.00</td>
</tr>
<tr>
<td>Delhi (ST)</td>
<td>0.81</td>
</tr>
</tbody>
</table>

Delhi (ST) being an inefficient unit has the peer efficient units namely, BEST, Bangalore and Chandigarh. The target values and the potential improvements for Delhi (ST), are given in Table 3.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Actual values</th>
<th>Target values</th>
<th>Potential improvement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operating cost (I)</td>
<td>9706.35</td>
<td>5137.06</td>
<td>−47.08</td>
</tr>
<tr>
<td>Taxes &amp; interest (I)</td>
<td>573.01</td>
<td>283.16</td>
<td>−50.58</td>
</tr>
<tr>
<td>Average number of buses held (I)</td>
<td>2088</td>
<td>1689.71</td>
<td>−19.08</td>
</tr>
<tr>
<td>Administration &amp; actts. staff (I)</td>
<td>3491</td>
<td>861.54</td>
<td>−75.32</td>
</tr>
<tr>
<td>Other staff (I)</td>
<td>19374</td>
<td>9054.71</td>
<td>−53.26</td>
</tr>
<tr>
<td>Accidents (O)</td>
<td>80</td>
<td>80</td>
<td>0</td>
</tr>
<tr>
<td>Average number of buses on road (O)</td>
<td>1610</td>
<td>1610</td>
<td>0</td>
</tr>
<tr>
<td>Total revenue (O)</td>
<td>5664.99</td>
<td>5664.99</td>
<td>0</td>
</tr>
<tr>
<td>Passengers carried (O)</td>
<td>1517.29</td>
<td>1522.25</td>
<td>0.33</td>
</tr>
</tbody>
</table>
Table 3 clearly indicates that Delhi (ST) should drastically decrease its inputs in order to achieve an efficient level of outputs.

- The operating cost should go down by almost 47% of the current levels. Also, the taxes and interest paid by Delhi (ST) should further go down by almost 50%.
- The maximum reduction is required in Administration and Accounts staff, almost 75%. This means that Delhi (ST) is unnecessarily paying the salaries and other benefits to the employees who are not even needed by the unit. Other staff should also decrease by almost 53%.
- There should be a decrease of 19% in the average number of buses held by Delhi (ST). This means that it should put this much more buses on the roads instead of keeping them in the depots. Delhi (ST) has almost 32.2% of overage buses. The unit should replace these buses.
- An increase of 0.33% is required in the number of passengers carried by Delhi (ST). If this is achieved other outputs would automatically increase.

6. Data analysis (Phase III)

Delhi has strong and vibrant economy. The economy has been divided into 31 different sectors. Out of these, sectors like Energy, Transport, Water supply, Medical & public health, Education & urban department are considered as core sectors because they not only provide infrastructure support but also help in covering all socio-economic development of Delhi. In Phase III of the analysis, the Transport Department of Delhi Government is considered as a sector and the efficiencies are calculated for the various sectors with regard to the budgetary allocations and the expenditures by these sectors. Data was collected for the twelve sectors namely, Energy, Industries, Transport, Science technology & environment, General economic services, General education, Technical education, Medical, Housing, Urban development, Public health and rural development. An outlay for the 9th five year plan and for the year 2001-02 have been considered as the two inputs and the expenditures for the 9th five year plan and for the year 2001-02 have been taken as the two outputs. It was observed that six of them turned out to be efficient and six were inefficient with the Transport Department showing an efficiency score of 0.88. The efficiency scores of all the sectors are given in Table 4.
Table 4
Relative efficiencies

<table>
<thead>
<tr>
<th>Sectors</th>
<th>Relative efficiencies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy</td>
<td>1.00</td>
</tr>
<tr>
<td>Industries</td>
<td>1.00</td>
</tr>
<tr>
<td>General education</td>
<td>1.00</td>
</tr>
<tr>
<td>Technical education</td>
<td>1.00</td>
</tr>
<tr>
<td>Housing</td>
<td>1.00</td>
</tr>
<tr>
<td>Urban development</td>
<td>1.00</td>
</tr>
<tr>
<td>General Economic Services</td>
<td>0.98</td>
</tr>
<tr>
<td>Medical</td>
<td>0.95</td>
</tr>
<tr>
<td>Public health</td>
<td>0.94</td>
</tr>
<tr>
<td>Rural development</td>
<td>0.92</td>
</tr>
<tr>
<td>Transport</td>
<td>0.88</td>
</tr>
<tr>
<td>Science &amp; technology</td>
<td>0.78</td>
</tr>
</tbody>
</table>

It was observed that the peer units for the Transport sector were Energy, Urban development and housing. The inputs for the Transport sector are high while the outputs are low as compared to the peer units making the Transport sector an inefficient unit.

The potential improvements for the Transport sector are given in Table 5.

Table 5
Potential improvements for the transport sector

<table>
<thead>
<tr>
<th>Variables</th>
<th>Actual values</th>
<th>Target values</th>
<th>Potential improvement</th>
</tr>
</thead>
<tbody>
<tr>
<td>9th five year plan (I)</td>
<td>315840.00</td>
<td>278053.61</td>
<td>−11.96</td>
</tr>
<tr>
<td>Plan outlay 2001-02 (I)</td>
<td>63660.00</td>
<td>56043.86</td>
<td>−11.96</td>
</tr>
<tr>
<td>9th five year plan expense (O)</td>
<td>234470.37</td>
<td>234470.37</td>
<td>0</td>
</tr>
<tr>
<td>Expenses 2001-02 (O)</td>
<td>55258.85</td>
<td>55258.85</td>
<td>0</td>
</tr>
</tbody>
</table>

The Table 5 clearly indicates that either the inputs for the Transport sector should be reduced or the outputs should be increased by almost 12% in order to achieve a level of efficiency.
7. Conclusions and future prospects of the study

The study in this paper has been made in three different phases. The following observations and conclusions can be drawn from the analysis.

- The public transport system of Delhi as compared to other states has turned out to be efficient but definitely it is not the best.

- Delhi, when compared with other metropolitan cities turned out to be an inefficient unit with an efficiency score of 0.81. It means that the inputs need to be decreased and outputs need to be increased in order to achieve a level of efficiency. If the inputs are further reduced, then the performance is again going to be affected. Instead of reducing the inputs drastically, the DTC (ST) should put more buses on the roads and thus increase its efficiency. More buses on the roads would automatically increase the number of passengers carried and the revenue earned. As a consequence to it, taxes and interest would come down.

- The results of the analysis of the DEA technique for the Transport Sector of Delhi Government projected the fact that if the budgetary allocations to this department are decreased by 12% or the output is increased by 12% then the optimal level of efficiency by the department can be attained. Reducing the budgetary allocations of this department would be directly reflected in the facilities provided to the commuters. If the department can increase the output then a better scenario can be observed on the roads of Delhi. The Transport Department should come up with more projects for the benefit of the consumers in such a way that they are able to spend the amount allocated to them. Proper planning should be done in the beginning of the financial year and all efforts should be made to complete the projects in the targeted or the stipulated time period. More money can be spent on the purchase and maintenance of the buses so that the performance of both the Transport sector and Delhi (ST) can be improved.

The analysis projected in this study is just a small step in improving the efficiency of DTC. The management has to use this information and initiate actions to provide better services to commuters and reduce the use of personalised vehicles on the roads of Delhi.
References


